Neural Mass model, Jansen and Rit 1995

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July 12, 2016

Introducctión

The Jansen-Rit model is a neural population model of a local cortical circuit. The single area model contains three interconnected neural populations: one for the pyramidal neurons and two for excitatory and inhibitory interneurons forming feedback loops (single area). A multiple area models includes interactions between multiples neural masses.

Model

- First each Neural mass has an average membrane potential (state variable)
- This average membrane potential is the results of different imputs
- In Neural mass model these imputs represents average pulse density or firing rate, we have that average membrane potential and average pulse densite are the two main quantities of this model.
- these two quantities are converted via two transformation, the first is a called Post Synaptic Potential function (PSP) and the second is a Potential-to-rate function.

Model

- That means, each imput to a Neural mass is converted from an average pulse density via PSP function to an average potential membrane, each of those potential is multiplied by some constants modeling the average number of synapses to the population.

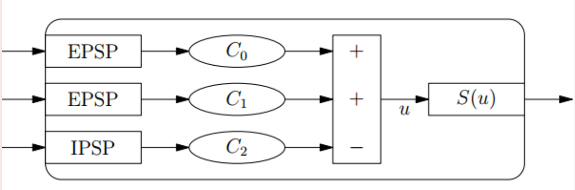


Figure: scheme of the transformations

Model

- Based in the model of Jansen and Rit the PSP function is a second diferential operator, wich is given by:

$$\begin{cases} \frac{\partial x}{\partial t} = y \\ \frac{\partial y}{\partial t} = Qqz - 2qy - q^2 x \end{cases}$$

  Here $z(t)$ is the imput to the PSP (firing rate) ,$x(t)$ is the output and the constants depend of the exitatory or inhibitory case

- Jansen used also the next potential-to-rate function:

$$S(v) = \frac{2e_0}{(1 + \exp^{r(v_0 - v)})}$$

Model

In this work is we are considering the Neural mass like three blocks: pyramidal neurons, local excitatory and local inhibitory neurons. The scheme is showed in the next figure:

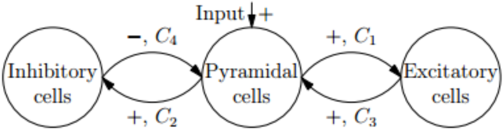


Figure: scheme of the single Neural mass model of Jansen

We have the following O.D.E

$$
\begin{cases}
\frac{\partial^2 y_0(t)}{\partial t^2} = AaS(C_2 y_1(t) - C_4 y_2(t)) - 2a\frac{\partial y_0(t)}{\partial t} \\
\qquad\qquad -a^2 y_0(t) \\
\frac{\partial^2 y_1(t)}{\partial t^2} = Aa(p/C_2 + S(C_1 y_0)) - 2a\frac{\partial y_1(t)}{\partial t} \\
\qquad\qquad -a^2 y_1(t) \\
\frac{\partial^2 y_2(t)}{\partial t^2} = BbS(C_3 y_0)) - 2b\frac{\partial y_2(t)}{\partial t} - b^2 y_2(t)
\end{cases}
$$

$A = 3.25$, $B = 22$, $C1 = 135$, $C2 = 0.8C1$, $C_3 = C_4 = 0.25C_1$, $e_0 = 2.5$, $v_0 = 6$, $r = 0.56$ y p entre 120 y 300. y_0, y_1 and y_2 represening the firing rate of pyramidal, excitatory and inhibitory neurones.

The Fourier Transform

Lemma 1: Let f a periodic function, that is, $\exists T > 0 : f(t+T) = f(t) \forall t \in \mathcal{R}$ then can be written as Fourier's Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos(\omega_n x) + b_n sen(\omega_n x)$$

Con $\omega_n = \frac{n2\pi}{T}$, a_0, a_n and $b_n \in \mathcal{R}$. Note that $\omega_1 = \frac{2\pi}{T}$
Definición 1: given a function $f \in L^1(\mathcal{R})$, We define the Fourier's Transform of f as the following aplication:

$$\Im\{f\} : \xi \to \hat{f}(\xi) = \int_{\mathcal{R}} f(x) e^{-2\pi i \xi x} dx$$

.

