3. Paschen Law

FB242 Gas discharges: physical mechanisms and applications

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Self -sustained discharges

Different stages of the discharge current at steady state:

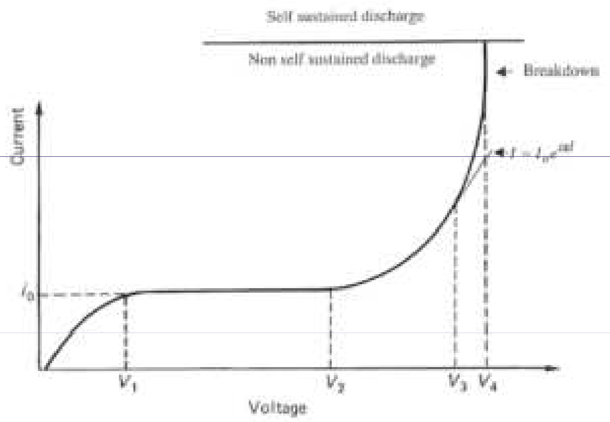
1. Initial increase in current. (Electron Collection efficiency increases)

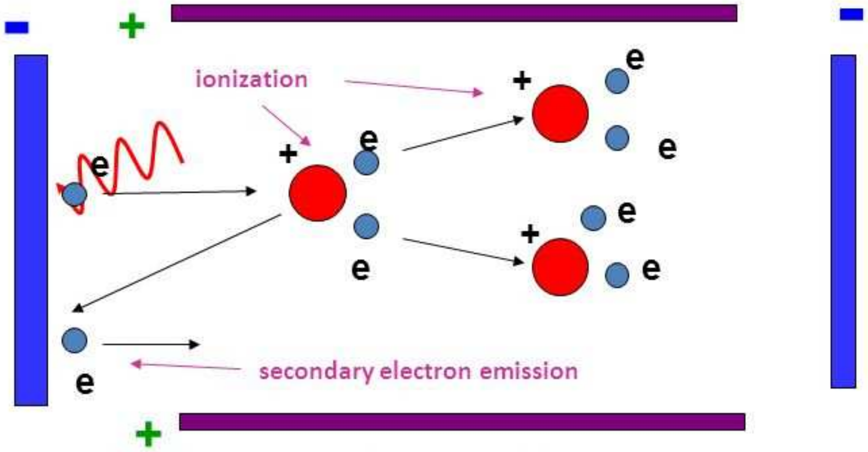
2. Current remains constant. (All the electrons reach the anode)

3. Current increases exponentially. (Primary Ionization sets in)

4. Current increases faster than exponential. (Secondary ionization set in)

5. Discharge becomes self sustained.

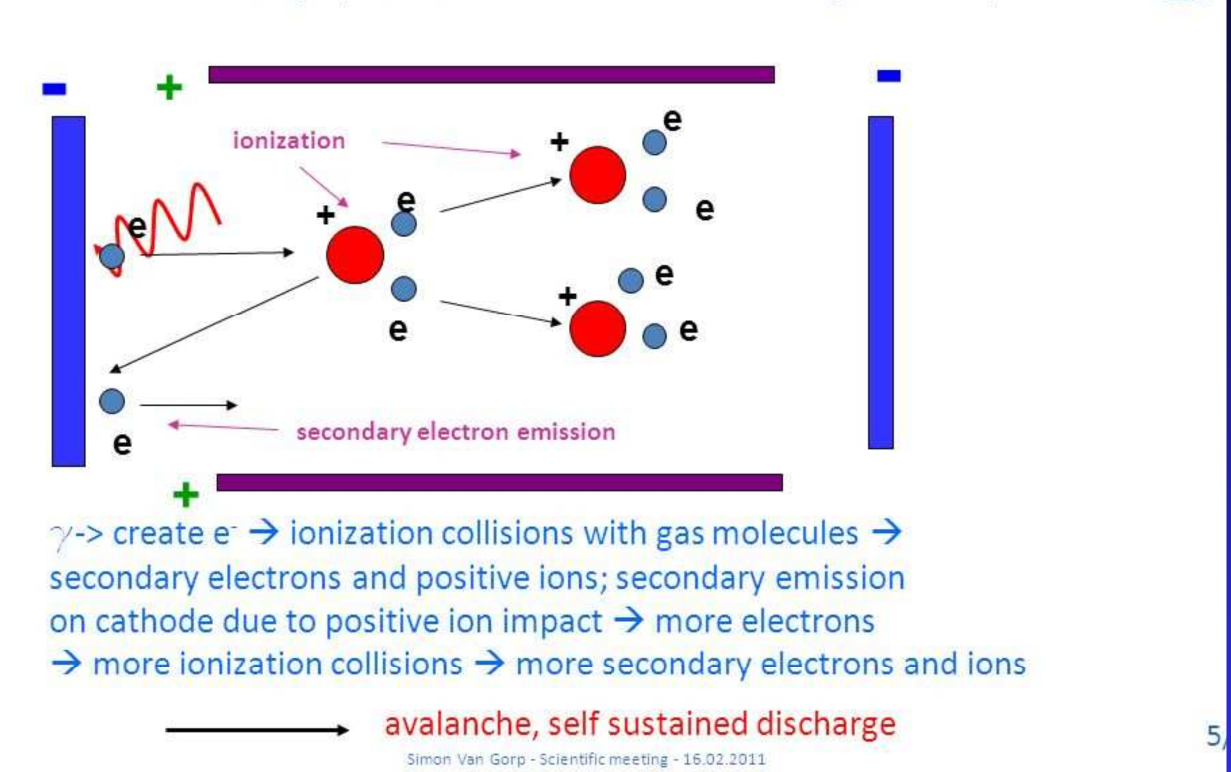




γ-> create e^- → ionization collisions with gas molecules → secondary electrons and positive ions; secondary emission on cathode due to positive ion impact → more electrons → more ionization collisions → more secondary electrons and io

→ avalanche, self sustained discharge

Townsend secondary ionization coefficient γ



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→ avalanche, self sustained discharge

Simon Van Gorp - Scientific meeting - 16.02.2011

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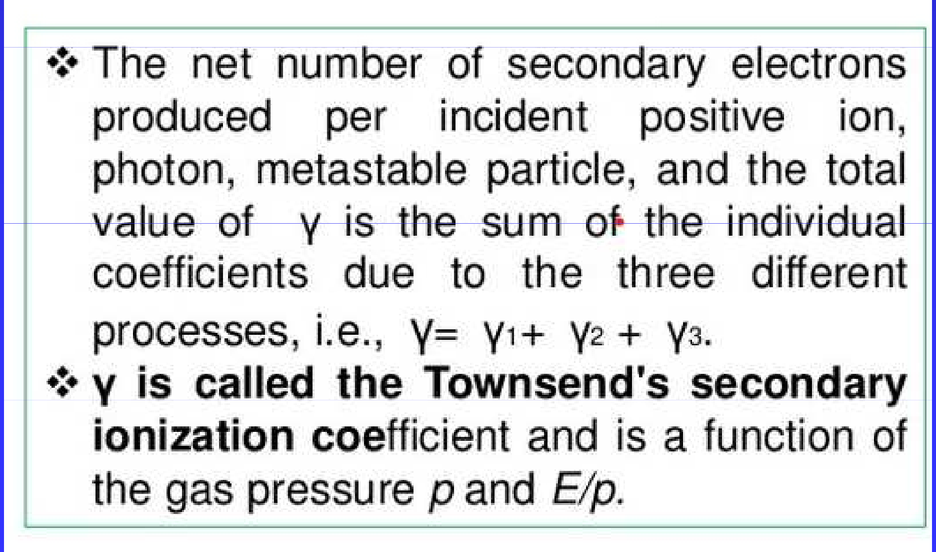
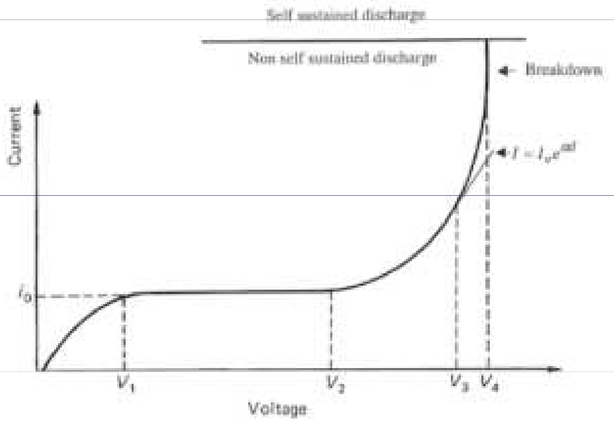


- The net number of secondary electrons produced per incident positive ion, photon, metastable particle, and the total value of γ is the sum of the individual coefficients due to the three different processes, i.e., $\gamma = \gamma_1 + \gamma_2 + \gamma_3$.
- **γ is called the Townsend's secondary ionization coe**fficient and is a function of the gas pressure *p* and *E/p*.

