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Physical laboratory 4

LED spectra

Task and goals

Determine the relationship between voltammograms and LED spectra. Compare to a similar relation between the voltage at the X-ray tube and the spectrum of the bremsstrahlung X-ray radiation.
Measure the emission spectra of different colored LEDs for different currents, and determine the wavelength by independent interference method. Measure the voltammetric characteristics of different colored LEDs. Measure the X-ray bremsstrahlung spectra for different voltages at the X-ray.
Verify the value of the ratio of fundamental physical constants hc/e.

Recommended procedure and tasks for measurements

The student has three weeks to make the measurement. Recommended Procedure:

- First and second week – spectra and voltammograms of LEDs. Spectra can be measured in several ways: fiber spectrophotometer, Newtonian glass, and simple grating spectroscope. Use at least two methods.
- Week three – the spectrum of bremsstrahlung x-ray as a function of voltage on the x-ray tube.

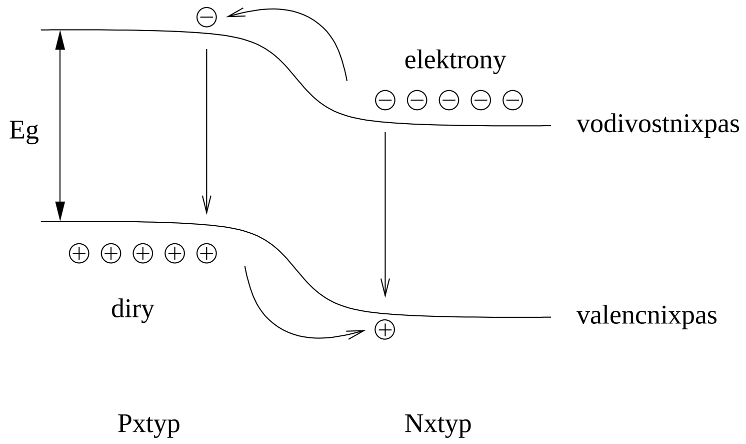
The output of the practicum will be presented to the instructor in the form of graphs and measured dependencies, including an estimate of uncertainties if possible and appropriate.

Voltage characteristics of LEDs

The first systematic measurement of Planck's constant was made in 1912 by Robert Millikan, best known for measuring the elementary charge, in which he observed the movement of charged oil droplets in an electrostatic field. The Planck's constant value of $h = 6.57 \times 10^{-34} : \mathrm{J,s}$ was determined by careful observation of the photoelectric effect on the surface of metals in a vacuum.

To approximate the value of Planck's constant in this problem, the practitioner will use the relationship between the characteristic voltage required to light a light-emitting diode (LED) and the color of the light emitted. In this way, we can find the value of Planck's constant with an error of tens of percent.

Like other types of diodes, the LED is based on a PN transition between a semiconductor type P-type and N-type. A depleted region – a layer of space charge – forms at the interface that prevents majority electrons and holes from penetrating the interface. If we apply a voltage in the permeable direction to the PN transition, it allows an additional electrostatic field to allow charge carriers to more easily cross the depleted region, and PN current begins to flow through the transition. Thus, in both regions (P and N type) dynamically increase the concentration of minority carriers, which tend to recombine with the major carriers. For the production of LEDs, semiconductors with a direct bandgap of suitable width (GaAs, $Ga_{1-x}Al_xAs$, GaP, GaN), which allow radiative recombination in the visible wavelength range, or in the near IR or UV region.



The width of the forbidden band is directly related to the photon energy of the emitted light and to the the voltammetric characteristics of the diode, which brings a correlation between these the two characteristics of the LED. We now discuss this relationship quantitatively and show how it can be used to approximate the value of Planck's constant.

