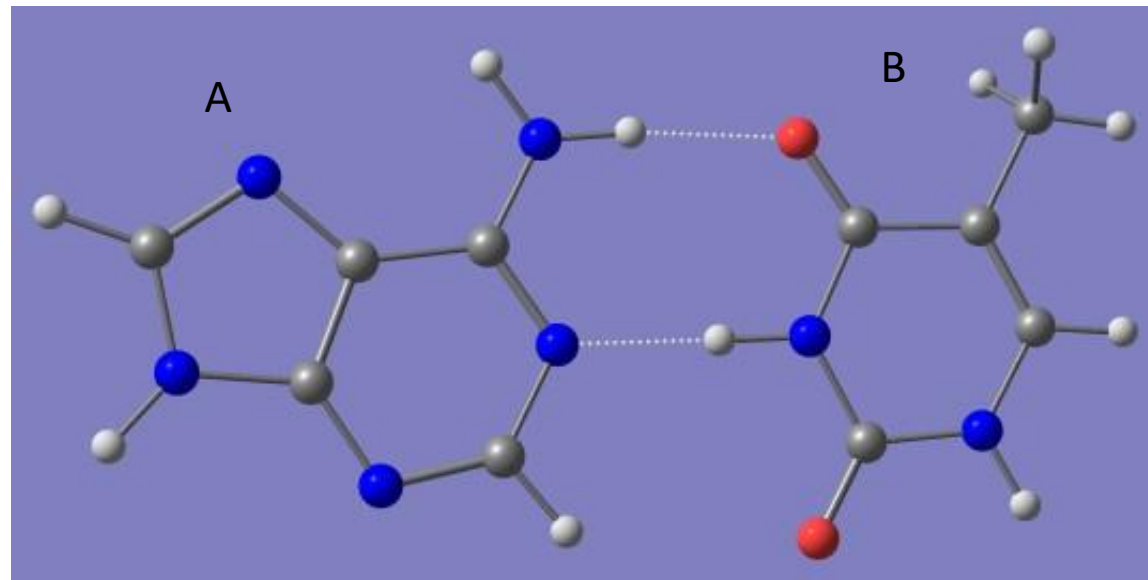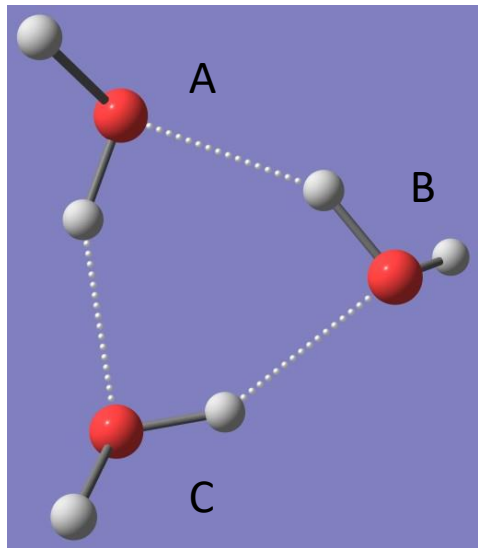# Interaction energies in non-covalent complexes, various decomposition schemes

Prepared by Radek Marek group
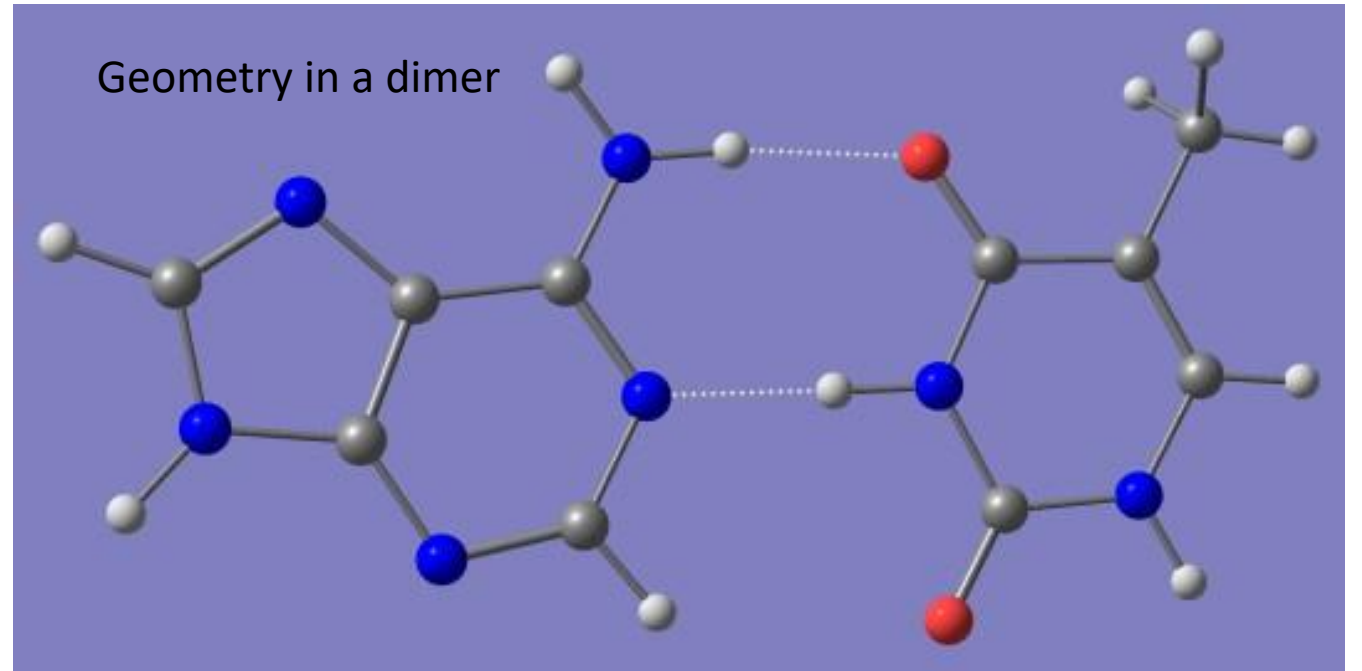
Interaction energy