Townsend criterion for self-sustained discharge

Different stages of the discharge current at steady state:



1. Initial increase in current. (Electron Collection efficiency increases)

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Let n be the number of electron moving towards the anode at distance x. After moving a distance of *dx* the number of electrons increases by *dn*

$$dn = n\alpha dx$$

If the number of electrons leaving the cathode per second is n_0 and the number reaching the anode is n_d, the solution of the above equation (neglecting electron attachment) is

$$n_d = n_o e^{\alpha d}$$

Consequently, The current inside the tube is given by

$$I_d = I_o e^{\alpha d}$$

Note that the discharge is not self sustained. That is, it needs the support of the external agency for its continuation.

Townsend assumed that at this stage positive ions striking the cathode have enough energy to liberate electrons from the cathode. Let γ be the number of electrons liberated by a positive ion from the cathode.

Let n_+ be the number of electrons liberated from the cathode per second by positive ion bombardment. Then the number of electrons reaching the anode at steady state per second is

$$n = (n_o + n_+)e^{\alpha d}$$

The number of positive ions created per second by this electron flow is

$$n_p = n - (n_o + n_+)$$

Thus

$$n_+ = \{n - (n_o - n_+)\}\gamma$$

Note that the current is not self-sustained

Substituting this in the first Equation we obtain

$$n = \frac{n_o e^{\alpha d}}{1 - \gamma\left(e^{\alpha d} - 1\right)}$$ leading to $$I = \frac{I_o e^{\alpha d}}{1 - \gamma\left(e^{\alpha d} - 1\right)}$$

γ is the Townsend's secondary ionisation coefficient.

In addition to positive ion bombardment there are other physical processes taking place inside the discharge tube that increase the number of electrons, for example

Ionization by positive ions (not that efficient)

Photo emission from the cathode

Ionization of the gas by photons

Incident of metastable ions on the cathode

Irrespective of the secondary ionisation process under consideration the final expression for the current has the same form. Indeed one can include all of them in a single formula as follows:

$$i = i_o \frac{e^{\alpha d}}{1 - \gamma_i\left[e^{\alpha d} - 1\right]}$$

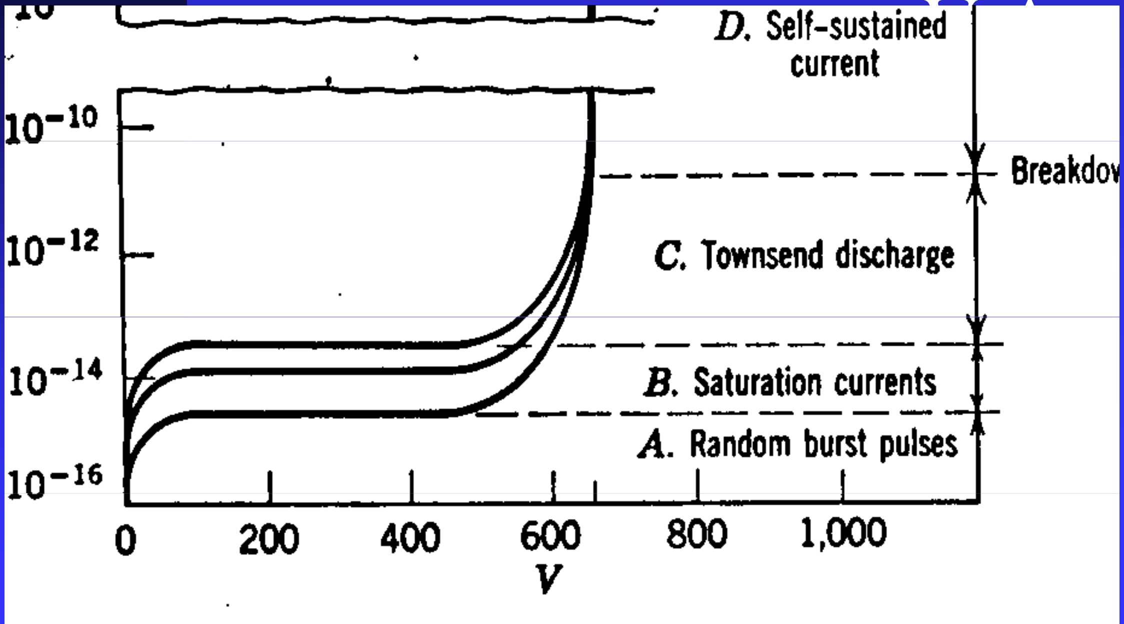
At stationary (i,e, stable) self-sustained discharge the discharge current *I* **is no more dependent on the external ionization current I_0**

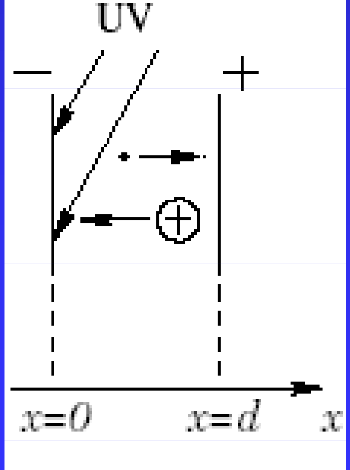
$$I = I_0 e^{\alpha d} / (1-\gamma(e^{\alpha d} - 1))$$

is growing mathematically infinitely. Physically, however:

$\gamma(e^{\alpha x} - 1) \geq 1$ **Townsend criterion for self-sustained discharge**

V-A characteristics for three different I_0 **values :**





Secondary electron emission

is critical for the existence od self-sustained discharges

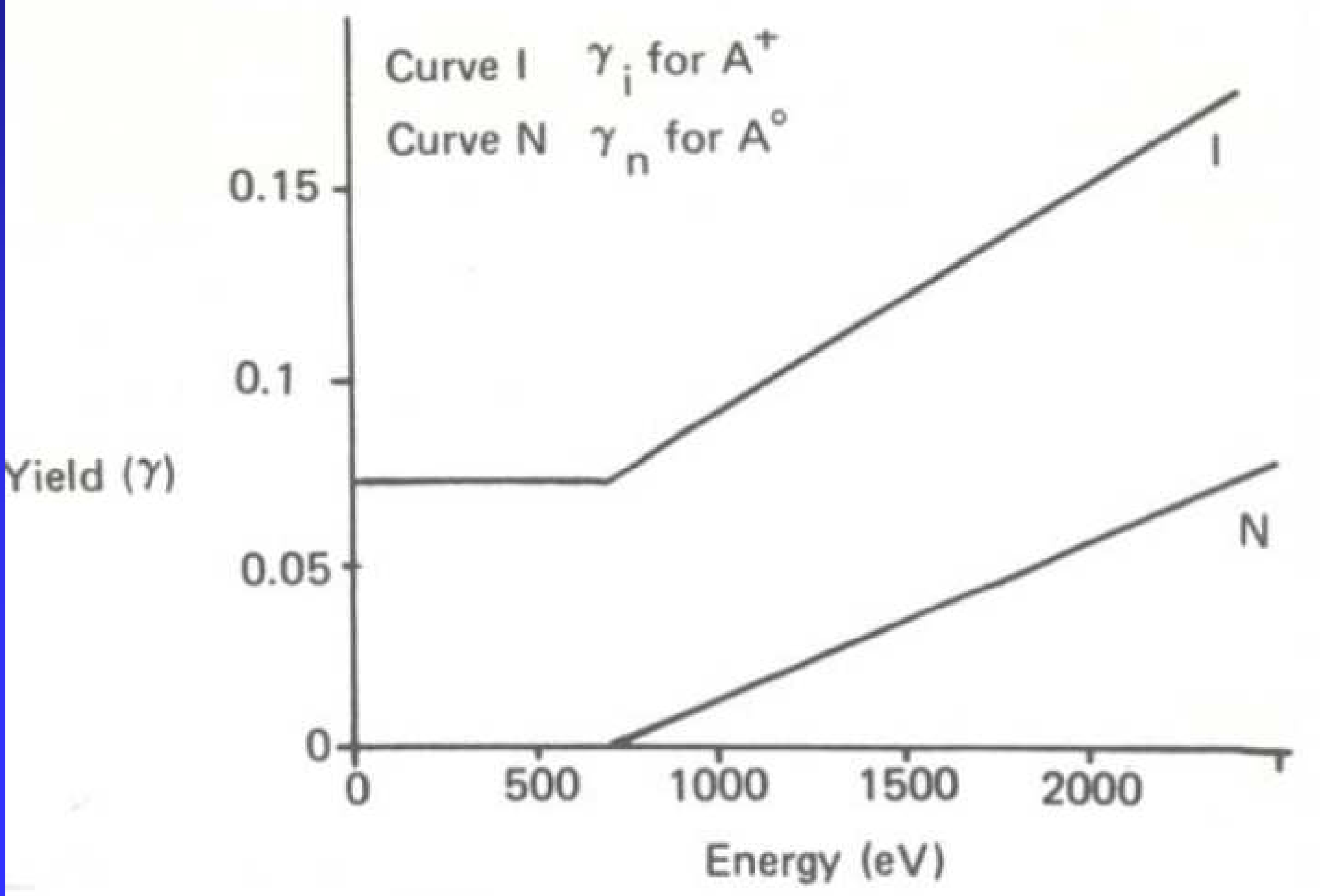
- There are two categories of the electron emission from the cathode
- Secondary el. emission due to the bombardmend of cathode by partices generated by the collision of primary electrons wit the gas atoms and molecules
  - Positive ions (ionization energy > $2E_w$)
  - Photons
  - Excited metastable atoms and molecules

  Collective electron emission, the first of all:
  - Termoemission
  - Electron emission by strong el. field (cold emission)

Sec. el. mission by positive ions is due to their potential, not kinetic emergy !