An ideal diode has a voltammetric characteristic, i.e., the current dependence I flowing through the diode on the voltage U applied to it, given by Shockley's equation

$$I(U) = I_s \left[\exp\left(\frac{eU}{k_B T} \right) - 1 \right] ; , \tag{1}$$

where I_s is the saturation current, e the elementary charge, T the temperature and k_B Boltzmann constant. The saturation current depends on the width of the forbidden band (a detailed discussion can be found in e.g., the textbook [1]), which leads to an approximate equation

$$I(U) \approx B \exp\left(-\frac{E_g - eU}{k_B T} \right) , \tag{2}$$

where B is a constant determined by doping and transition geometry. In practice, a series of highly luminous diodes with approximately the same parameters (e.g., a maximum operating current of about 20 : mA) for which it is possible to can be expected to have approximately the same value of the constant B.

Under this assumption, the preceding equation for the prescribed value of the current the relationship between the voltage across the diode U_f and the width of the forbidden band

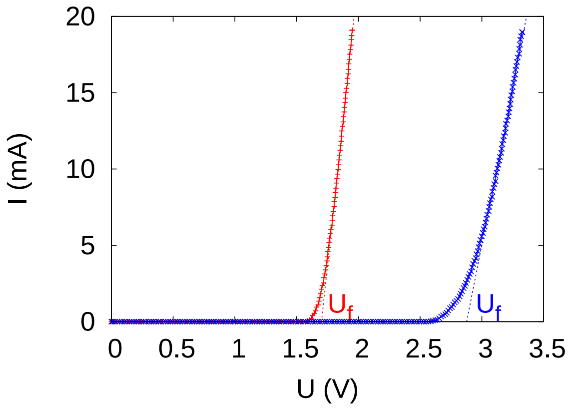
$$eU_f|_I = E_g + \Delta E|_I \; . \tag{3}$$

The offset $\Delta E|_I$ is universal for all diodes with the same value of B.

It remains to find the relation between the width of the forbidden band and the photon wavelength emitted by the LEDs. In the case of a semiconductor with a direct bandgap, the radiative recombination by electron hopping out of the conduction band to the valence band while emitting a photon. We ignore the participation of recombination centers, which are essential for the LED function are practically necessary for the function of the LEDs.

The energy of the emitted photons is then approximately equal to the width of the forbidden band E_g, which determines the frequency and wavelength of the emitted radiation: $hf = hc/\lambda = E_g$. When combined with the previous relationship between U_f and E_g, we get the relationship

$$U_f|_I = \frac{hc}{e} \lambda^{-1} + \Delta U|_I ; . \tag{4}$$



Obrázek 1: V-A characteristics of the red and blue LEDs with the voltage U_f indicated.

This session offers the possibility of experimentally determining the constant hc/e from the directive dependence of U_f on λ^{-1}. If we know the values of c and e from other experiments, the value of Planck's constant h can be calculated.

For higher currents flowing through a diode, its voltammogram is affected by the diode's DC resistance R

$$I(U) = I_s \left[\exp\left(\frac{e(U - RI)}{k_B T} \right) - 1 \right] . \tag{5}$$

From here, we can derive an approximate relation for the voltammetric characteristic for high currents

$$I(U) = \left\{ \begin{array}{ll} 0 & \mathrm{for} U < U_f \\ \frac{U - U_f}{R} & \mathrm{for} U \geq U_f \end{array} \right. , \tag{6}$$

where the energy eU_f can be approximately equal to the width of the forbidden band $eU_f \approx E_g$. The energy of the emitted photons is approximately equal to the width of the forbidden band E_g, which determines the frequency and wavelength of the emitted radiation: $hf = hc/\lambda = E_g$

$$U_f \approx \frac{hc}{e} \lambda^{-1} , \tag{7}$$

where we can easily determine the Plack constant.