$$\Delta E^{ABC} = E^{ABC} - [E^A + E^B + E^C]$$

$$\Delta E^{AB} = (E^{AB}) - (E^A + E^B).$$

$$\Delta E^{ABC} = \Delta E^{AB} + \Delta E^{AC} + \Delta E^{BC} + \Delta E^3 = E^{AB} - [E^A + E^B] + E^{AC} - [E^A + E^C] + E^{BC} - [E^B + E^C] + \Delta E^3$$

$$\Delta E^3 = E^{ABC} - E^{AB} - E^{AC} - E^{BC} + E^A + E^B + E^C$$

Deformation energy

Monomer geometry

Geometry in a dimer
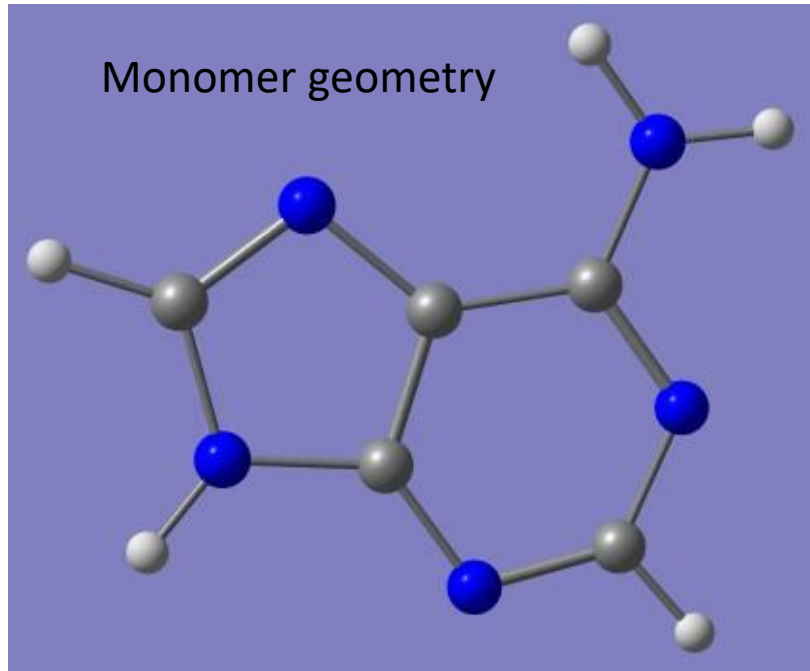
$$\Delta E^{DEF} \doteq (E^A - E^{Ai}) + (E^B - E^{Bi})$$

$$\Delta E^{T(AB)} = \Delta E^{AB} + \Delta E^{DEF}$$

Deformation (preparation) energy

Binding (total) energy

Basis sets

- Set of functions representing atomic orbitals

- Molecular orbital = linear combination of atomic orbitals (MO-LCAO)

- Minimal basis set (one function per orbital)

- Gaussian-type and Slater-type functions

- 6-311++G(d,p)

- LANL2DZ

- aug-cc-pVTZ

- Diffuse and polarization functions

An s orbital is polarized by using a p-type orbital
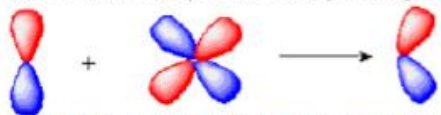
A p orbital is polarized by mixing in a d-type orbital

Basis Set	Bond Length (Å)
6-31G(d)	0.93497
6-31G(d,p)	0.92099
6-31+G(d,p)	0.94208
6-31++G(d,p)	0.92643
6-311G(d,p)	0.91312
6-311++G(d,p)	0.91720
6-311G(3df,3pd)	0.91369
6-311++G(3df,3pd)	0.91739

bond length of HF
- experimental: 0.917 A
- calculated at MP4 level

Formaldehyde in 3-21 basis set

atom	primitives	contracted
H1	3s	1s
H2	3s	1s
C	6s,3p	3s,2p
O	6s,3p	3s,2p
Total	36	22