D. B. Medved, P. Mahadevan and J. K. Layton, *Phys. Rev.* **129**, 2086 (1963).

Figure 1.3: Secondary electron emission coefficient as a function of energy for argon ion and atom bombardment of molybdenum [66].



SECONDARY ELECTRON EMISSION **219**

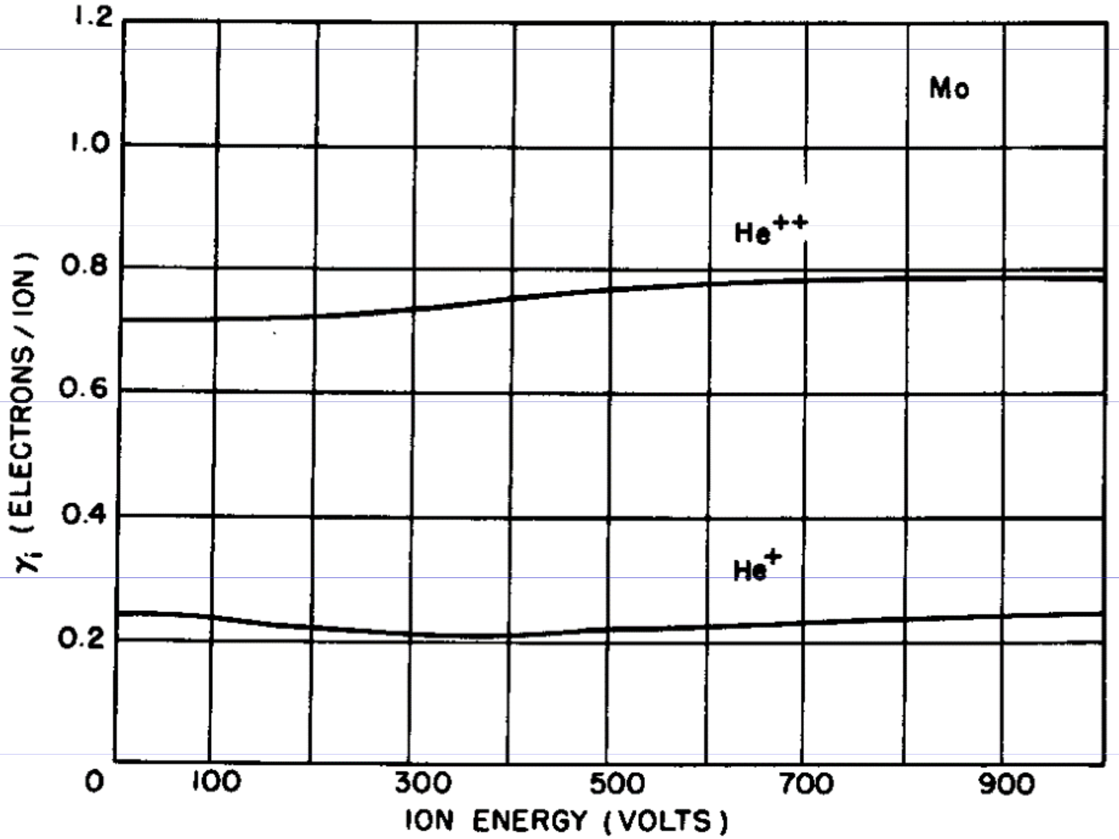
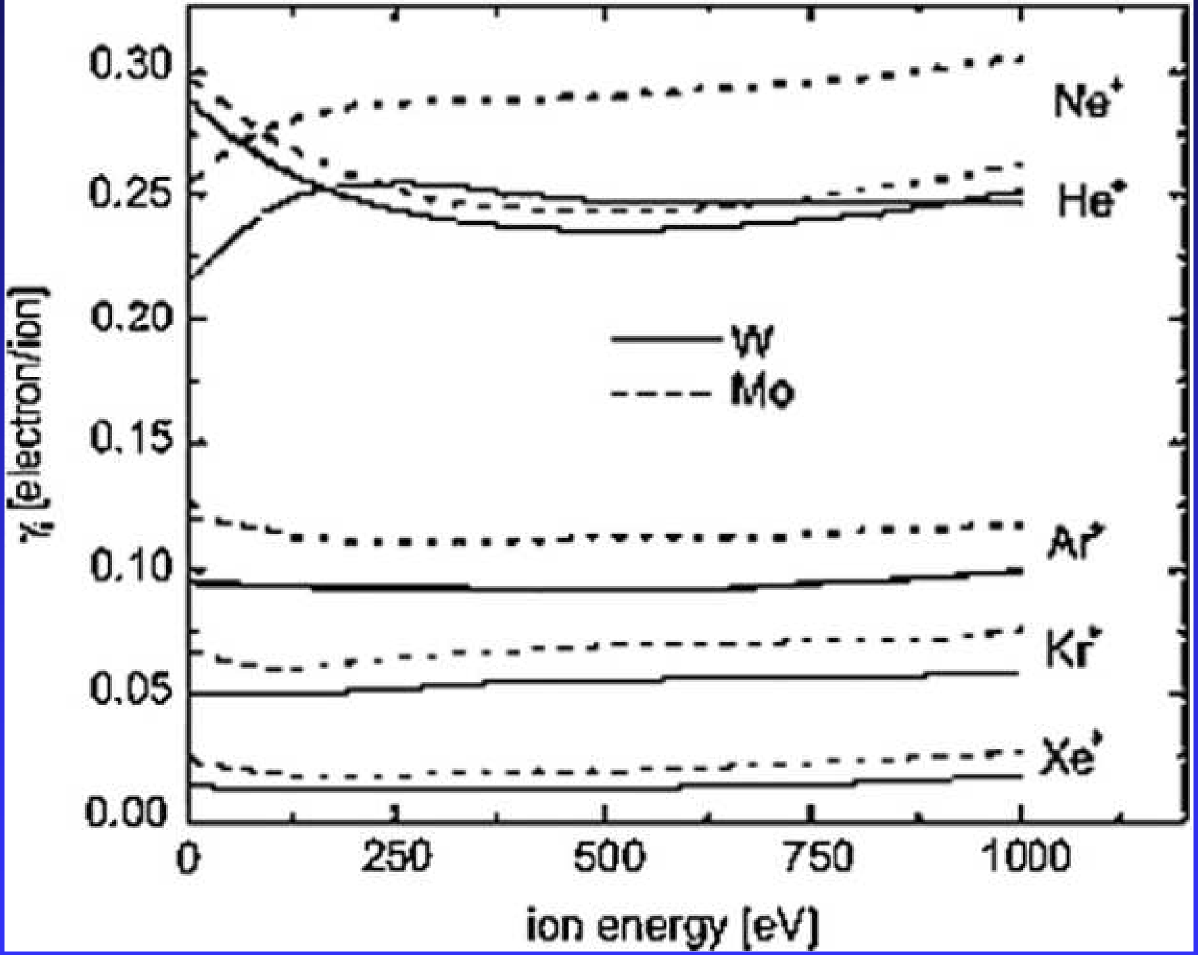


Fig. 7.9 Total electron yield of He^{++} and He^{+} ions on atomically clean molybdenum.

