Ústav fyziky a technologií plazmatu
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F7544 Experimentální metody 1

High frequency measurements using Vector Network Analyser

Task summary:

Perform measurements of simple devices (connectors, cables, their assemblies) up to GHz frequencies using VNA. Observe strange phenomena, investigate resonances, analyse frequency dependence of reflection and transmission coefficients. Perform fancy and complex matrix gymnastics. Learn what-to-do and what-not-to-do at high frequencies. Enjoy.

Transmission line theory

Classical circuit analysis is based on two Kirchoff's laws:

current The sum of currents in a network of conductors meeting at a junction is zero.

voltage The sum of voltages around any closed loop is zero.

While they seem obvious for any physicist, they are not. Actually, they are making one very big assumption, i.e. that propagation of signal is instantaneous and happens only via the conductors. In fact, they are the low frequency asymptotic solution of more general Maxwell equations and they are usable only if the dimension of circuit L is negligible in comparison with wavelength λ.

Infinite homogeneous transmission line

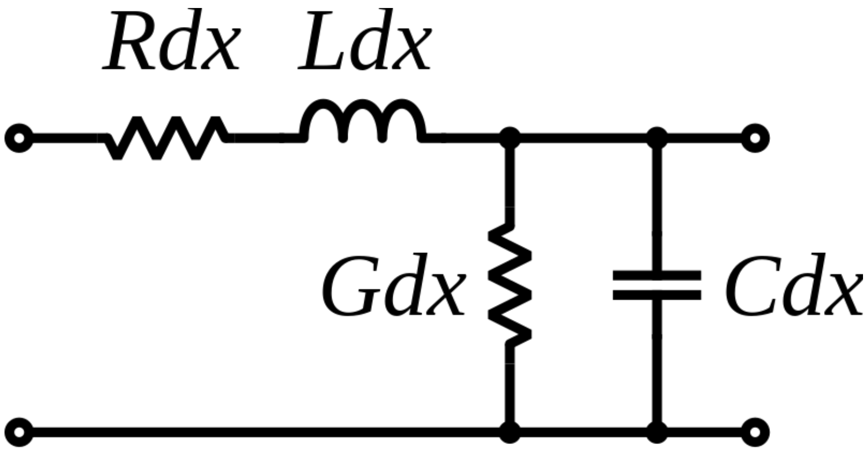
Still, even at higher frequencies ($L \approx \lambda$) some simplification (vs. full solution of Maxwell equations) can be done. It is extension of lumped element circuit theory. We consider the classical R, L, C components as point-like, but we must introduce a new element – transmission line, which takes into account a new class of wave phenomena – finite propagation time, reflection, wave impedances, etc. Originally, the theory was developed for long distance transmission lines used for telegraphy.

Homogeneous two conductor transmission line can be constructed using distributed element circuit theory from infinite number of elementary R, L, C, G components.

The current I and voltage U are not constant along the transmission line, so they are functions of both time and position. They are related by telegraph (or telegrapher's) equations

$$\frac{\partial}{\partial x} V(x,t) = -L\frac{\partial}{\partial t}I(x,t) - R\, I(x,t)$$
$$\frac{\partial}{\partial x} I(x,t) = -C\frac{\partial}{\partial t}V(x,t) - G\, V(x,t)$$

If the problem is nice and linear, the solution of telegraph equations is a superposition of infinite number of waves with different frequencies and propagating both forward and backward ($U_\omega{=}U_{0\omega}\mathrm{e}^{i\omega t\pm\gamma x}$, $I_\omega{=}I_{0\omega}\mathrm{e}^{i\omega t\pm\gamma x}$). For each such wave, its current and voltage at certain position



Obrázek 1: A section of transmission line modelled using lumped elements.

x are linked by the complex impedance $Z(x)$. Generally, this impedance depends on frequency and can be different for forward and backward wave. For infinite transmission line, this complex impedance is called *characteristic impedance* and depends only on frequency, not on position or direction of propagation

$$Z_\mathrm{c} = \sqrt{\frac{R + i\omega L}{G + i\omega C}}$$

Spatial wave propagation constant γ is complex (it consists of attenuation constant α and phase constant β) and frequency dependent

$$\gamma = \alpha + i\beta = \sqrt{(R + i\omega L)(G + i\omega C)}$$

Most of real transmission lines are constructed in a way that R, G can be neglected (approximation of lossless transmission line). In that case, the characteristic impedance and spatial propagation constant become much simpler