Number of contracted gaussians in the function describing a core orbital

Number of contracting gaussians forming the second valence function

3-21G

Number of contracting gaussians forming the first valence function

$$\varphi_i = \sum_{\mu=1}^{36} g_\mu c_{\mu i} \quad ; i=1,36$$

Pople and Dunning basis sets

3-21G(d,p)

atom	primitives	contracted
H1	3s,1p	2s,1p
H2	3s,1p	2s,1p
C	6s,3p,1d	3s,2p,1d
O	6s,3p,1d	3s,2p,1d
total	54	40

Dunning

$N = D, T, Q, 5, 6$

cc-pVNZ

polarized

correlation consistent

$$g(\alpha, \vec{r}) = cx^n y^m z^l e^{-\alpha r^2}$$ Gaussian-type

$$s(\zeta, \vec{r}) = cx^n y^m z^l e^{-\zeta r}$$ Slater-type

Anatomy of the Basis Set

```
C    0
S    6   1.00
          3047.52490              0.183470000E-02
          457.369510              0.140373000E-01
          103.948690              0.688426000E-01
          29.2101550              0.232184400
          9.28666300              0.467941300
          3.16392700              0.362312000
SP   3   1.00
          7.86827240             -0.119332400           0.689991000E-01
          1.88128850             -0.160854200           0.316424000
         0.544249300              1.14345640            0.744308300
SP   1   1.00
         0.168714400              1.00000000            1.00000000
```

https://www.basissetexchange.org/ - Basis set exchange library

Basis set vs. Accuracy

	HF	MP2	QCISD	QCISD(T)	... FCI
6-31G(d)					
6-31G(d,p)					
⋮					
6-311+G(2df,p)					
⋮					
∞					Exact

Increasing electron correlation →

↓ Increasing basis set size

Basis set superposition error (BSSE)

Subsystem

Subsystem

A

B

Super system

A

B

See larger basis set rather than isolated subsystems

Absolute value of interaction energy goes up.

Counterpoise (CP) method

Ghost orbitals for monomers
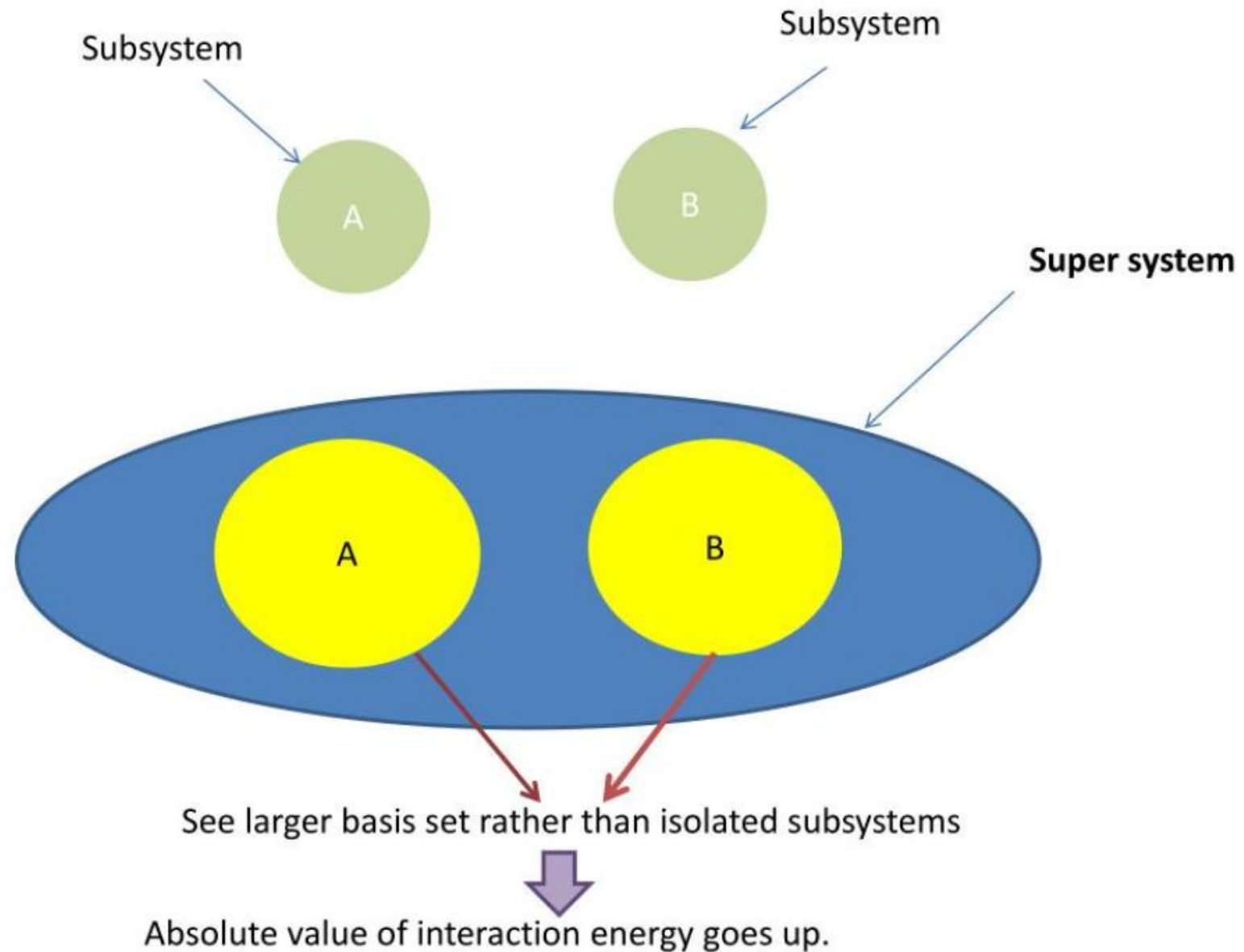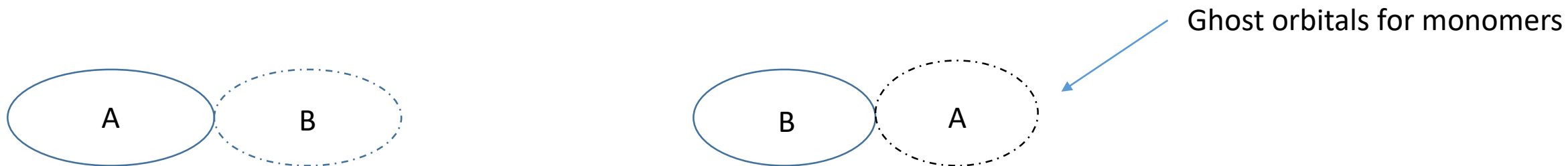
The uncorrected interaction energy is

