E2011: Theoretical fundamentals of computer science
Topic 2: Boolean algebra

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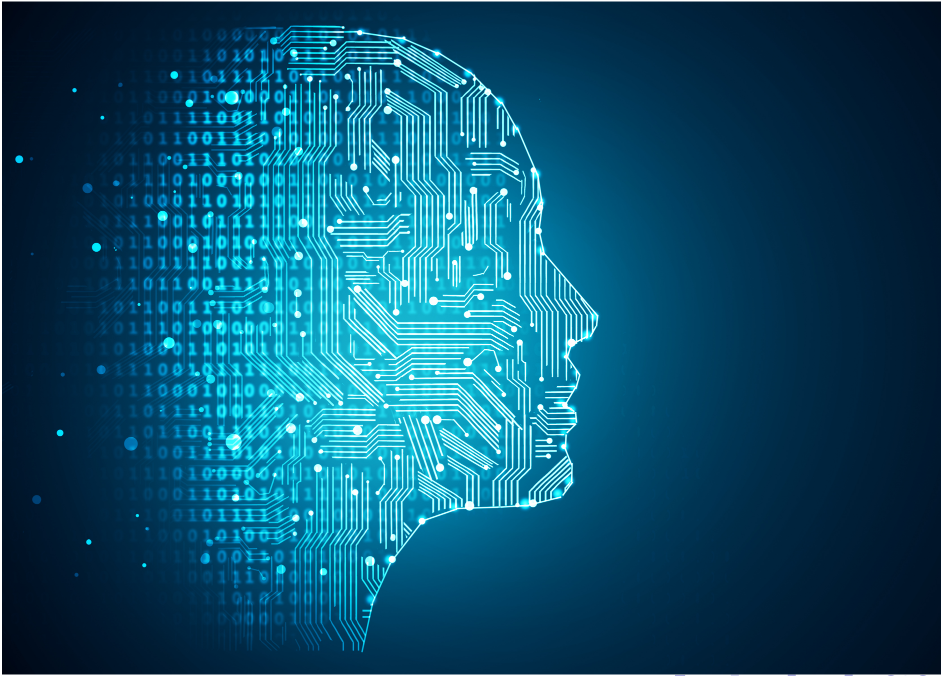
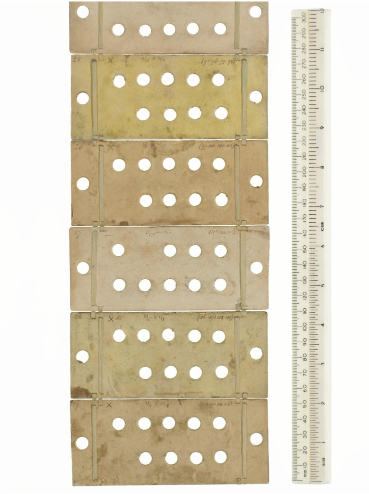


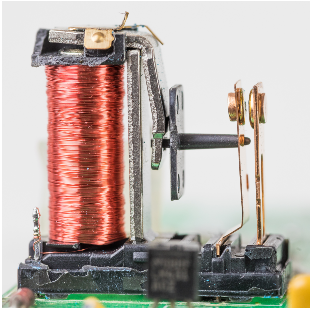
Outline

Introduction: "0/1"

Babbage's punched cards

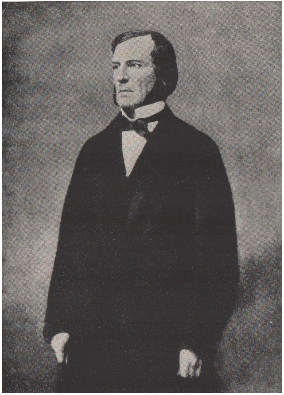


Basic relay device



George Boole (1815-1864)

- 1844: "On a general method in analysis"; gold prize in mathematics from Royal Society
- logical system: "An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities" $\longrightarrow$ "algebra of logic"