$$Z_\mathrm{c} = \sqrt{\frac{L}{C}}$$
$$\gamma = i\omega\sqrt{LC}$$

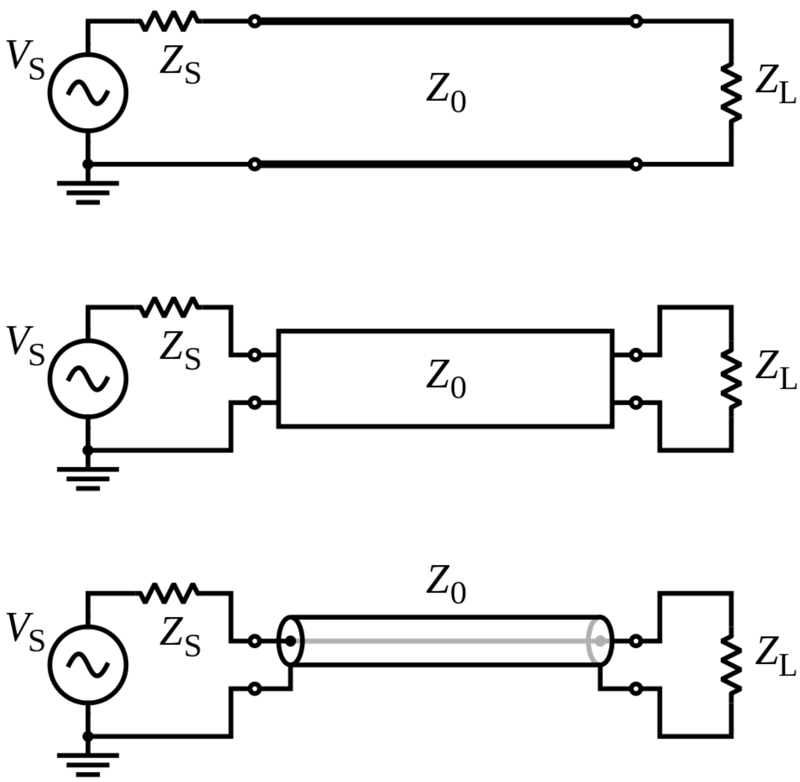
The characteristic impedance of two conductor lossless homogeneous transmission line depends on geometry (e.g. diameter and distance of the two conductors) and on the dielectric separating them. The velocity of propagation (both phase and group velocities are the same as the lossless line is not dispersive) depends only on the dielectric (and for a vacuum dielectric it is equal to the speed of light c).

While the transmission line characteristic impedance has units Ω, it is definitely not some resistance. I.e. lossless line has purely real Z_c, but no Joule heating losses.

The theory of two wire transmission line, forward and backward waves, characteristic impedance, propagation constant, etc. can be actually generalised to different number of conductors (e.g. 0 – free space propagation, 1 – hollow waveguides, 3 – tri-phase lines in electric power distribution, etc.) and even more exotic electromagnetic field configurations (e.g. non TEM waves).

Terminated transmission line

A typical (easiest but still of practical interest) case for an analysis is a finite length (d) transmission line with characteristic impedance Z_0 terminated by the load impedance Z_L and carrying a sinusoidal steady state signal with single frequency ω.



Obrázek 2: Loaded finite transmission line – different schematics styles.

Ratio of complex voltage and current (i.e. impedance) is different at different positions x (which is measured from the line end, so x=0 corresponds to the place of load impedance) along the transmission line

$$Z(x) = Z_0 \frac{Z_\mathrm{L} + Z_0 \tanh(\gamma\, x)}{Z_0 + Z_\mathrm{L} \tanh(\gamma\, x)}$$

For the lossless line (Z_0 is real, $\gamma = i\beta$), it simplifies to

$$Z(x) = Z_0 \frac{Z_\mathrm{L} + iZ_0 \tan(\beta\, x)}{Z_0 + iZ_\mathrm{L} \tan(\beta\, x)}$$

From the point of view of observer the lossless transmission line of characteristic impedance Z_0 and length d transforms the load impedance Z_L at its end to impedance Z_in at its input

$$Z_\mathrm{in} = Z_0 \frac{Z_\mathrm{L} + iZ_0 \tan(\beta\, d)}{Z_0 + iZ_\mathrm{L} \tan(\beta\, d)}$$

When condition $Z_0 = Z_\mathrm{L}$ is met, the line behaves like infinite transmission line and dependence of impedance on position disappears. This case is called ideally terminated line or *matched line.*