$$E_{\text{interaction}}^{AB} = E_{AB}^{\text{Opt}} - E_{A}^{\text{Opt}} - E_{B}^{\text{Opt}}$$

The counterpoise interaction energy is

$$E_{\text{interaction}}^{AB} = E_{AB}^{\text{Opt}} - E_{A}^{\text{Opt}} - E_{B}^{\text{Opt}} + \delta_{AB}^{\text{BSSE}}.$$

The basis set for the subsystems contain the basis function of the whole molecule

Practice1: counterpoise correction in Gaussian

```
/home/yevg_yur/Water_dimer_MP2/w2.com - yevg_yur@fundy.ncbr.muni.cz - Editor - WinS(

Encoding ▾   □ Color ▾

%Mem=16384MB
%nprocshared = 7
# rmp2/aug-cc-pVDZ counterpoise=2

Water dimer CP

0,1 0,1 0,1
O(Fragment=1)            -1.62893        -0.04138         0.37137
H(Fragment=1)            -0.69803        -0.09168         0.09337
H(Fragment =1)           -2.06663        -0.73498        -0.13663
O(Fragment =2)            1.21457         0.03172        -0.27623
H(Fragment =2)            1.44927         0.91672        -0.58573
H(Fragment =2)            1.72977        -0.08038         0.53387
```

Water dimer from http://www.begdb.org/ web page, section
"Water clusters – updated", system name water-2-Cs

Practice 1: Counterpoise correction in Gaussian: output

```
    4  O    -0.000000
        Mulliken charges condensed to fragments:
    1       0.000000
    2      -0.000000
Electronic spatial extent (au):  <R**2>=          93.4042
Charge=              -0.0000 electrons
Dipole moment (field-independent basis, Debye):
   X=            1.2156    Y=            0.0009    Z=           -1.6024  Tot=           2.0113
Quadrupole moment (field-independent basis, Debye-Ang):
   XX=          -3.5624    YY=          -4.2713    ZZ=          -6.8094
   XY=           0.0014    XZ=          -2.9721    YZ=           0.0010
Traceless Quadrupole moment (field-independent basis, Debye-Ang):
   XX=           1.3186    YY=           0.6098    ZZ=          -1.9283
   XY=           0.0014    XZ=          -2.9721    YZ=           0.0010
Octapole moment (field-independent basis, Debye-Ang**2):
  XXX=         -22.8233   YYY=           0.0020   ZZZ=          -1.2167  XYY=          -5.3888
  XXY=           0.0024   XXZ=          -5.7172   XZZ=          -9.6777  YZZ=          -0.0005
  YYZ=          -1.2037   XYZ=           0.0018
Hexadecapole moment (field-independent basis, Debye-Ang**3):
 XXXX=         -79.7064 YYYY=          -6.0810 ZZZZ=          -7.3380 XXXY=           0.0038
 XXXZ=         -10.8619 YYYX=           0.0033 YYYZ=           0.0001 ZZZX=          -1.6532
 ZZZY=           0.0005 XXYY=          -8.8972 XXZZ=         -16.1078 YYZZ=          -2.1568
 XXYZ=           0.0031 YYXZ=          -2.0208 ZZXY=          -0.0010
N-N= 9.106981851130D+00 E-N=-1.987109173839D+02  KE= 7.589137640258D+01
CPIOFr:  IOpCl= 0 IRwI=-1 IRwCP=  731 ICalc=  4 LCPTot=      11458 Len1MO=      29304 IndFrg=      157978
CPIOFr:  IOpCl= 0 IRwI=-2 IRwCP=  731 ICalc=  0 LCPTot=      11458 Len1MO=      29304 IndFrg=       40762
Counterpoise corrected energy =    -152.528876344028
               BSSE energy =         0.001330539401
           sum of fragments =    -152.521763258480
         complexation energy =       -5.30 kcal/mole (raw)
         complexation energy =       -4.46 kcal/mole (corrected)
Unable to Open any file for archive entry.
1\1\GINC-IVAVIK\SP\RMP2-FC\Aug-CC-pVDZ\H4O2\YEVG_YUR\11-Oct-2023\0\\#
rmp2/aug-cc-pVDZ counterpoise=2\\Water dimer CP\\0,1\O,0,-1.62893,-0.0
```