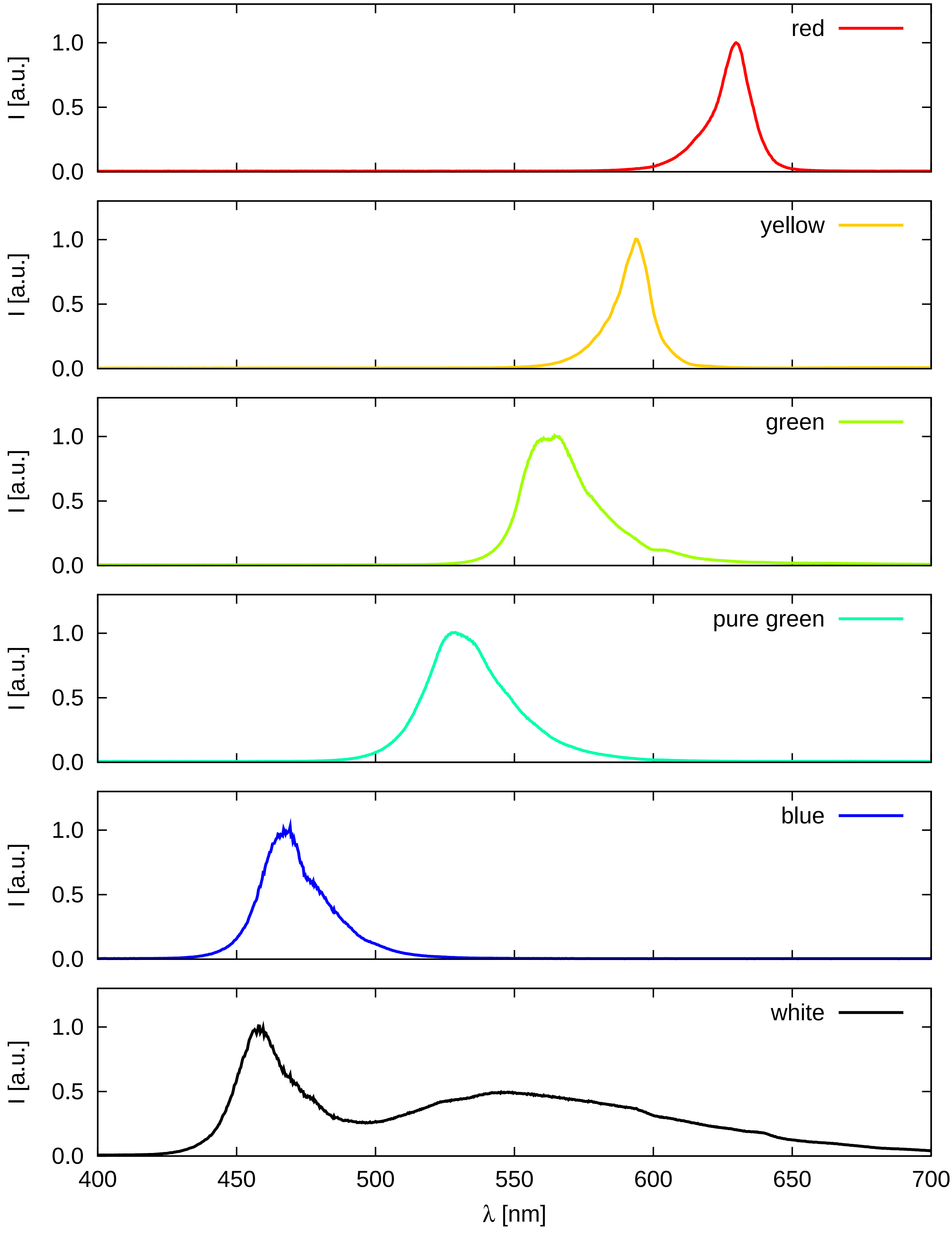
Úkoly

1. Determine the wavelengths of radiation of each LED in the series using diffraction grating. Since the emission lines are very broad, we take the wavelength corresponding to the peak
2. Measure the voltammetric characteristics of the LED.
3. From the voltammograms of each LED, subtract U_f and construct a graph of the dependence of U_f on λ^{-1}, from which we can obtain the value of the constant hc/e.