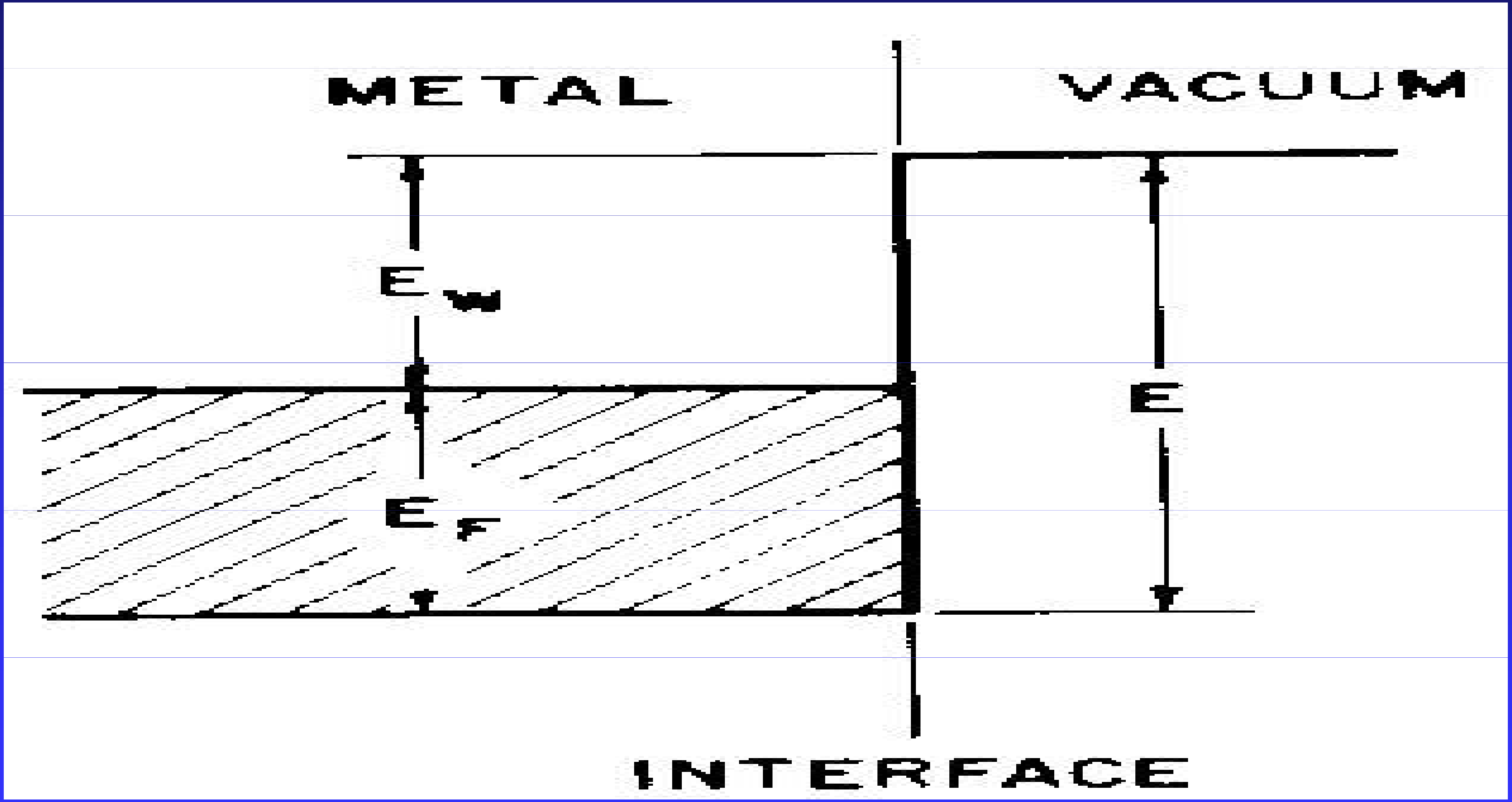
H. D. Hagstrum (1953)

PHYREV J1 V89 P244

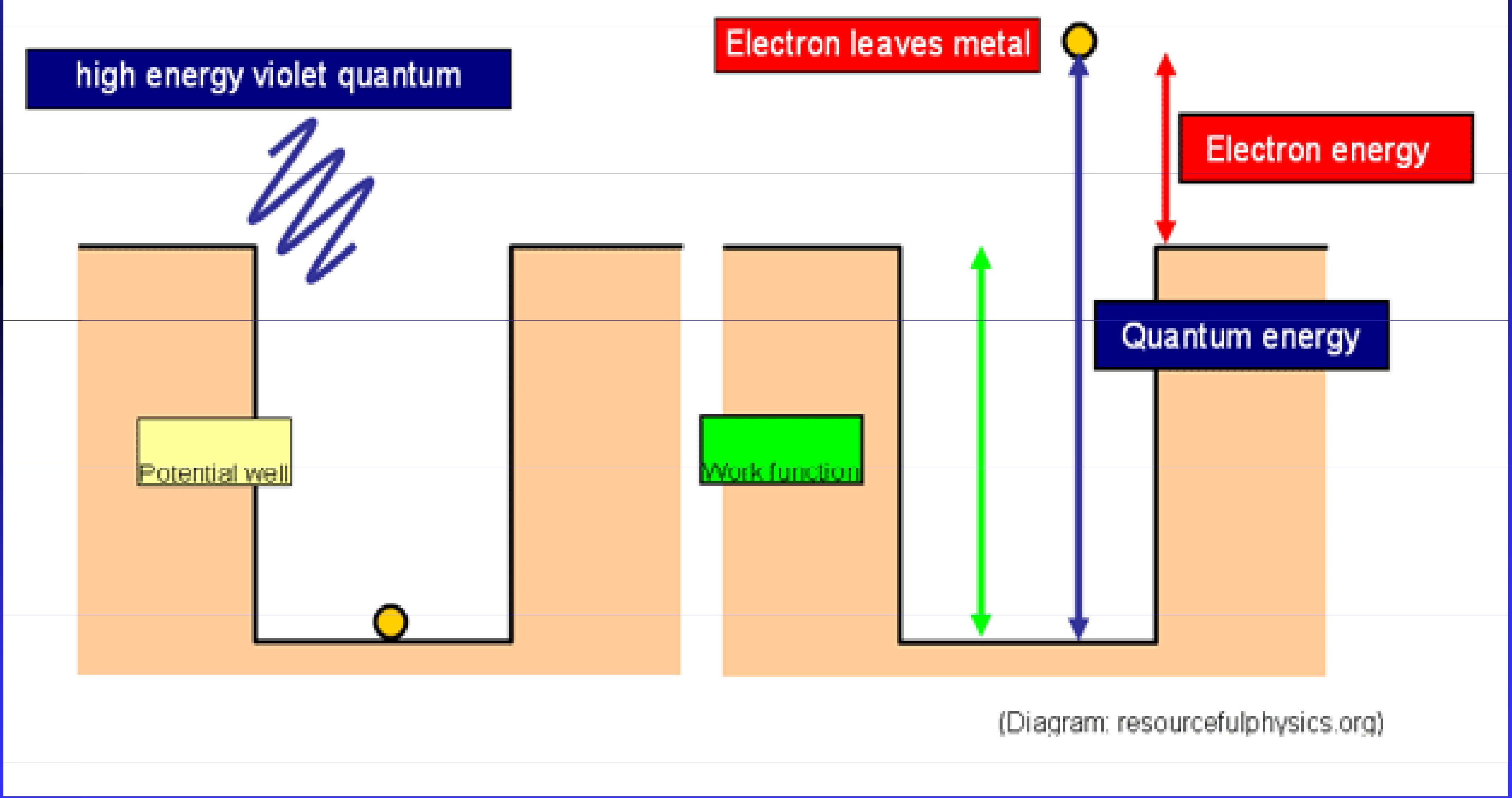
Effect of the cathode material (ionization energy > $2E_w$)