Wave propagation theory (and thus also its special case – transmission line theory) shows that if the line is not ideally terminated (i.e. $Z_0 \neq Z_\mathrm{L}$), there will be reflections. It is most often described by the voltage reflection coefficient, i.e. ratio of voltage complex amplitudes of reflected and forward waves in a steady sinusoidal state

$$\Gamma = \frac{V_\mathrm{r}}{V_\mathrm{f}}$$

Its generally complex, as both V_r and V_f are phasors. Similarly, a current reflection coefficient could be defined. As both of them are essentially the same (sometimes with opposite sign, depending on definition), it is rarely used. Reflection coefficient Γ is not constant along the transmission line (surely, because the phases of forward and backward waves are also changing along the line). For the lossless line, magnitude $|\Gamma|$ is constant along the line while for a lossy line, $|\Gamma|$ exponentially decreases when going from the end (where Z_L is placed) to its begining.

Voltage reflection coefficient at the transmission line end depends on impedances

$$\Gamma = \frac{Z_\mathrm{L} - Z_0}{Z_\mathrm{L} + Z_0}$$

There are three remarkable special cases:

- short circuit, $Z_\mathrm{L} = 0$, Γ=-1
- open circuit, $Z_\mathrm{L} = \infty$, Γ=+1
- matched line, $Z_\mathrm{L} = Z_0$, Γ=0

Similarities with Fresnel equations for reflection coefficient in optics and observed inversion of phase when $n_2 > n_1$ are obvious.

Reflection on transmission lines can be described also by other parameters. Historically, voltage standing wave ratio (VSWR) was easily measurable even at very high frequencies. It is a ratio between the maximum (anti-node) voltage and minimum (node) voltage on the line with a standing wave (i.e. superposition of forward and backward waves)

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Notice, that all information about phase shift of reflected wave is lost by using the absolute value, However, the position of the node can be easily measured simultaneously with VSWR using a slotted line technique.

The power reflected at the mismatch is given by Γ^2. This power reflection coefficient is often expressed as *return loss* in logarithmic scale and with a unit of decibel (dB)

$$RL = -20 \log |\Gamma| \qquad \text{[dB]}$$

Theory of two port networks

In many cases in physics and engineering and especially in circuit signal propagation, an abstraction of *two port network* is extremely useful. Consider a arbitrary network consisting of linear elements (typically R, L, C but sometimes even more exotic elements are permitted) which has two well defined ports (each with two poles) to the outside world. Moreover, consider a steady state, either static (DC) or sinusoidal steady state at certain frequency ω. This consideration is actually not very limiting as thanks to the network linearity even arbitrary waveforms can be Fourier separated to a superposition of single frequency signals.

Any such linear two port network can be mathematically described by 2×2 complex matrix which transforms currents and voltages. Physical properties of the network like reciprocity (transmitting the same going from port 1 to port 2 as going from port 2 to port 1), symmetry (same impedances at port 1 and port 2) and losslessness (no resistors to dissipate power) reflect in mathematical properties of the matrix.

There are many possibilities, how to choose which pair of parameters is transformed to the other pair. Among the most used are for example

impedance matrix

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

admitance matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

hybrid matrix, typical in transistor equivalent networks

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

ABCD matrix, also known as chain, cascade or transmission matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For very high frequencies, the current and voltage are ill-defined. For this reason, the abstraction of power waves going in (a) and out (b) of ports 1 and 2 is much more suitable (and curiously, this power wave abstraction works well for all frequencies, even DC). Power and phase of the signal are easily measurable even at optical frequencies.

scattering matrix suitable even for very high frequencies

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Physical meaning of s-parameters is straightforward:

s_{11} corresponds to reflection coefficient Γ at port 1

s_{22} corresponds to reflection coefficient Γ at port 2

s_{21} corresponds to transmission coefficient going from port 1 to port 2

s_{12} corresponds to transmission coefficient going from port 2 to port 1

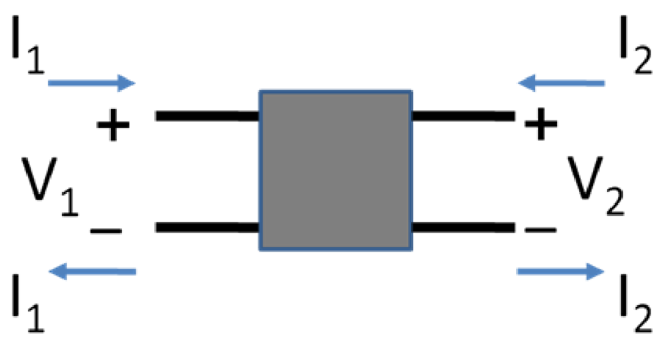
In the four statements above, it is intentionally written 'corresponds to' and not 'is', as termination of unused port must be taken into account.