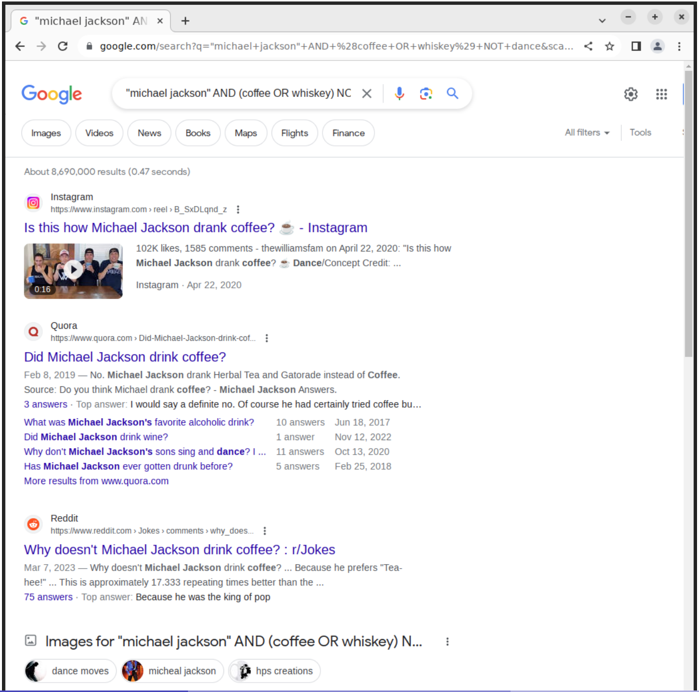


Victor Shestakov (1907-1987)

- Moscow State University (1934)
- theory of electric switches based on Boolean logic
- algebraic logic model for 2-, 3-, 4-poles switches

Claude Shannon (1916-2001)

- "father of information theory"
- MIT
- thesis on theory of electrical circuits based on Boolean algebra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Books
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Flights
Finance
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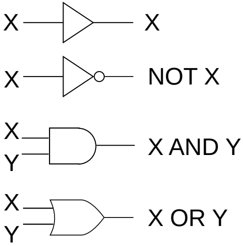
About 8,690,000 results (0.47 seconds)
Instagram
https://www.instagram.com › reel › B_SxDLqnd_z
Is this how Michael Jackson drank coffee? ☕ - Instagram
0:16
102K likes, 1585 comments - thewilliamsfam on April 22, 2020: "Is this how Michael Jackson drank coffee? ☕ Dance/Concept Credit: ...
Instagram · Apr 22, 2020
Quora
https://www.quora.com › Did-Michael-Jackson-drink-cof...
Did Michael Jackson drink coffee?
Feb 8, 2019 — No. Michael Jackson drank Herbal Tea and Gatorade instead of Coffee. Source: Do you think Michael drank coffee? - Michael Jackson Answers.
3 answers · Top answer: I would say a definite no. Of course he had certainly tried coffee bu...
What was Michael Jackson's favorite alcoholic drink? 10 answers Jun 18, 2017
Did Michael Jackson drink wine? 1 answer Nov 12, 2022
Why don't Michael Jackson's sons sing and dance? I ... 11 answers Oct 13, 2020
Has Michael Jackson ever gotten drunk before? 5 answers Feb 25, 2018
More results from www.quora.com
Reddit
https://www.reddit.com › Jokes › comments › why_does...
Why doesn't Michael Jackson drink coffee? : r/Jokes
Mar 7, 2023 — Why doesn't Michael Jackson drink coffee? ... Because he prefers "Tea-hee!" ... This is approximately 17.333 repeating times better than the ...
75 answers · Top answer: Because he was the king of pop
Images for "michael jackson" AND (coffee OR whiskey) N...
dance moves
micheal jackson
hps creations

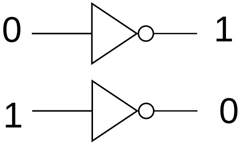
Fundamentals

- binary logic: "tertium non datur": law of excluded middle
- symbolism: 0: FALSE, 1: TRUE
- *variables*: stand for one of the two possible values, are usually represented by letters (or strings)
- *operators*: nary functions of variables, usually unary or binary