Work function of metals E_w



Photoemission

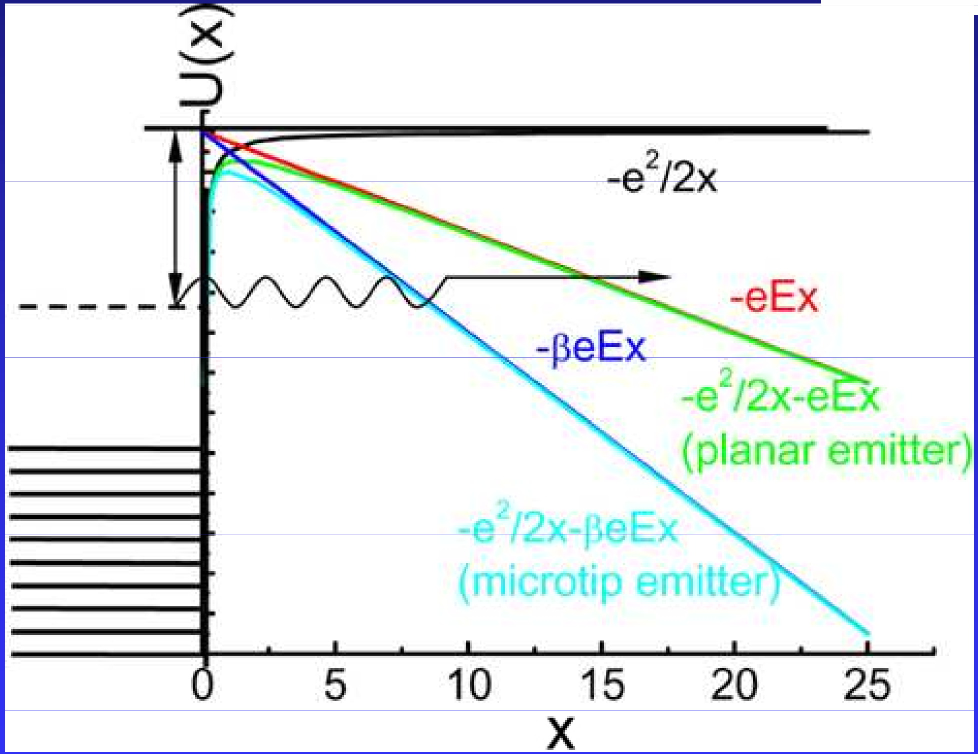


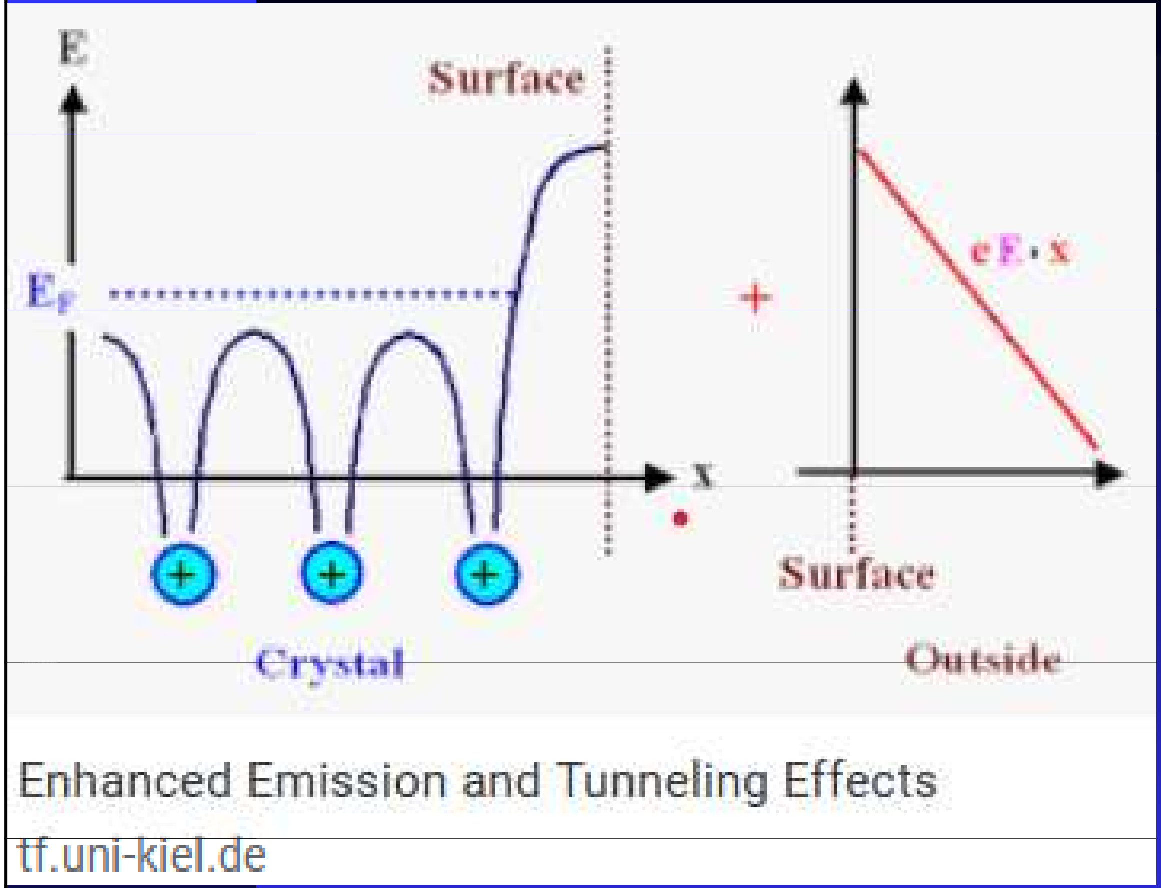
(Diagram: resourcefulphysics.org)

Field emission – tunel effect

Fowler-Nordheim equation rovnica:

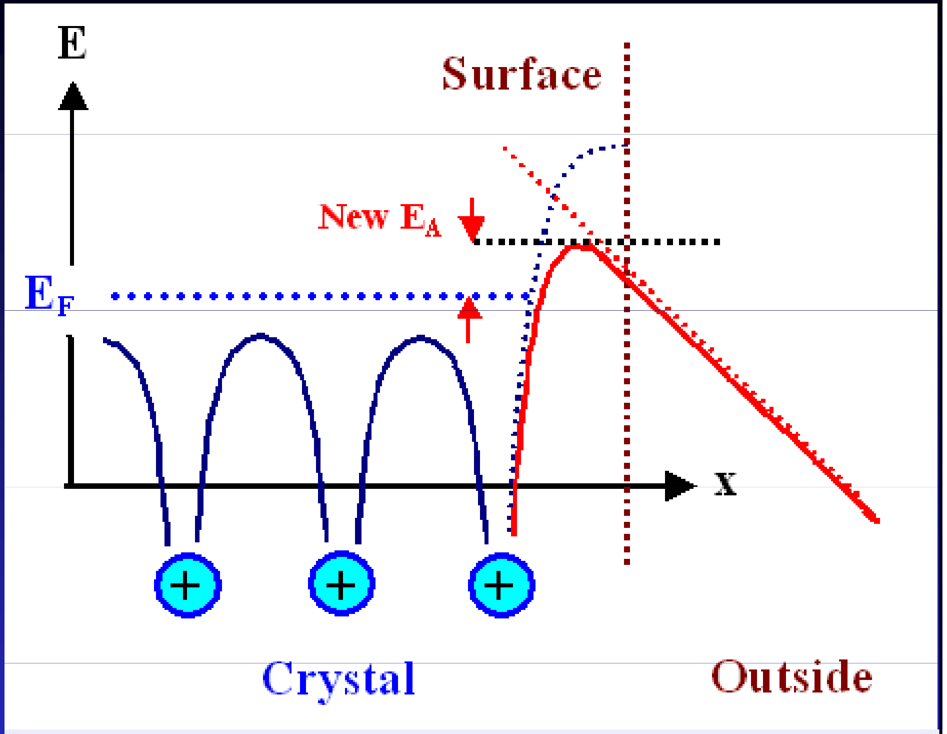
$$J = A\frac{(\beta E)^2}{\phi}\exp\left(-\frac{\beta\phi^{3/2}}{\beta E}\right)$$

