APPENDIX 1

The steps in derivation of the probability p(n,x) for the production of an avalanche of n electrons at the distance x from the cathode are as follows:

Let N(x) is the number of electrons emitted from the cathode which pass the distance x' from the cathode without any ionizing collision. Then

$$dN(x') = -\alpha.N(x)dx'$$

$$N(x') = N_0 .\exp(-\alpha x')$$

where N_0 = N(x'=0). As a consequence of this

$$p(1,x') = N(x')/N_0 = \exp(-\alpha x') \qquad (1)$$

Let the probability that the avalanche contains n-1 at x' is

$$p(n-1,x') \qquad (2)$$

The probability that one and only one of these electrons will ionize in the region between x' and x'+dx' can be found from the binomial distribution considering that W(k,l) = p(n-1, 1), and y = α.dx .Thus

$$p(n-1,1) = (n-1).\alpha.dx'(1-\alpha.dx')^{n-2}$$

for dx' ≈ 0

$$p(n-1,1) \cong (n-1).\alpha.dx' \qquad (3)$$

The number of electrons in the avalanche has now increased from n-1 to n. The probability that none of these electrons will ionize in the region between x'+dx'(≅ x') and x is:

$$[p(1,x-x')]^n = [\exp\{-a.(x-x')\}]^n = \exp\{-n.a.(x-x')\} \qquad (4)$$

where p(1,x-x') is the probability that a single electron will not ionize between x' and x. If we take the product of expressions (2),(3) and (4), and integrate over x ,for n>1, we get

$$p(n,x) = \int_o^x p(n-1,x').p(n-1,1).[p(1,x-x')]^n dx'$$

$$p(n,x) = \int_o^x p(n-1,x').(n-1).\alpha.\exp\{-n.\alpha.(x-x')\}dx' \qquad (5)$$

The solution of the equation (5) is:

$$p(n,x) = \exp(-n.\alpha.x).\left(\exp\{\alpha.x\}-1\right)^{n-1} =$$

$$= \left(\frac{1}{n}\right)^n.\left(n-1\right)^{n-1} = \frac{1}{n}.\left(1-\frac{1}{n}\right)^{n-1}$$