Line: 1/787   Column: 1   Character: 32 (0x20)   Encoding: 1252 (ANSI - L

Practice 1: Counterpoise (CP)-corrected interaction energy in Gaussian

- Build any small non-covalent complex of your choice in Avogadro , GaussView or ChemCraft

- Perform geometry optimization and frequency calculation at the RHF/6-311G(d,p) computational level

- Obtain counterpoise-corrected interaction energy and BSSE value for optimized geometry

- Repeat the same procedure using RHF/3-21G computational level. How the geometry, interaction energy and BSSE values are different?
- Visualize the output files in GaussView

Energy decomposition analysis (EDA)

■ **Energy decomposition analysis (EDA)** methods bridge the gap between electronic structure calculations and conceptual interpretations of molecular interaction energy (*i.e.* $\Delta E_{int}(\zeta)$).
(NOTE: Theoretically, Morokuma-Kitaura model and ASM can be regarded as a kind of EDA. However, in this presentation, the term of EDA is used only for the decomposition of $\Delta E_{int}(\zeta)$.)

■ **EDA is the Kohn-Sham molecular orbital (KS-MO) theory-based approach**, enabling to further analyze $\Delta E_{int}(\zeta)$ as follows:

$$\Delta E_{int}(\zeta) = \Delta E_{elstat}(\zeta) + \Delta E_{Pauli}(\zeta) + \Delta E_{oi}(\zeta)$$

ΔE_{elstat}: electrostatic energy (the Classic Coulomb interaction between the unperturbed charge distributions of the deformed reactants, usually attractive)
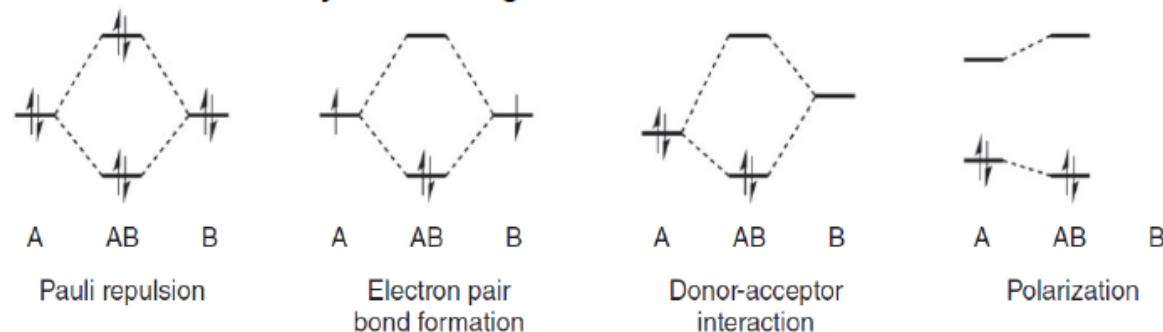→ further analyzed by Hirshfeld surface analysis, molecular electrostatic potential (MEP) *etc.*
ΔE_{Pauli}: Pauli repulsion (destabilizing interaction between occupied orbitals; *i.e.* steric repulsion)
ΔE_{oi}: polarization (empty-occupied orbital) and charge transfer (donor-acceptor interactions including HOMO-LUMO interaction)
→ further decomposed into contributions from each irreducible representations Γ of the point group to which the molecular system belongs.

$$\Delta E_{oi}(\zeta) = \sum_{\Gamma} E_{oi(\Gamma}^{\Gamma}$$

| A | AB | B | A | AB | B | A | AB | B | A | AB | B |

Pauli repulsion   Electron pair bond formation   Donor-acceptor interaction   Polarization

Orbital interaction diagrams for the most commonly appearing interactions.

■ **In EDA, not only equilibrium structures but** transition states (TS) and nonstationary points along intrinsic rection coordinate (IRC) can be analyzed.

For representative reviews on EDA, see: (a) von Hopffgarten, M.; Frenking, G. *WIREs Comput. Mol. Sci.* **2012**, *2*, 43. (b) Su, P.; Tang, Z.; Wu, W. *WIREs Comput. Mol. Sci.* **2020**, *10*, e1460.