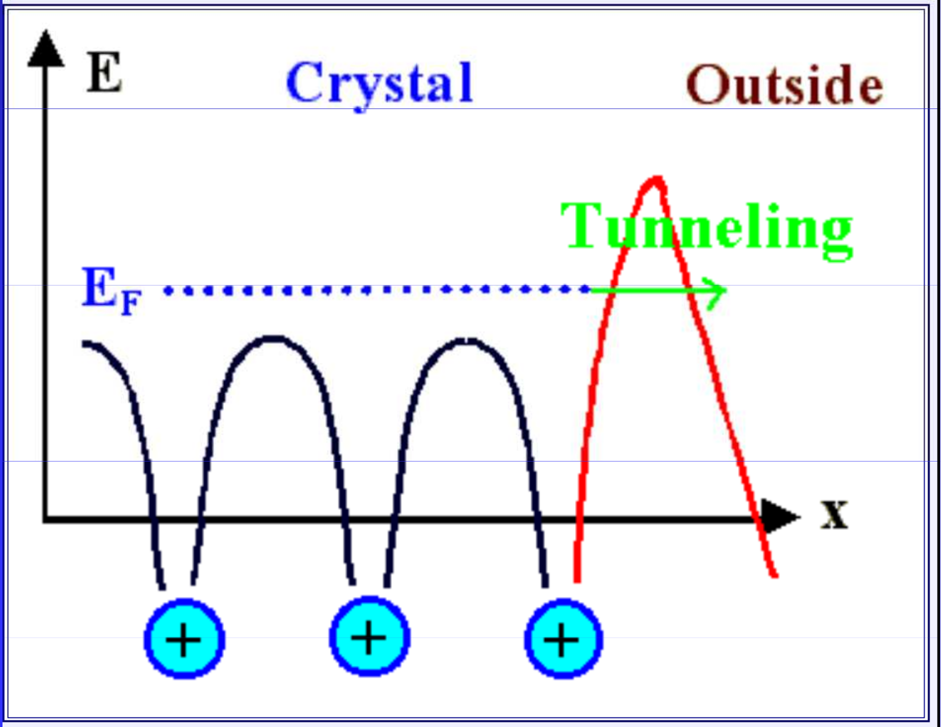




Enhanced Emission and Tunneling Effects

tf.uni-kiel.de



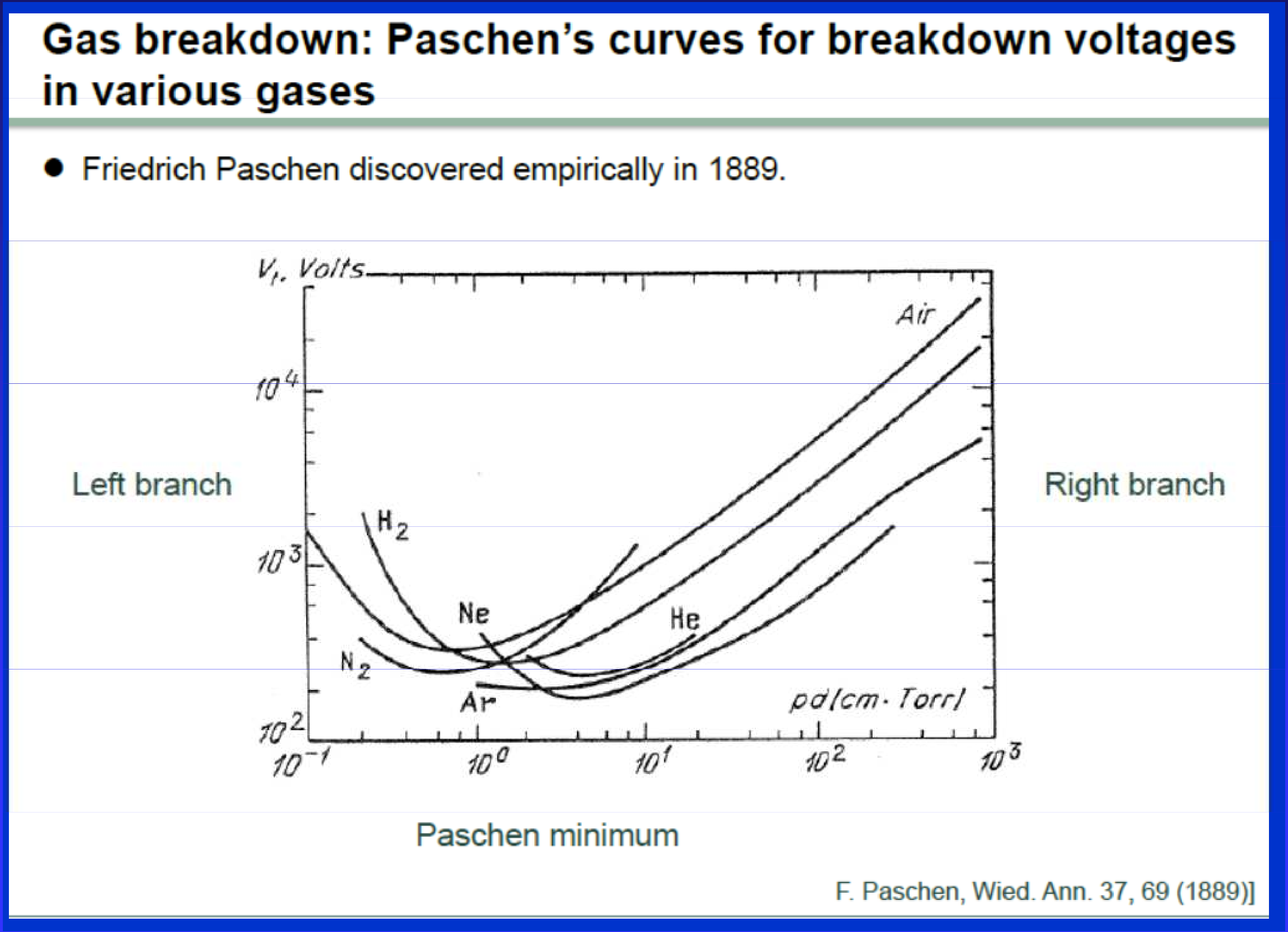


https://www.tf.uni-kiel.de/matwis/amat/elmat_en/kap_2/backbone/r2_3_2.html

$\gamma(e^{\alpha x} - 1) \geq 1$ Townsend ctiterion for self-sustained discharge

Paschen´s law – Paschen´s curve

Paschen’s law indicates that under a uniform electric field, for various discharge gaps with different values of gas pressure and gap distance, the breakdown voltage will be the same if the product pd is kept the same.



Gas breakdown: Paschen’s curves for breakdown voltages in various gases

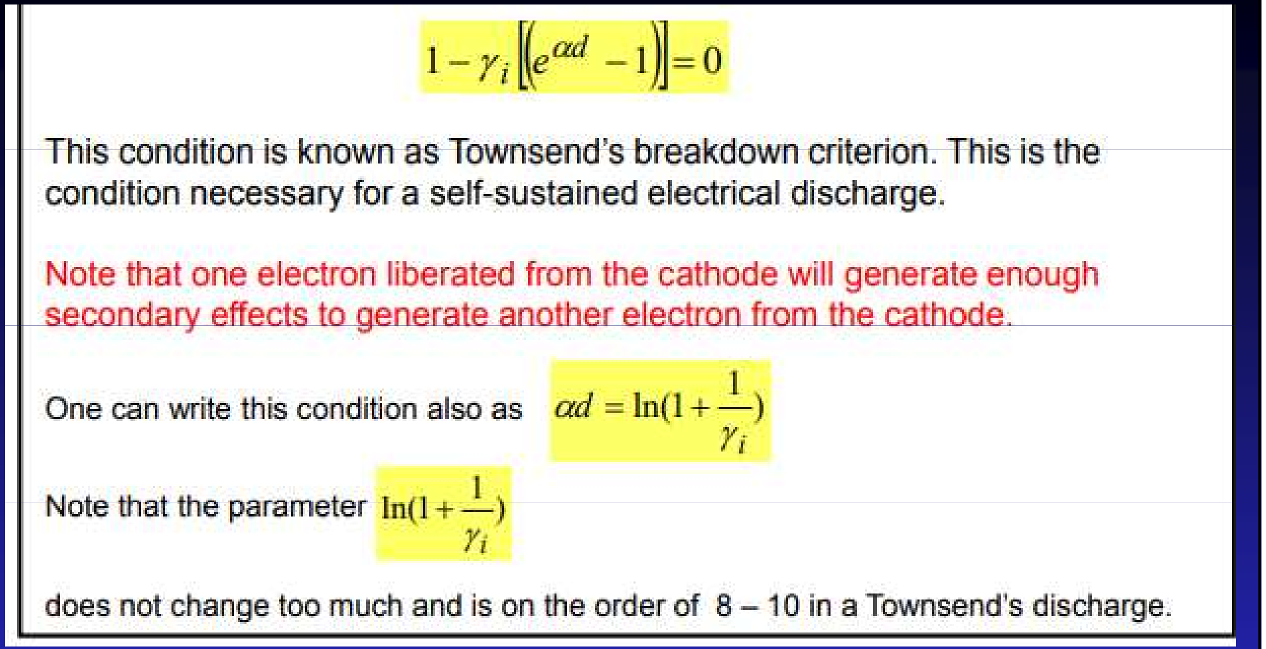
- Friedrich Paschen discovered empirically in 1889.

Left branch

Right branch

Paschen minimum

F. Paschen, Wied. Ann. 37, 69 (1889)]



$$1-\gamma_i\left[\left(e^{\alpha d}-1\right)\right]=0$$

This condition is known as Townsend's breakdown criterion. This is the condition necessary for a self-sustained electrical discharge.

Note that one electron liberated from the cathode will generate enough secondary effects to generate another electron from the cathode.

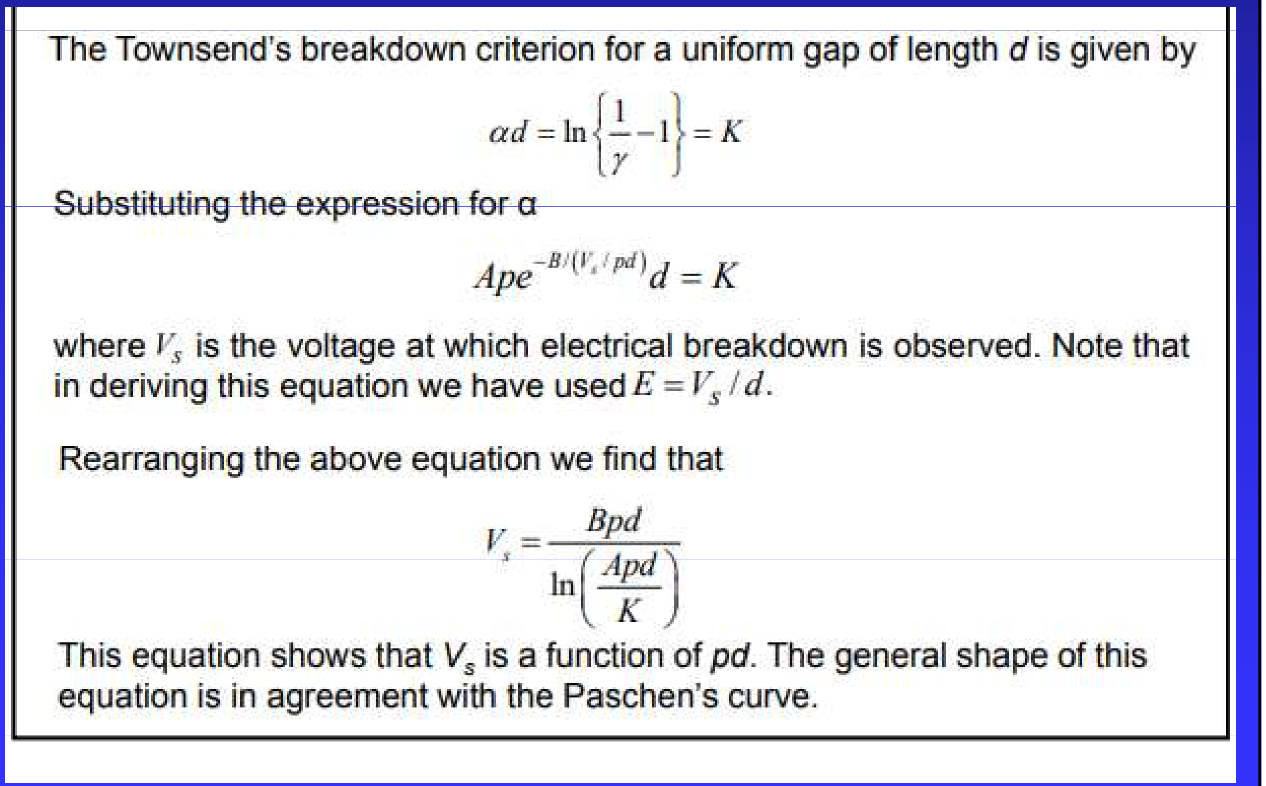
One can write this condition also as $\alpha d=\ln(1+\frac{1}{\gamma_i})$