Wavelength measurement by interference on Newton glasses

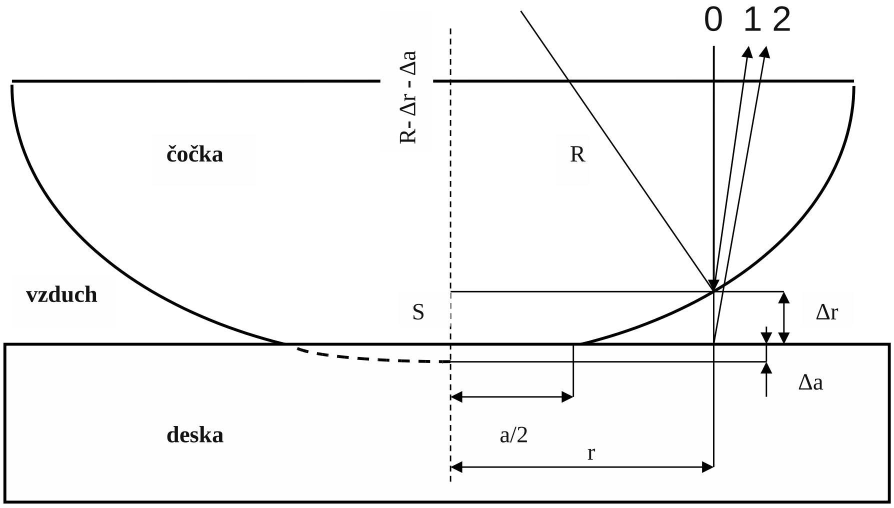
Theory

The wavelength of light is measured using the interference effect on a thin air gap between a plane glass plate and a lens of radius R. When observed in reflected or transmitted light, we see alternating light and dark circular bands of increasing radius r, called Newton's rings. At the



Obrázek 2: Emission spectra of LEDs of different colors available in practice. For LEDs labeled „white“ and „pure green“ intrinsic PN transition at wavelengths in the blue to UV region and the resulting color is achieved by phosphorescence.

contact of a spherical lens with a plane glass plate, the lens and plate deform slightly, and the point contact results in a flat circular contact which appears as a dark circular area in reflected light and as a bright circular area in transmitted light, the so-called Hertzian spot, whose radius $a/2$ depends on the attractive force. The situation in the plane of the section is shown in figure 3.



Obrázek 3: Schematic drawing of interfering rays on Newton's glasses.

We assume that a plane monochromatic wave of wavelength *lambda* incident perpendicularly on the plane refractive surface of the lens proceeds to the spherical refractive surface, where it is partially reflected and reversed in phase. A part of the wave proceeds further through the air gap, and at the air-plate interface, it is partially reflected without a change of phase and proceeds back with the same phase. At points on a circle of radius r centered at the point where the lens touches the plate, this is shown by three rays: the incoming „0“ and two reflected ones, „1“ and „2“. The outgoing waves interfere, and the resulting intensity depends on the phase difference of the waves or the path difference of the two reflected beams. According to the figure, the path difference Δ of the rays „1“ and „2“ with respect to the phase change is equal to

$$\Delta = 2\Delta r + \frac{\lambda}{2} \tag{8}$$

The minimum of light intensity occurs on circles with radii r_k for which the path difference is equal to an odd multiple of $\lambda/2$, i.e.