Differential (deformation) density

$\Delta \rho < 0$ ▮ charge depletion
$\Delta \rho > 0$ ▮ charge accumulation

$$\Delta \rho(r) = \rho^{mol.}(r) - \sum_{i=1}^{N_{at}} \rho_i^{at.}(r)$$

H-H

charge transfer
from 1s orbitals

$\Delta \rho$
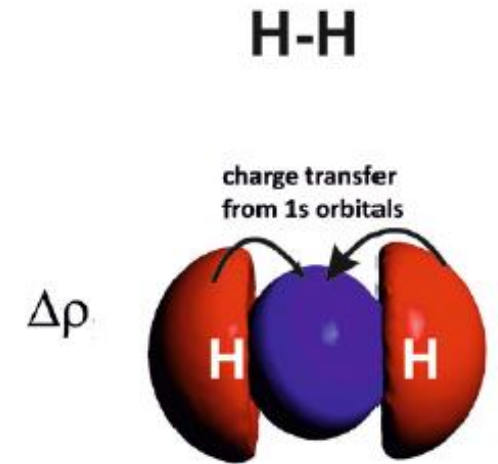
The positive values the point of density accumulation in the molecule (relative to isolated atoms).
When the molecule is formed from atoms the density flows from the area of negative value towards the area with positive value

The Natural Orbitals for Chemical Valence (NOCV)

NOCV's $(\psi_i = \sum\limits_i C_{ij} * \lambda_j)$ **diagonalize the deformation density matrix:**

$$\Delta P C_i = v_i C_i \quad ; \quad i = 1, M$$

where $\Delta P = P - P_0$, **density matrix of the combined molecule,**
P_0- **density matrix of the considered molecular fragments.**

NOCV's also decompose the deformation density $\Delta\rho$:

$$\Delta\rho(r) = \sum_{k=1}^{M} v_k \psi_k^2(r) \quad\longrightarrow$$

useful qualitative data
by inspection of the sign
of $\Delta\rho$: negative (outflow),
positive (inflow) of density

NOCV's are in pairs:

$$\Delta\rho(r) = \sum_{k=1}^{M/2} v_k [-\psi_{-k}^2(r) + \psi_k^2(r)] = \sum_{k=1}^{M/2} \Delta\rho_k(r)$$

Practice 2: NH3→ BH3 dative bond in the ammoniaborane

- Build NH3BH3 molecule (shown by instructor)
- Symmetrize the system by clicking on ⭐
- Select **Task → Geometry Optimization**
- Select **XC functional → GGA:BP86**
- Select **Basis set → DZP**
- Select **Numerical quality → Good**
- Run the calculation with **File → Run** (save with the name ammoniaborane)
- After the geometry optimization is completed, Select **Yes → New Job**
- Set **Task → Single Point**
- Go in the panel bar to **Model → Regions**
- Select the atoms of NH3 (holding Shift), click the + button, rename region to NH3, the same procedure for BH3
- Go in the panel bar to **MultiLevel → Fragments**
- Check the 'Use fragments' check box
- **Properties → ETS-NOCV**
- Select an **Closed-Shell** ETS-NOCV analysis
- EDA results and NOCV channels visualization (shown by instructor)
- **Homework**: optimize NH3 and BH3 separately at the same computational level, find the total deformation energy

NBO: Brief Overview

- NBO is a calculated bonding orbital with maximum electron density.
- Hierarchical Atomic/Molecular Orbitals in the NBO analysis:

$$AO \rightarrow NAO \rightarrow NHO \rightarrow NBO \rightarrow NLMO \rightarrow MO$$
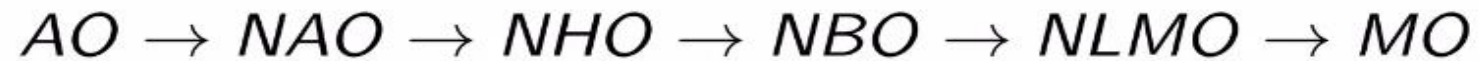
  - NAO: Natural Atomic Orbitals
  - NHO: Natural Hybrid Orbitals
  - NBO: Natural Bond Orbitals
  - NLMO: Natural (Semi-)Localized Molecular Orbitals

Natural Atomic Orbitals

- "Natural" atomic orbitals are the optimal (maximum occupancy), effective AOs in the molecular environment, derived by:
  - diagonalizing the *localized block* of the full density matrix of a given molecule,
  - which is associated with basis functions on that atom
- Thus they meet simultaneous requirements of **orthonormality** and **maximum occupancy**.

- For isolated atoms: NAOs coincide with NOs.
- For a polyatomic molecule: the NAOs mostly retain one-centre character (in contrast to NOs that become delocalized over all nuclear centers).