Note that the parameter $\ln(1+\frac{1}{\gamma_i})$

does not change too much and is on the order of 8 – 10 in a Townsend's discharge.

$$\frac{\alpha}{p}=Ae^{-B/(E/p)}$$

$$E = V_s/d$$



The Townsend's breakdown criterion for a uniform gap of length *d* is given by

$$\alpha d=\ln\left\{\frac{1}{\gamma}-1\right\}=K$$

Substituting the expression for α

$$Ape^{-B/(V_s/pd)}d=K$$

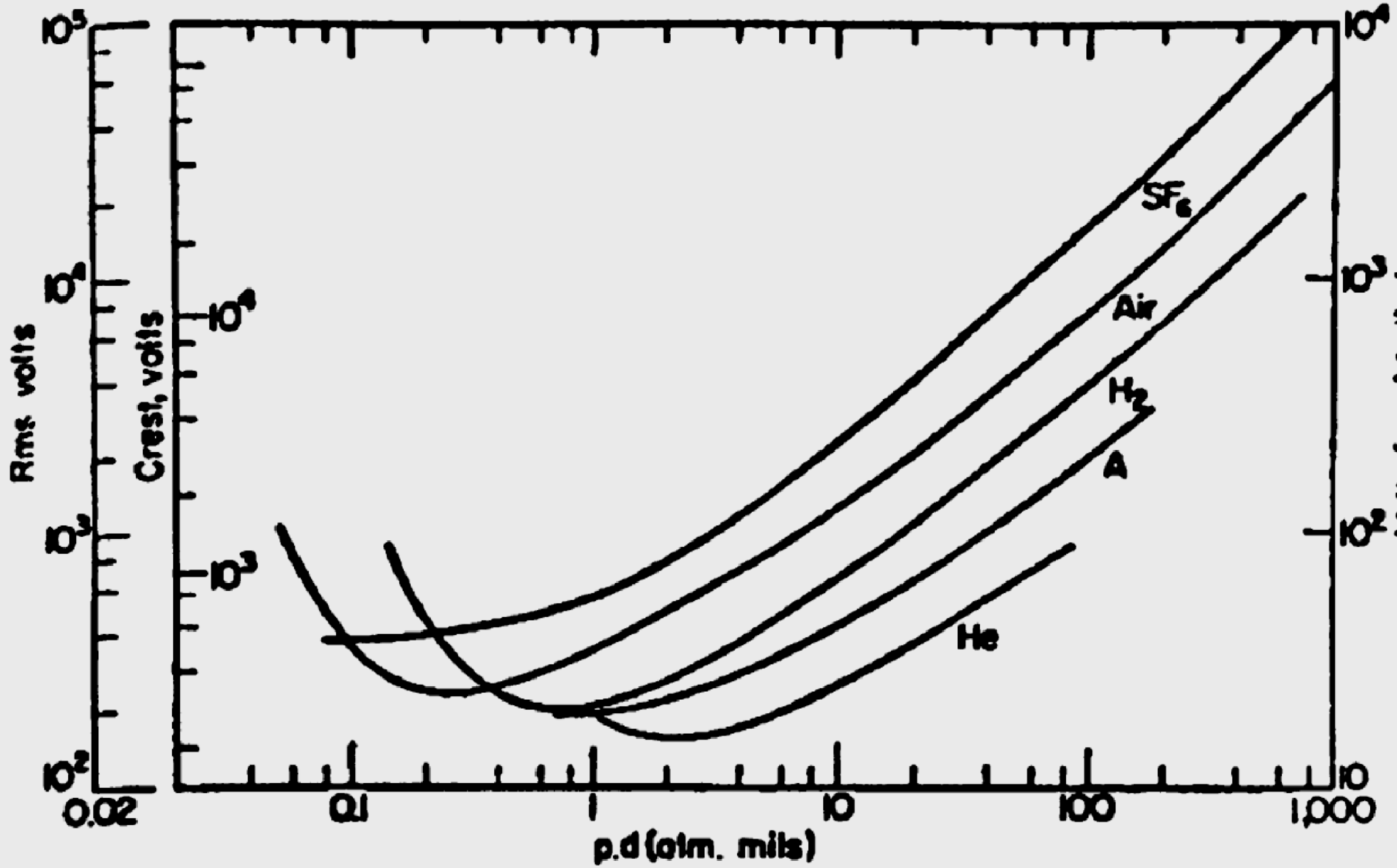
where V_s is the voltage at which electrical breakdown is observed. Note that in deriving this equation we have used $E=V_s/d$.

Rearranging the above equation we find that

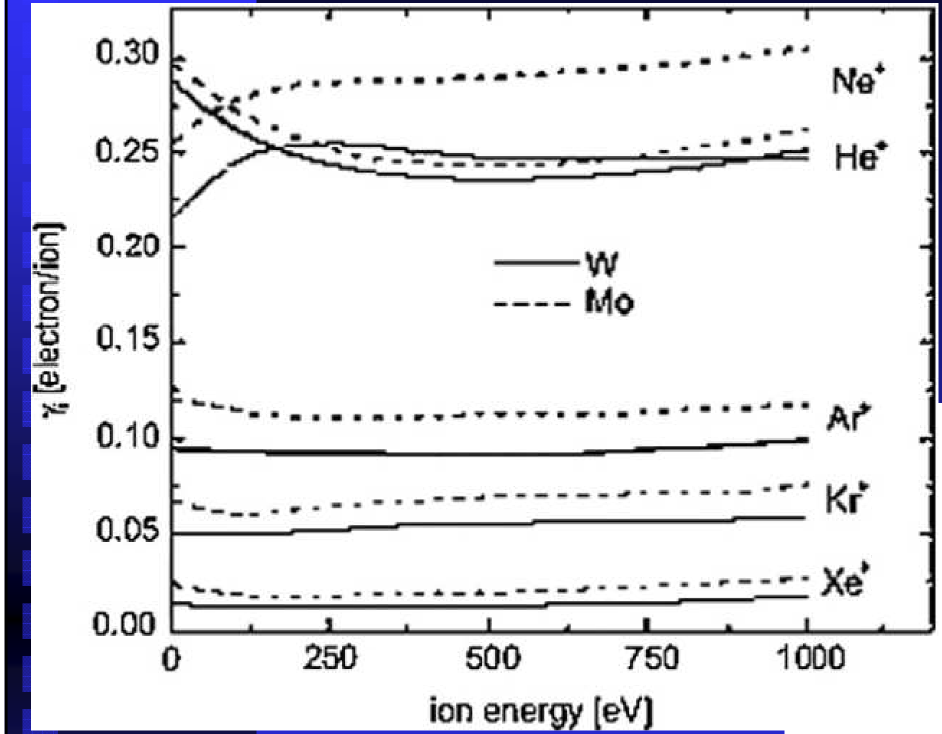
$$V_s=\frac{Bpd}{\ln\left(\frac{Apd}{K}\right)}$$

This equation shows that V_s is a function of *pd*. The general shape of this equation is in agreement with the Paschen's curve.

Paschen´s curves for different gases



Paschen´s curves for different cathode materials



$$U_z = \frac{A.pd}{\ln\left[\dfrac{B.pd}{\ln\left(+\dfrac{1}{\gamma}\right)}\right]} = f(pd)$$