Mathematically, the s-parameters are transforming complex amplitudes but practically, we are dealing with signal power and phase. Therefore, the absolute value of s-parameter is often expressed in logarithmic scale and in dB units

$$10 \log |S_{ij}|^2 = 20 \log |S_{ij}| \qquad \text{[dB]}$$

For example, *insertion loss*, i.e. loss of power due to attenuation of signal when passing through a device from port 1 to port 2, is defined by

$$IL = -20 \log |S_{21}| \qquad \text{[dB]}$$



Obrázek 3: Two port device – schematics and definition of quantities.

Loss of power due to reflection at input (port 1) is *return loss* (which we already mentioned above)

$$RL = -20 \log |S_{11}|[\qquad [\mathrm{dB}]$$

When both amplitude and phase of s-parameter are to be plotted simultaneously vs frequency, a polar graph or Smith chart are usually chosen.

For reciprocal networks, the S-matrix is symmetric ($s_{21} = s_{12}$); for symmetrical networks $s_{11} = s_{22}$ and for lossless networks the S-matrix is unitary $[S]^* = [[S]^{\mathrm{T}}]^{-1}$. Examples of non-reciprocal devices are those using active components (amplifiers) or chirality breaking devices (e.g. Faraday effect).

For cascaded high frequency devices, an equivalent to ABCD matrix (which is low frequency only, as it uses U, I) should use complex amplitudes of forward and backward waves

T matrix scattering transfer matrix

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

Arbitrarily long cascade of two port devices is also a two port device itself and its resulting T-matrix can be calculated by simple matrix multiplication of T-matrices of each device in the cascade.

Conversion from S-matrix to T-matrix

$$t_{11} = \frac{1}{s_{21}}$$
$$t_{12} = \frac{-s_{22}}{s_{21}}$$
$$t_{21} = \frac{s_{11}}{s_{21}}$$
$$t_{22} = \frac{-\det\left(S\right)}{s_{21}}$$

where determinant $\det(X) = x_{11} \cdot x_{22} - x_{12} \cdot x_{21}$.

Conversion from T-matrix to S-matrix

$$s_{11} = \frac{T_{21}}{T_{11}}$$
$$s_{12} = \frac{\det\left(T\right)}{T_{11}}$$
$$s_{21} = \frac{1}{T_{11}}$$
$$s_{22} = \frac{-T_{12}}{T_{11}}$$

Experimental set-up

The device used for measuring scattering matrix is called *Vector Network Analyser*. In our case, it will be Rohde&Schwarz ZVL, which operates in frequency range 10 kHz – 15 GHz and has two ports. It is bidirectional, as it can measure the complex 2×2 S-matrix in one go, without any device reconnecting or reorienting.

Before the actual measurement, the VNA must be calibrated by certified precision calibration kit, using Open, Short and Match for each port and then Through between the ports.

In most tasks, we will use devices with SMA connector, which is affordable and should be usable up to 18 GHz. Nearly all devices-under-test we will use are passive, reciprocal and symmetric, which should essentially reduce number of measurements by factor 2..

Tasks

1. get acquainted with the VNA
2. calibrate the VNA in the range 10 kHz–10 GHz
3. measure the S-parameters of simple SMA-to-SMA piece, discuss frequency response, reciprocity, symmetry, losslessness
4. verify the suitability of BNC connectors
5. verify the suitability of N connectors
6. using T-matrix, remove the influence of SMA-to-BNC pieces
7. observe S-matrix of BNC T-piece with and without open coaxial cable, find resonances, calculate lengths
8. terminate the cable from the previous task, compare theoretical and experimental results
9. determine an unknown RLC impedance from measured S parameters
10. repeat previous task, but with longer cable, remove its effect by T-matrix
11. determine the dimensions of waveguide from transmission curve
12. measure the gain and operating frequency range of microwave amplifier

Recommended literature

D. Pozar: Microwave engineering
Tysl, Růžička: Teoretické základy mikrovlnné techniky

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