$$\Delta = (2k+1)\frac{\lambda}{2}, \text{resp.} \Delta r_k = k\frac{\lambda}{2}, eq2 \tag{9}$$

where $k = 1, 2, ...$ is the order of the minimum. The size of the air gap Δr between the lens and the plate at a distance r from the point of contact S is determined from geometry:

$$(R - \Delta r - \Delta a)^2 + r^2 = R^2 \tag{10}$$

$$(R - \Delta a)^2 + \left(\frac{a}{2}\right)^2 = R^2 \tag{11}$$

The height of the circular canopy Δa formed by the deformation of the spherical surface of the lens is determined from (11) assuming that $2R \gg \Delta a$

$$\Delta a = \frac{a^2}{8R} \tag{12}$$

Assuming $2R \gg \Delta r + \Delta a$ we get from the equation (10)

$$2R(\Delta r + \Delta a) = r^2$$

and using (12) to obtain for the radius of the circle r on which the size of the air gap Δr

$$r^2 = 2R\Delta r + \frac{a^2}{4} \tag{13}$$

If the size of the air gap satisfies the equation (**??**), we obtain the equation for the radii of the circles r_k with the minimum light intensity

$$r^2 = \lambda Rk + \frac{a^2}{4} \tag{14}$$

The equation (14) shows that the square of the radius of the dark ring is a linear function of the order of the minimum k. Plotting the dependence (14) on the graph gives the equation of the line ($Y = r_k^2$ and $X = k$)

$$Y = A + BX \tag{15}$$

and from the constants A, B, we can determine the wavelength and radius of the Hertzian spot:

$$\lambda = \frac{B}{R}\frac{a}{2} = \sqrt{A} \tag{16}$$

If we only want to determine the wavelength, we can determine it from the squared difference of pairs of radii r_k and r_n by (14) as follows:

$$\lambda = \frac{r_k^2 - r_n^2}{R(k-n)} \tag{17}$$

Measurement progress

To measure the radii of Newton's rings, we use a microscope with a top illumination and a measuring eyepiece. Two lenses of the same radius of curvature are mounted in a metal fixture with a cylindrical hole into which the microscope objective is inserted freely so that the interference rings can be focused. In this case, the size of the air gap is twice that of the lens-planar plate arrangement. The equation (14) for the radii of the circles r_k with minimum intensity then transitions to the relation

$$r^2 = \frac{1}{2}\lambda Rk + \frac{a^2}{4} \tag{18}$$

and the relation (16) for determining the wavelength from the directive line to

$$\lambda = \frac{2B}{R}\frac{a}{2} = \sqrt{A}. \tag{19}$$

Similarly, the relation (17) for determining the wavelength from the squared difference of pairs of radii r_k and r_n goes to:

$$\lambda = 2\frac{r_k^2 - r_n^2}{R(k-n)} \tag{20}$$

The equation (18) shows that the square of the radius of the dark ring is a linear function of the order of the minimum of k. For illumination, we can use a sodium lamp or a luminescent diode. Since we use the microscope to determine the radii of the Newtonian rings in parts of the eyepiece scale, we must first determine the microscope magnification $Z = y'/y$ using a test slide, where y is the distance of the indentations on the test slide in $mathrmm$ and y' is the distance of the indentations in parts. We determine the actual size of the Newton rings in μm as $r_k = r'_k/Z$. To make the measurements, place the fixture on the microscope stage, focus the interference rings, and gently move the fixture or microscope stage to place the rings in the center of the field of view. The size of the rings is determined by the two extreme positions on the ring, the difference of which determines the diameter of the ring. Proceed from the smallest to the largest ring as far as the scale of the eyepiece will allow.

Remark:

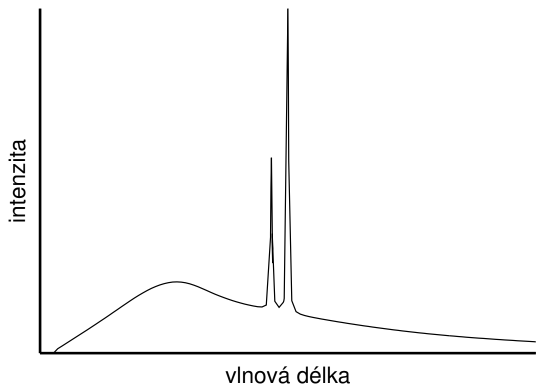
To determine the wavelength of light, we need to know the radius of curvature of the refractive surface of the lens R. If its value is unknown or is not given with sufficient accuracy, but we have a source of monochromatic radiation of a known wavelength, e.g., sodium lamp with wavelength $\lambda = 589{,}30{,}mathrmnm$, we can determine the radius of the lens R from the equations (18) or (20) by measuring the radii of the Newton rings.