Lemma 2: Let f a periodic function , then:

$$\arg\max_{\mathcal{R}^+} \hat{f}(\xi) = \omega$$

That's why we use the Fourier Transform, to get the frequency of the cylcles.

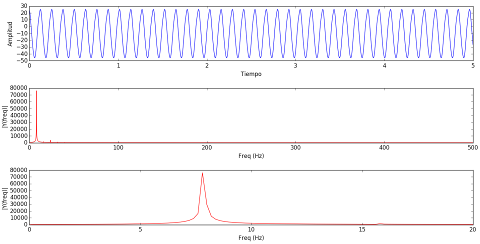
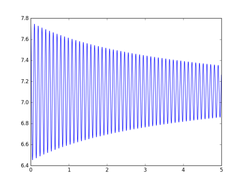


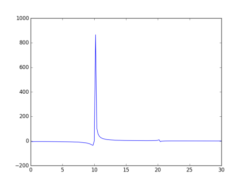
FIGURE: F. Transform of a periodic function

Some problems

- not periodic function



- F.T.



Solutions

- I used some literature values of parameters to get periodic oscillations
- Sometimes I used Matcont to find Hopf Bifurcations

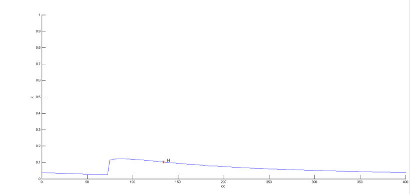


Figure: Hopf bifurcation of equilibrium point

I moved parameters C, p usually.

Some frequency Maps

With the correct parameters values i got the following Frequency maps:

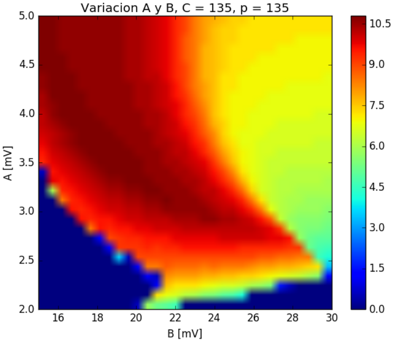


Figure: Frequency maps

The main idea of all this is to find zones where the frequency change, for example.

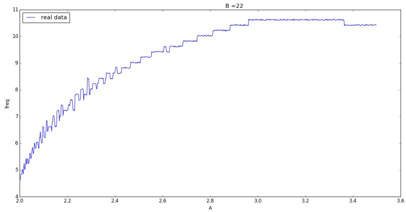


FIGURE: Moving A

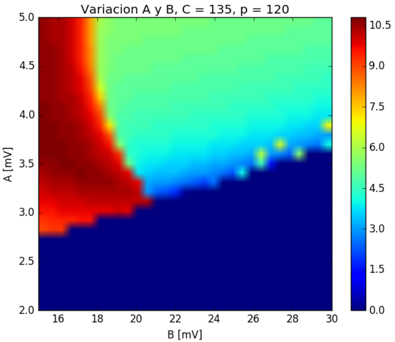


FIGURE: Another Frequency map, changing the p value

We got a decreasing relation to the frequency

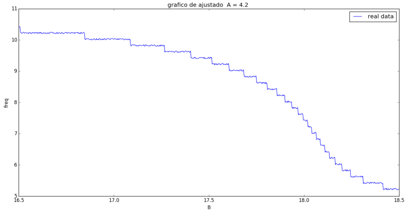


Figure: In this case we are moving B

Coupled system

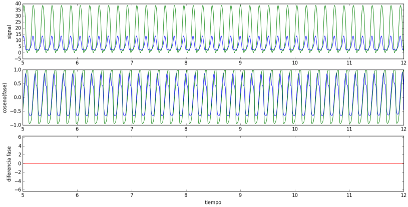


Figure: Phased signales