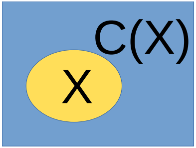
- variables: X, Y
- *negation*: **NOT**, $\neg X$
- *conjunction*: **AND**, $X \wedge Y$
- *disjunction*: **OR**, $X \vee Y$



Equivalence with sets and number operations

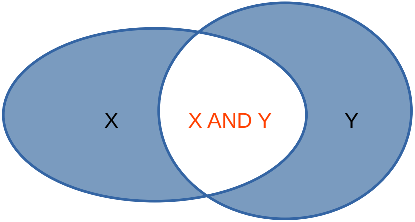


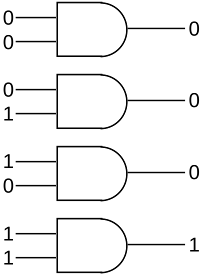
- negation: $\neg X \equiv \overline{X} \equiv C(X)$



Equivalence with sets and number operations

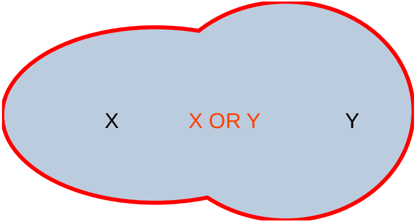
- conjunction:
  $X \wedge Y \equiv X \cdot Y \equiv X \cap Y$

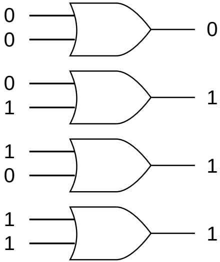




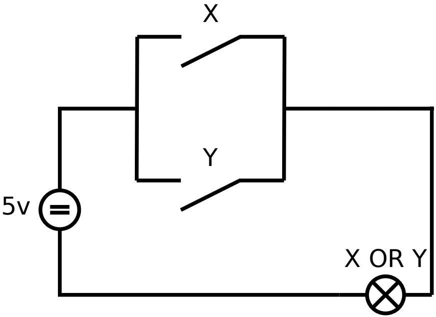
Equivalence with sets and number operations

- disjunction:
  $X \vee Y \equiv X + Y \equiv X \cup Y$





Example:



- *commutative law*:

$$X \wedge Y = Y \wedge X \text{ or } X \cdot Y = Y \cdot X$$

$$X \vee Y = Y \vee X \text{ or } X + Y = Y + X$$

- in the following we will use the usual algebraic notation, and skip $\cdot$ when not necessary
- *associative law*:

$$(XY)Z = X(YZ)$$

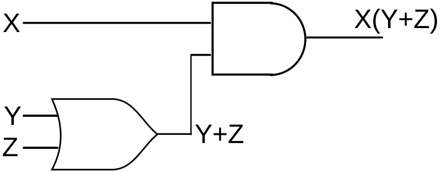
$$(X + Y) + Z = X + (Y + Z)$$

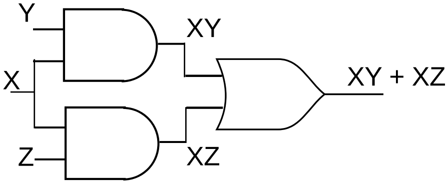
- *distributive law*

$$X(Y + Z) = XY + XZ$$

Example:

$$X(Y + Z) = XY + XZ$$





Truth tables for functions (and circuits)

Each logic function is fully described by enumerating all possible inputs and corresponding outputs (2^n values for n distinct inputs/variables).

NOT

X	$\overline{X}$
0	1
1	0

AND

X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

OR

X	Y	$X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

Fundamental laws and theorems

- $\overline{\overline{X}} = X$
- **OR** operations:

$$X + 0 = X$$
$$X + 1 = 1$$
$$X + X = X \text{ (idempotence)}$$
$$X + \overline{X} = 1$$

- **AND** operations:

$$X \cdot 0 = 0$$
$$X \cdot 1 = X$$
$$X \cdot X = X \text{ (idempotence)}$$
$$X \cdot \overline{X} = 0$$

