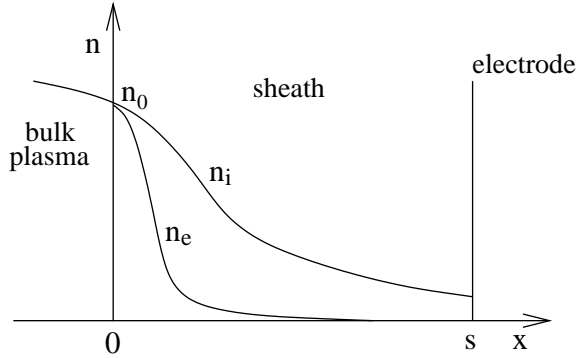


1 DC sheath bez srážek, Bohmova rychlost



$$\Phi(0) = 0 \quad (1)$$

$$\frac{d\Phi}{dx} \approx 0 \quad (2)$$

$$n_e = n_0 e^{\frac{q\Phi}{kT_e}} \quad (3)$$

$$n_i v_i = n_0 v_B \quad (4)$$

$$\frac{1}{2} m_i v_i^2 + q\Phi = \frac{1}{2} m_i v_B^2 \quad (5)$$

Řešíme Poissonovu rovnici $\Delta\Phi = -\frac{\rho}{\varepsilon_0}$:

$$\frac{d^2\Phi}{dx^2} = -\frac{q}{\varepsilon_0} (n_i - n_e) = \frac{n_0 q}{\varepsilon_0} \left(e^{q\Phi/kT_e} - \frac{1}{\sqrt{1 - \frac{2q\Phi}{m_i v_B^2}}} \right) \quad (6)$$

$$\frac{d}{dx} \left(\frac{d\Phi}{dx} \right)^2 = 2 \frac{d\Phi}{dx} \frac{n_0 q}{\varepsilon_0} \left(e^{q\Phi/kT_e} - \frac{1}{\sqrt{1 - \frac{2q\Phi}{m_i v_B^2}}} \right)$$

$$E^2 - E_0^2 = \frac{2n_0 q}{\varepsilon_0} \int_0^\Phi \left(e^{q\Phi'/kT_e} - \frac{1}{\sqrt{1 - \frac{2q\Phi'}{m_i v_B^2}}} \right) d\Phi' \quad (7)$$

Protože levá strana rovnice (7) je nezáporná a $d\Phi' < 0$, musí platit

$$e^{q\Phi/kT_e} - \frac{1}{\sqrt{1 - \frac{2q\Phi}{m_i v_B^2}}} \leq 0,$$

z čehož vyplývá

$$v_B^2 \geq \frac{1}{m_i} \frac{2q\Phi}{1 - e^{-2q\Phi/kT_e}}.$$

Tato nerovnost musí platit i pro malé hodnoty potenciálu Φ , takže

$$v_B^2 \geq \frac{kT_e}{m_i}. \quad (8)$$

Rychlost iontů $\sqrt{kT_e/m_i}$ se nazývá Bohmova rychlost.

2 Plovoucí potenciál

$$\begin{aligned} \frac{1}{4} n_0 \sqrt{\frac{8kT_e}{\pi m_e}} e^{\frac{q\Phi_{fl}}{kT_e}} &= n_0 \sqrt{\frac{kT_e}{m_i}} \\ q\Phi_{fl} &= -\frac{kT_e}{2} \ln \frac{m_i}{2\pi m_e} \end{aligned} \quad (9)$$

3 Child-Langmuirův zákon pro bezsrážkový sheath

Rovnici (7) upravíme pro předpoklady $n_e \approx 0$, $\frac{1}{2}m_i v_B^2 \ll q\Phi$ a $E_0 \ll E$ a za využití $j = qn_0 v_B$ na

$$\begin{aligned}
 E^2 &\approx -\frac{2n_e q}{\varepsilon_0} \int_0^\Phi \frac{d\Phi'}{\sqrt{-\frac{2q\Phi'}{m_i v_B^2}}} = -\frac{\sqrt{2qm_i}}{\varepsilon_0} n_0 v_B \int_0^\Phi \frac{d\Phi'}{\sqrt{-\Phi'}} = \frac{j}{\varepsilon_0} \sqrt{\frac{2m_i}{q}} 2\sqrt{-\Phi} \\
 \frac{d\Phi}{dx} &= -\sqrt{\frac{j}{\varepsilon_0} 2\sqrt{\frac{2m_i}{q}}} (-\Phi)^{\frac{1}{4}} \\
 (-\Phi)^{\frac{3}{4}} &= \frac{3}{2} \sqrt{\frac{j}{\varepsilon_0} \sqrt{\frac{m_i}{2q}}} x \\
 j &= \frac{4\varepsilon_0}{9} \sqrt{\frac{2q}{m_i}} \frac{U_{sh}^{\frac{3}{2}}}{s^2}
 \end{aligned} \tag{10}$$

4 Srážkový sheath

$$j = qn_i v_i = qn_i \mu_i E \tag{11}$$

Pro jednoduchost předpokládejme konstantní pohyblivost iontů μ_i :

$$\begin{aligned}
 \frac{d^2\Phi}{dx^2} &= -\frac{qn_i}{\varepsilon_0} = -\frac{j}{\varepsilon_0 \mu_i \frac{d\Phi}{dx}} \\
 2 \frac{d\Phi}{dx} \frac{d^2\Phi}{dx^2} &= \frac{2j}{\varepsilon_0 \mu_i} \\
 \frac{d\Phi}{dx} &= -\sqrt{\frac{2j}{\varepsilon_0 \mu_i}} x \\
 -\Phi &= \frac{2}{3} \sqrt{\frac{2j}{\varepsilon_0 \mu_i}} x^{\frac{3}{2}} \\
 j &= \frac{9\varepsilon_0 \mu_i}{8} \frac{U_{sh}^2}{s^3}
 \end{aligned} \tag{12}$$

5 Sheath s konstantní koncentrací iontů

$$E = \frac{qn_0 x}{\varepsilon_0} \tag{13}$$

$$U_{sh} = \frac{qn_0 s^2}{2\varepsilon_0} \tag{14}$$