Natural population analysis (NPA)

NPA

```
WARNING:   Population inversion found on atom  C    1
           Population inversion found on atom  C    2
           Population inversion found on atom  C    3
           Population inversion found on atom  C    4
           Population inversion found on atom  C    5
           Population inversion found on atom  C    6
           Population inversion found on atom  H    8
           Population inversion found on atom  H    9
           Population inversion found on atom  H   11
           Population inversion found on atom  H   12


Summary of Natural Population Analysis:

                                            Natural Population
                      Natural  ---------------------------------------------
          Atom  No    Charge       Core      Valence     Rydberg      Total
         ------------------------------------------------------------------
           C    1    -0.20116    1.99925     4.18319     0.01872     6.20116
           C    2    -0.20116    1.99925     4.18319     0.01872     6.20116
           C    3    -0.20116    1.99925     4.18319     0.01872     6.20116
           C    4    -0.20116    1.99925     4.18319     0.01872     6.20116
           C    5    -0.20116    1.99925     4.18319     0.01872     6.20116
           C    6    -0.20116    1.99925     4.18319     0.01872     6.20116
           H    7     0.20116    0.00000     0.79679     0.00205     0.79884
           H    8     0.20116    0.00000     0.79679     0.00205     0.79884
           H    9     0.20116    0.00000     0.79679     0.00205     0.79884
           H   10     0.20116    0.00000     0.79679     0.00205     0.79884
           H   11     0.20116    0.00000     0.79679     0.00205     0.79884
           H   12     0.20116    0.00000     0.79679     0.00205     0.79884
         ==================================================================
         * Total *    0.00000   11.99551    29.87986     0.12463    42.00000
```

29772,1   93%

Natural bond orbitals

```
   (Occupancy)   Bond orbital/ Coefficients/ Hybrids
--------------------------------------------------------------
  1. (1.98219) BD ( 1) C   1 - C   2
          ( 50.00%)   0.7071* C   1 s( 36.00%)p 1.77( 63.89%)d 0.00(  0.09%)
                                    f 0.00(  0.02%)
                          0.0000  0.6000 -0.0089  0.0023  0.7060
                         -0.0304 -0.0031 -0.3731 -0.0172  0.0023
                          0.0000  0.0000  0.0000 -0.0189 -0.0128
                          0.0000  0.0000  0.0000  0.0000  0.0009
                          0.0076 -0.0148 -0.0077  0.0000 -0.0082
                          0.0043  0.0000  0.0000  0.0025 -0.0089
          ( 50.00%)   0.7071* C   2 s( 36.00%)p 1.77( 63.89%)d 0.00(  0.09%)
                                    f 0.00(  0.02%)
                          0.0000  0.6000 -0.0089  0.0023 -0.6762
                          0.0003  0.0035  0.4249 -0.0349 -0.0016
                          0.0000  0.0000  0.0000 -0.0102 -0.0130
                          0.0000  0.0000  0.0000  0.0000  0.0159
                          0.0073 -0.0148 -0.0077  0.0000  0.0079
                         -0.0050  0.0000  0.0000  0.0025  0.0089
  2. (1.66388) BD ( 2) C   1 - C   2
          ( 50.00%)   0.7071* C   1 s( 0.00%)p 1.00( 99.95%)d 0.00(  0.02%)
                                    f 0.00(  0.03%)
                          0.0000  0.0000  0.0000  0.0000  0.0000
                          0.0000  0.0000  0.0000  0.0000  0.0000
                          0.9997  0.0102  0.0088  0.0000  0.0000
                          0.0021  0.0069 -0.0096 -0.0091  0.0000
                          0.0000  0.0000  0.0000 -0.0159  0.0000
                          0.0000 -0.0001  0.0006  0.0000  0.0000
          ( 50.00%)   0.7071* C   2 s( 0.00%)p 1.00( 99.95%)d 0.00(  0.02%)
                                    f 0.00(  0.03%)
                          0.0000  0.0000  0.0000  0.0000  0.0000
                          0.0000  0.0000  0.0000  0.0000  0.0000
                          0.9997  0.0102  0.0088  0.0000  0.0000

30050,1                                                    94%
```

Natural Bond Orbitals

```
     (Occupancy)   Bond orbital/ Coefficients/ Hybrids
--------------------------------------------------------------------------------
   1. (1.98219) BD ( 1) C   1 - C   2
              ( 50.00%)   0.7071* C   1 s( 36.00%)p 1.77( 63.89%)d 0.00(  0.09%)
                                          f 0.00(  0.02%)
                            0.0000  0.6000 -0.0089  0.0023  0.7060
                           -0.0304 -0.0031 -0.3731 -0.0172  0.0023
                            0.0000  0.0000  0.0000 -0.0189 -0.0128
                            0.0000  0.0000  0.0000  0.0000  0.0009
                            0.0076 -0.0148 -0.0077  0.0000 -0.0082
                            0.0043  0.0000  0.0000  0.0025 -0.0089
              ( 50.00%)   0.7071* C   2 s( 36.00%)p 1.77( 63.89%)d 0.00(  0.09%)
                                          f 0.00(  0.02%)
                            0.0000  0.6000 -0.0089  0.0023 -0.6762
                            0.0003  0.0035  0.4249 -0.0349 -0.0016
                            0.0000  0.0000  0.0000 -0.0102 -0.0130
                            0.0000  0.0000  0.0000  0.0000  0.0159
                            0.0073 -0.0148 -0.0077  0.0000  0.0079
                           -0.0050  0.0000  0.0000  0.0025  0.0089
   2. (1.66388) BD ( 2) C   1 - C   2
              ( 50.00%)   0.7071* C   1 s(  0.00%)p 1.00( 99.95%)d 0.00(  0.02%)
                                          f 0.00(  0.03%)
                            0.0000  0.0000  0.0000  0.0000  0.0000
                            0.0000  0.0000  0.0000  0.0000  0.0000
                            0.9997  0.0102  0.0088  0.0000  0.0000
                            0.0021  0.0069 -0.0096 -0.0091  0.0000
                            0.0000  0.0000  0.0000 -0.0159  0.0000
                            0.0000 -0.0001  0.0006  0.0000  0.0000
              ( 50.00%)   0.7071* C   2 s(  0.00%)p 1.00( 99.95%)d 0.00(  0.02%)
                                          f 0.00(  0.03%)
                            0.0000  0.0000  0.0000  0.0000  0.0000
                            0.0000  0.0000  0.0000  0.0000  0.0000
                            0.9997  0.0102  0.0088  0.0000  0.0000
```