Fundamental laws and theorems

- dual of distributive law:

$$X + YZ = (X + Y)(X + Z)$$

Proof:

$$\begin{aligned}
(X + Y)(X + Z) &= XX + XZ + YX + YZ \\
&= X + XZ + YX + YZ && \because XX = X \\
&= X(1 + Z) + YX + YZ \\
&= X + YX + YZ && \because 1 + Z = 1 \\
&= X(1 + Y) + YZ \\
&= X + YZ && \because 1 + Y = 1
\end{aligned}$$

Fundamental laws and theorems

- dual of distributive law:

$$X + YZ = (X + Y)(X + Z)$$

Proof (by brute force approach - truth table):

X	Y	Z	$X + Y$	$X + Z$	YZ	$(X + Y)(X + Z)$	$X + YZ$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	0	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

Fundamental laws and theorems

- absorption law:

$$X + XY = X$$
$$X(X + Y) = X$$

- identity theorem:

$$X + \overline{X}Y = X + Y$$
$$X(\overline{X} + Y) = XY$$

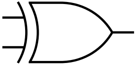
- De Morgan's theorem:

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$
$$\overline{XY} = \overline{X} + \overline{Y}$$

Other operators

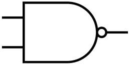
XOR
"exclusive OR"

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0



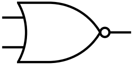
NAND
"negated-AND"

X	Y	$\overline{XY}$
0	0	1
0	1	1
1	0	1
1	1	0



NOR
"negated-OR"

X	Y	$\overline{X+Y}$
0	0	1
0	1	0
1	0	0
1	1	0



Truth table ⟶ function ⟶ circuit

Consider the following truth table:

X	Y	Z	$F(X,Y,Z)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

What is the corresponding logic function?

Method

Write the function as a sum of products (i.e. disjunction of conjunctions): for each "1" in the function column, take the sum (OR) of the corresponding *fundamental product* (ANDs). Then simplify the expression.

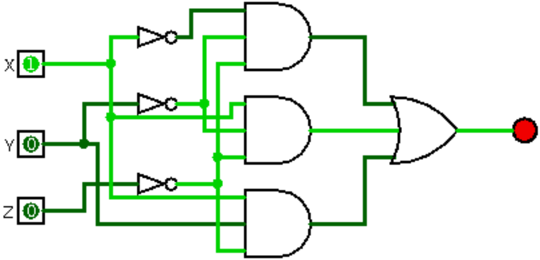
X	Y	Z	$F(X,Y,Z)$	products
0	0	0	1	$\overline{X} \cdot \overline{Y} \cdot \overline{Z}$
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	1	$X \cdot \overline{Y} \cdot \overline{Z}$
1	0	1	0	
1	1	0	1	$X \cdot Y \cdot \overline{Z}$
1	1	1	0	

X	Y	Z	$F(X,Y,Z)$	products
0	0	0	1	$\overline{X} \cdot \overline{Y} \cdot \overline{Z}$
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	1	$X \cdot \overline{Y} \cdot \overline{Z}$
1	0	1	0	
1	1	0	1	$X \cdot Y \cdot \overline{Z}$
1	1	1	0	

$$F(X,Y,Z) = \overline{X} \cdot \overline{Y} \cdot \overline{Z} + X \cdot \overline{Y} \cdot \overline{Z} + X \cdot Y \cdot \overline{Z}$$

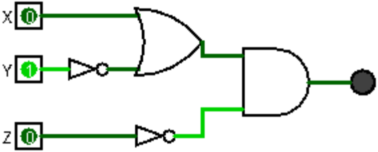
Implementation:

$$F(X, Y, Z) = \overline{X} \cdot \overline{Y} \cdot \overline{Z} + X \cdot \overline{Y} \cdot \overline{Z} + X \cdot Y \cdot \overline{Z}$$



Simplification:

$$
\begin{aligned}
F(X,Y,Z) &= \overline{X}\cdot\overline{Y}\cdot\overline{Z} + X\cdot\overline{Y}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z} \\
&= (\overline{X}+X)\overline{Y}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z} \\
&= \overline{Y}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z} \\
&= \overline{Z}(\overline{Y}+XY) \\
&= \overline{Z}(X+\overline{Y})
\end{aligned}
$$



Questions?