**The matrix formalism for light propagation in layered structures.**
The propagation of light in a layered structure of homogeneous media with planar interfaces produces no diffraction, and can, hence, be described by impact ($E_{\mathrm{i}}$), reflected ($E_{\mathrm{r}}$) and transmitted ($E_{\mathrm{t}}$) wave amplitudes of the electric field using simple matrix formalism (see Fig. 1):

$$\begin{bmatrix} E_{\mathrm{i}} \\ E_{\mathrm{r}} \end{bmatrix} = \mathbf{V}_0^{-1}\mathbf{R}_1^{-1}\ldots\mathbf{R}_m^{-1}\mathbf{V}_{\mathrm{f}} \begin{bmatrix} E_{\mathrm{t}} \\ 0 \end{bmatrix} \equiv \mathbf{P} \begin{bmatrix} E_{\mathrm{t}} \\ 0 \end{bmatrix}, \tag{1}$$

where matrices $\mathbf{R}_j$, $j$=1...$m$, belong to $j$-th layer of the structure and matrices $\mathbf{V}$ are connected with substrate (index $_0$) and ambient (index $_{\mathrm{f}}$).

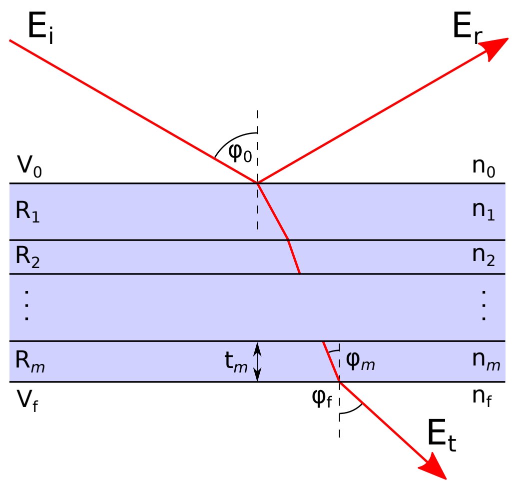


**Fig.** 1: Notation for light propagation through a layered structure. In SPR measurements, the light is impacting from substrate.

The expression

$$\mathbf{R}_j^{-1} = \begin{pmatrix} \cos\theta_j & -\dfrac{\mathrm{i}\sin\theta_j}{\tilde{n}_j} \\ -\mathrm{i}\,\tilde{n}_j\sin\theta_j & \cos\theta_j \end{pmatrix} \tag{2}$$

is formally polarization independent and det$\mathbf{R}_j$=1. Note, however, that definition of effective index of refraction $\tilde{n}_j$ within the $j$-th layer is polarization dependent itself:

$$\mathrm{s}: \tilde{n}_j = n_j \cos\varphi_j \qquad \mathrm{p}: \tilde{n}_j = \frac{n_j}{\cos\varphi_j}, \tag{3}$$

where $n_j$ is the index of refraction of $j$-th layer. For angle $\varphi_j$ at which the $j$-th layer is propagated the Snell's law brings

$$\cos\varphi_j = \sqrt{1 - \left(\frac{n_0}{n_j}\sin\varphi_0\right)^2}; \tag{4}$$

analogical formula can be used to obtain the angle $\varphi_{\mathrm{f}}$ in the ambient. The phase $\theta_j$ acquired within the $j$-th layer is given by

$$\theta_j = 2\pi\frac{t_j}{\lambda} n_j \cos\varphi_j, \tag{5}$$

where $t_j$ is the thickness of the $j$-th layer and $\lambda$ is the vacuum wavelength of incident light.
For the $\mathbf{V}$ matrices we generally have

$$\mathrm{s}: \mathbf{V} = \begin{pmatrix} 1 & 1 \\ \tilde{n} & -\tilde{n} \end{pmatrix} \qquad \mathrm{p}: \mathbf{V} = \cos\varphi \begin{pmatrix} 1 & 1 \\ \tilde{n} & -\tilde{n} \end{pmatrix},$$

which brings

$$\mathrm{s}: \mathbf{V}_{\mathrm{f}} = \begin{pmatrix} 1 & 1 \\ \tilde{n}_{\mathrm{f}} & -\tilde{n}_{\mathrm{f}} \end{pmatrix} \qquad \mathrm{p}: \mathbf{V}_{\mathrm{f}} = \cos\varphi_{\mathrm{f}} \begin{pmatrix} 1 & 1 \\ \tilde{n}_{\mathrm{f}} & -\tilde{n}_{\mathrm{f}} \end{pmatrix}, \tag{6}$$

$$\mathrm{s}:(\mathbf{V}_0)^{-1}=\frac{1}{2\tilde{n}_0}\begin{pmatrix}\tilde{n}_0 & 1\\ \tilde{n}_0 & -1\end{pmatrix}\qquad \mathrm{p}:(\mathbf{V}_0)^{-1}=\frac{1}{2\tilde{n}_0\cos\varphi_0}\begin{pmatrix}\tilde{n}_0 & 1\\ \tilde{n}_0 & -1\end{pmatrix}. \tag{7}$$

The overall propagation matrix $\mathbf{P}$ from eq. (**1**) allows to compute the complex reflectivity (distinguishing $r_\mathrm{s}$ and $r_\mathrm{p}$ by the appropriate entries of $\mathbf{P}$ for each of the polarizations) as well as the complex reflectance ratio $\rho=r_\mathrm{p}/r_\mathrm{s}$. To this end, we will denote the polarization-dependent components of $\mathbf{P}$ by the respective index:

$$\mathrm{p}:\quad \mathbf{P}\equiv\begin{pmatrix}a_\mathrm{p} & b_\mathrm{p}\\ c_\mathrm{p} & d_\mathrm{p}\end{pmatrix}\qquad \mathrm{s}:\quad \mathbf{P}\equiv\begin{pmatrix}a_\mathrm{s} & b_\mathrm{s}\\ c_\mathrm{s} & d_\mathrm{s}\end{pmatrix},$$

so that $r_\mathrm{s}=c_\mathrm{s}/a_\mathrm{s}$, $r_\mathrm{p}=c_\mathrm{p}/a_\mathrm{p}$ and $t_\mathrm{s}=1/a_\mathrm{s}$, $t_\mathrm{p}=1/a_\mathrm{p}$. Consequently,

$$R=rr^*,\qquad \text{but}\qquad T=\frac{n_\mathrm{f}\cos\varphi_\mathrm{f}}{n_0\cos\varphi_0}\,tt^*.$$