30050,1   94%

2nd Order Perturbation Analysis of the Fock Matrix

```
Second Order Perturbation Theory Analysis of Fock Matrix in NBO Basis

   Threshold for printing:   0.50 kcal/mol
                                                                 E(2)  E(j)-E(i) F(i,j)
           Donor NBO (i)                   Acceptor NBO (j)      kcal/mol  a.u.    a.u.
 ========================================================================================

within unit  1
  1. BD (   1) C   1 - C   2    /  72. RY*(   1) C   3                0.99    2.29   0.043
  1. BD (   1) C   1 - C   2    /  73. RY*(   2) C   3                1.28    3.52   0.060
  1. BD (   1) C   1 - C   2    / 147. RY*(   1) C   6                0.99    2.29   0.043
  1. BD (   1) C   1 - C   2    / 148. RY*(   2) C   6                1.28    3.52   0.060
  1. BD (   1) C   1 - C   2    / 252. BD*(   1) C   1 - C   6        1.87    1.25   0.043
  1. BD (   1) C   1 - C   2    / 253. BD*(   1) C   1 - H   7        0.81    1.12   0.027
  1. BD (   1) C   1 - C   2    / 254. BD*(   1) C   2 - C   3        1.87    1.25   0.043
  1. BD (   1) C   1 - C   2    / 255. BD*(   1) C   2 - H   8        0.81    1.12   0.027
  1. BD (   1) C   1 - C   2    / 258. BD*(   1) C   3 - H   9        2.71    1.12   0.049
  1. BD (   1) C   1 - C   2    / 264. BD*(   1) C   6 - H  12        2.71    1.12   0.049
  2. BD (   2) C   1 - C   2    /  75. RY*(   4) C   3                1.26    0.79   0.031
  2. BD (   2) C   1 - C   2    / 150. RY*(   4) C   6                1.26    0.79   0.031
  2. BD (   2) C   1 - C   2    / 257. BD*(   2) C   3 - C   4       20.61    0.28   0.068
  2. BD (   2) C   1 - C   2    / 262. BD*(   2) C   5 - C   6       20.61    0.28   0.068
  3. BD (   1) C   1 - C   6    /  47. RY*(   1) C   2                0.99    2.29   0.043
  3. BD (   1) C   1 - C   6    /  48. RY*(   2) C   2                1.28    3.52   0.060
  3. BD (   1) C   1 - C   6    / 122. RY*(   1) C   5                0.99    2.29   0.043
  3. BD (   1) C   1 - C   6    / 123. RY*(   2) C   5                1.28    3.52   0.060
  3. BD (   1) C   1 - C   6    / 250. BD*(   1) C   1 - C   2        1.87    1.25   0.043
  3. BD (   1) C   1 - C   6    / 253. BD*(   1) C   1 - H   7        0.81    1.12   0.027
  3. BD (   1) C   1 - C   6    / 255. BD*(   1) C   2 - H   8        2.71    1.12   0.049
  3. BD (   1) C   1 - C   6    / 261. BD*(   1) C   5 - C   6        1.87    1.25   0.043
  3. BD (   1) C   1 - C   6    / 263. BD*(   1) C   5 - H  11        2.71    1.12   0.049
  3. BD (   1) C   1 - C   6    / 264. BD*(   1) C   6 - H  12        0.81    1.12   0.027
  4. BD (   1) C   1 - H   7    /  28. RY*(   7) C   1                0.53    2.22   0.031
                                                                         31281,0-1      98%
```

Practice 3: NBO analysis

- Go to http://www.begdb.org/
- Download geometry of a non-covalent complex, e.g. A-T base pair in S22 - benchmark non-covalent complexes
- Perform NBO analysis, B3LYP/6-311++G(d,p) computational level, add Pop=NBO output=wfn external="/software/ncbr/softrepo/ncbr/nbo" to Gaussian root section
- Analyze natural charges of atoms invoved in non-covalent interactions, NBOs and characteristic donor-acceptor interactions
- **Homework:** try natural energy decomposition analysis for this complex