E2011: Theoretical fundamentals of computer science
Introduction to algorithms - Exercises

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Problem 1

Quadratic equations

Write the pseudocode for solving the second-degree equation

$$ax^2 + bx + c = 0, \qquad a, b, c \in \mathbb{R}$$

- identify the input and output
- express the solution
- check that all the input cases are covered

Algorithm 1 Quadratic equation solver

Input: $a, b, c \in \mathbb{R}$
Output: x_1, x_2 solutions of $ax^2 + bx + c = 0$

```
if a = 0 then
    if b = 0 then
        if c = 0 then
            print "Undet. eq."
        else
            print "Imp. eq."
        end if
    else
        x1 ← −c/b
        print "1st deg. eq.:" x1
    end if
else
    Δ ← b^2 − 4ac
    if Δ ≥ 0 then
        x1,2 ← (−b ± √Δ)/(2a)
        print "Solution(s): ", x1,2
    else
        x1 ← −b/(2a)
        x2 ← √(−Δ)/(2a)
        print x1 + i x2
    end if
end if
```

Problem 2

Series processing

Given a series (vector) of elements $[x_i], i = 1, \ldots, N$, compute:

- sum of all elements
- product of all non-zero elements
- cummulative sum of positive elements ($S_k = \sum_{i \in P} x_i$, where P is the set of indexes of positive elements)

Problem 3

Series processing

Let $[a_i], i = 1, \ldots, N, N \geq 3$ be a real-valued vector. Write the pseudocode to

- let the user input the values for $[a_i]$
- compute a new vector $[s_i]$ defined as

$$s_i = \begin{cases} 0 & i \in \{1, N\} \\ \frac{a_{i+1} - a_{i-1}}{2} & 0 < i < N \end{cases},$$

- display (output) the result

Solution to Problem 3

Input: $N \in \mathbb{N}, N \geq 3$ ▷ total number of elements
Output: $[s_i], i = 1, \ldots, N$, as required
  input N ▷ ask the user for the value of N
  input $[a_i], i = 1, \ldots, N$ ▷ ask the user for vector elements
  $[s_i] \leftarrow 0, i = 1, \ldots, N$ ▷ initialize the result vector
  for $i = 2, \ldots, N-1$ **do**
    $s_i \leftarrow \frac{a_{i+1} - a_{i-1}}{2}$
  end for
  print $[s_i], i = 1 \ldots, N$ ▷ show the result

Problem 4

Sieve of Eratosthenes

Write the pseudocode to find all prime numbers smaller than a given $N \in \mathbb{N}, N \geq 2$ using the algorithm "sieve of Eratosthenes".

(non-optimal) Solution to Problem 4

Input: $N \in \mathbb{N}, N \geq 2$
Output: P - the set of prime numbers $\leq N$

$P \leftarrow \emptyset$
$C \leftarrow \{2, \ldots, N\}$ ▷ candidates
$x \leftarrow 2$ ▷ first prime
while $C \neq \emptyset$ **do**
  $P \leftarrow P \cup \{x\}$ ▷ add current
  for $k = 1, \ldots, \lfloor \max(C)/x \rfloor$ **do**
    if $kx \in C$ **then**
      $C \leftarrow C \setminus \{kx\}$
    end if
  end for
  if $C \neq \emptyset$ **then**
    $x \leftarrow \min(C)$
  end if
end while