

E2011: Theoretical fundamentals of computer science

Topic 2: Boolean algebra - Exercises

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Some useful tools:

- **Logic.ly**: <https://logic.ly/demo/> - demo version is enough to try out some simple designs
- **LogiSim**: <http://www.cburch.com/logisim/download.html> - a free, portable (Java) application with many features
- **Digital**: <https://github.com/hneemann/Digital> - another educational tool for digital circuits

Exercise 1

Show that:

$$X(X + Y) = X \quad (\text{law of absorption})$$

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Proof:

$$\begin{aligned} X(X + Y) &= X \cdot X + X \cdot Y \\ &= X + X \cdot Y \\ &= X(1 + Y) \\ &= X \end{aligned}$$

Exercise 2

Show that:

$$XY + YZ + \bar{X}Z = XY + \bar{X}Z$$

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Proof:

$$\begin{aligned}XY + YZ + \bar{X}Z &= XY + (X + \bar{X})YZ + \bar{X}Z \\&= XY + XYZ + \bar{X}YZ + \bar{X}Z \\&= (XY + XYZ) + (\bar{X}YZ + \bar{X}Z) \\&= XY + \bar{X}Z\end{aligned}$$

Exercise 3

Prove de Morgan's theorem using truth table.

Exercise 4

Using de Morgan's theorem, expand

$$\overline{\overline{X + Y + Z}}$$

and construct the corresponding truth table.

Exercise 5

Identify the boolean function corresponding to the following truth table:

X	Y	Z	W	$F(X, Y, Z, W)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Exercise 5 - cont'd

Function identification:

$$F(X, Y, Z, W) = X\bar{Y}ZW + XY\bar{Z}\bar{W} + XY\bar{Z}W + XYZ\bar{W} + XYZW$$

Exercise 6

Simplify previous function.

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$$\begin{aligned}F(X, Y, Z, W) &= X\bar{Y}ZW + XY\bar{Z}\bar{W} + XY\bar{Z}W + XYZ\bar{W} + XYZW \\&= XZW(\bar{Y} + Y) + XY\bar{Z}(\bar{W} + W) + XYZ\bar{W} \\&= XZW + XY\bar{Z} + XYZ\bar{W} \\&= XY\bar{Z} + XZ(W + Y\bar{W}) \\&= XY\bar{Z} + XZ(Y + W) \\&= XY\bar{Z} + XYZ + XZW \\&= XY(Z + \bar{Z}) + XZW \\&= XY + XZW \\&= X(Y + ZW)\end{aligned}$$

Exercise 7

Design the logic circuits corresponding to the initial and simplified forms of the previous function, respectively.

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