Úkoly

1. Assemble the preparation with the lens and plate, insert it into the microscope objective, focus the interference fringes, and place the center of the rings in the center of the microscope field of view. Check the function of the measuring eyepiece and, if necessary, focus the scale with the eyepiece lens.
2. Illuminate the specimen with the LED (powered via a 4.5 V battery regulation resistor) and measure the diameters of all rings in the scale range. Determine the wavelength of the LED.
3. From the results of measurements 1 and 2, determine the diameter a of the Hertzian spot.

Bremsstrahlung radiation

Theory



Obrázek 4: Spectrum of X-rays as the sum of the continuous and characteristic spectra.

The incident electron builds up in the anti-cathode material an X-ray spectrum which has continuous and line component, Fig. 4.

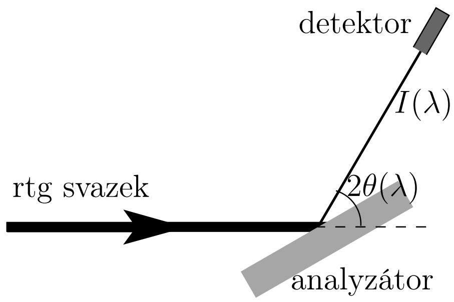
Bremsstrahlung radiation

The braking of the incident electron in the material produces the continuous component of the X-ray spectrum. anticathodes – *bremsstrahlung radiation.* The photon energy of the bremsstrahlung radiation is maximal when the entire kinetic energy of the incident electron is converted to photon energy. For the minimum wavelength (edge) of a continuous spectrum, the following holds

$$\lambda_{\min} = \frac{hc}{eU} \tag{21}$$

where U is the accelerating voltage in the rtg lamp. The maximum intensity is the bremsstrahlung radiation for wavelengths from approximately $1.5\,\lambda_{\min}$ to $1.8\,\lambda_{\min}$ (the value also depends on the type of X-ray tube).

Measurement progress



Obrázek 5: Scheme of an energy-sensitive measurement with a crystal analyzer.

The spectrum is measured by diffracting the collimated radiation on a single crystal (analyzer); see Fig. 5. At a given angle θ between the incident radiation and the crystallographic plane, the a diffraction for the wavelength spectrum for which the Bragg diffraction condition (cubic crystals)

$$2a\sin\theta = \lambda\sqrt{N},\ \ N = h^2 + k^2 + l^2\ . \tag{22}$$

The dependence of the diffracted intensity on the angle θ measured by rotating the crystal with can be converted to a wavelength dependence of the intensity. In doing so, it is necessary to the effect of the superposition of higher orders of diffraction on the analyzer must be taken into account.

We will therefore measure the spectrum as the intensity dependence of the Bragg angle of the analyzer, which we then convert to wavelength. We will measure the dependencies for the series of voltages on the X-ray and the current flowing through it. We analyze the dependencies of the minimum wavelength and maximum intensity of the continuous spectrum and the maximum of the characteristic spectrum. We perform measurements with an inserted nickel filter and analyze its effect on the spectrum.

Experimental equipment

X-ray source with copper, molybdenum, and tungsten anti-cathode, goniometer, LiF analytical single crystal (lattice constant 4.028 Å, surface plane (001)), X-ray ionization detector (dead time $\tau = 90\ \mu$s), control computer.

Reference

[1] S.M. Sze: *Physics of semiconductor devices*, John Wiley and Sons Inc., New York (1981).

[2] R.A. Millikan, Phys. Rev. **7**, 355 (1916)