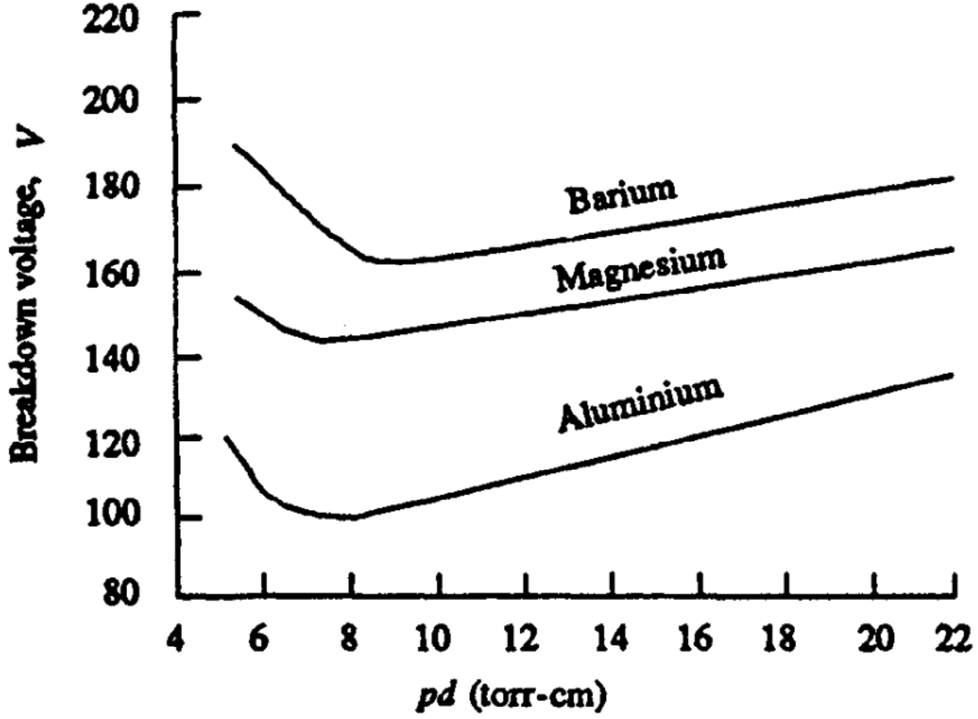
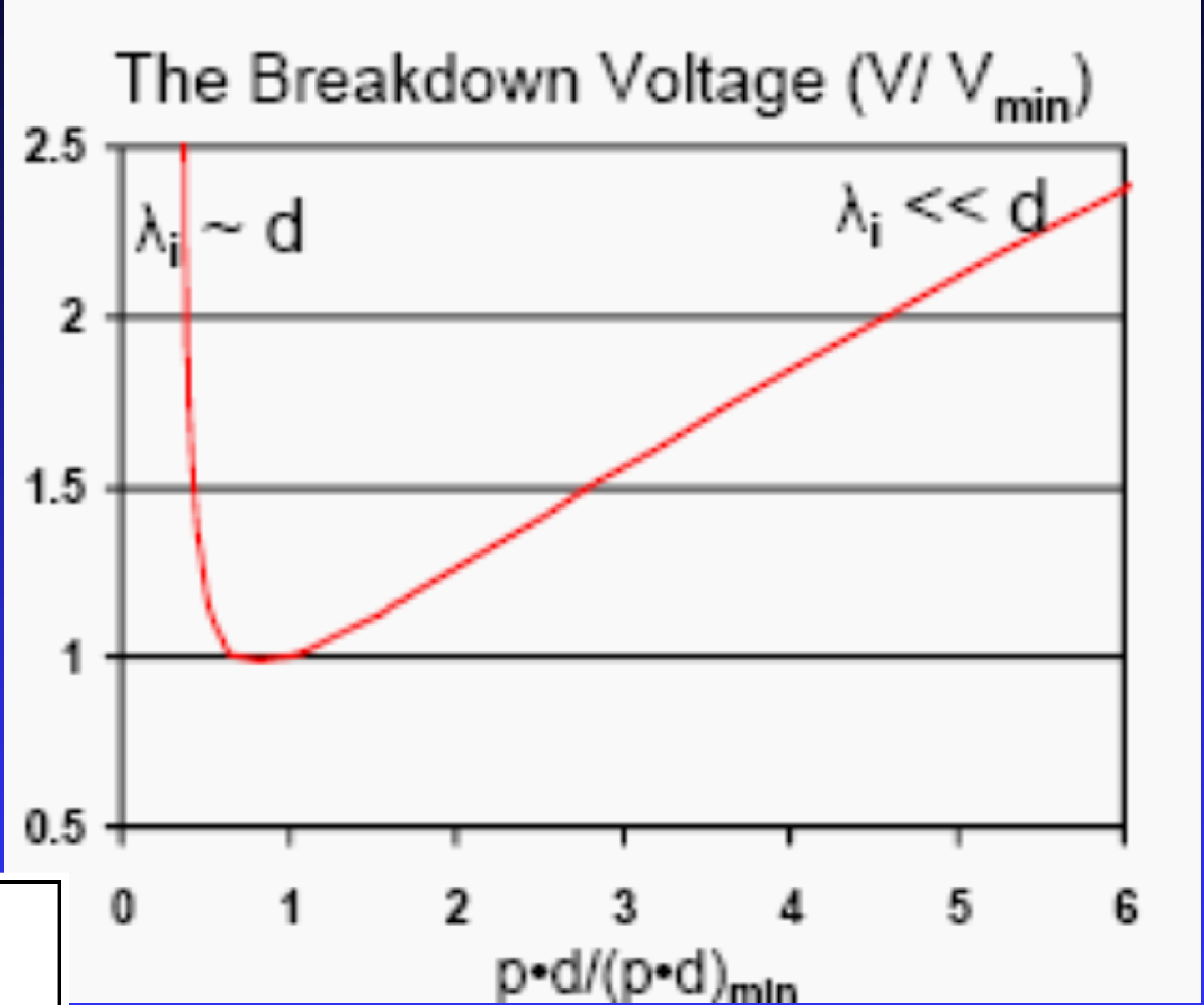
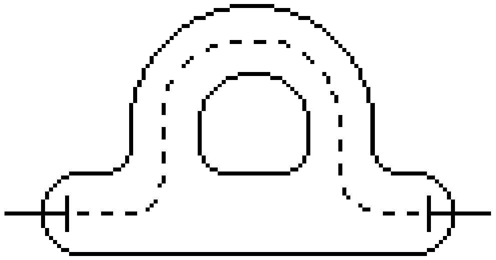


Fig. 2.15 Dependence of breakdown voltage on the cathode materials



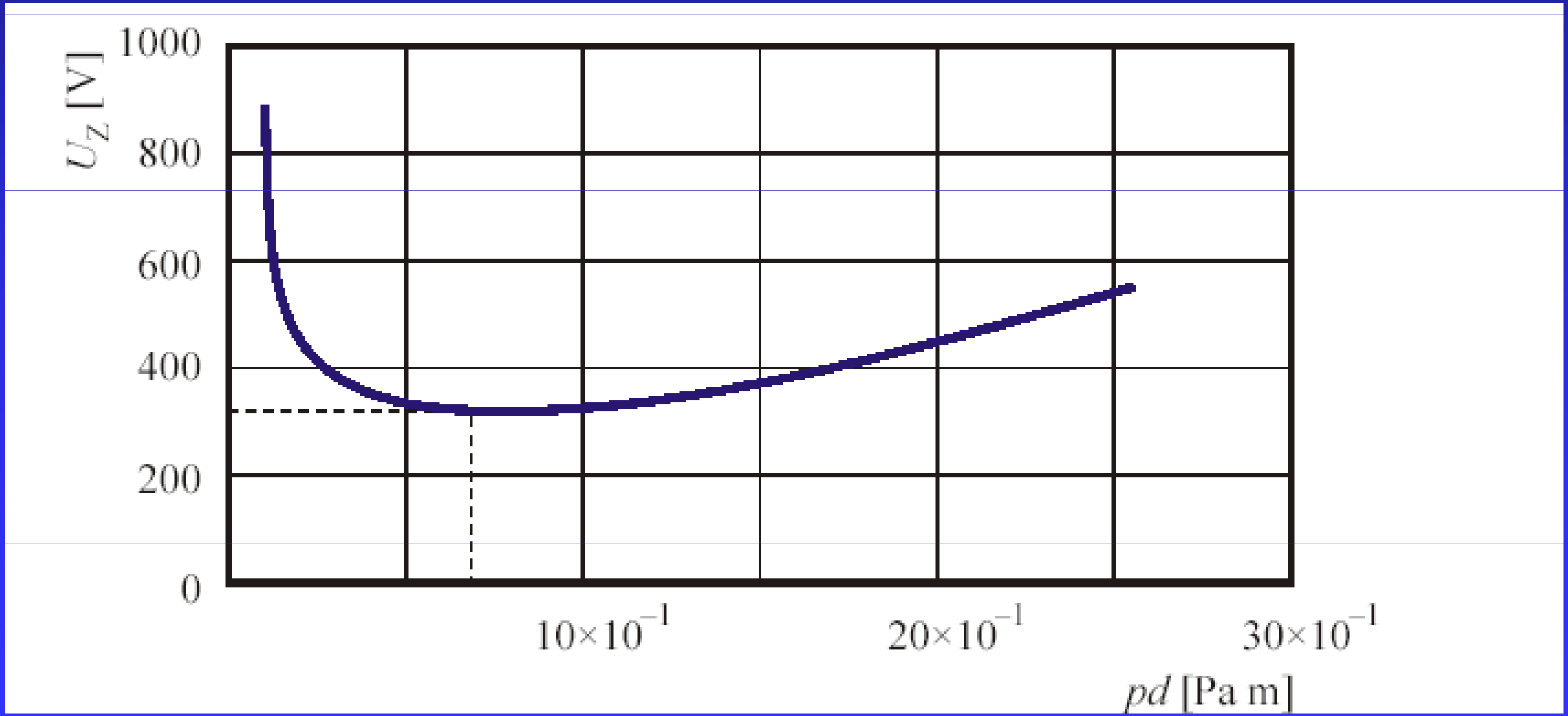


$pd < (pd)_{min}$

Long Path Breakdown

Paschen´s curve for air

Brekdown voltage for ambient air is approximately 25 kV/cm



Similarity of gas discharges

The similarity laws, which are designed to extrapolate discharge properties from one discharge to the other, are very helpful in predicting discharge properties. The principal advantages of the similarity laws include predicting the discharge properties by manipulating gas pressure and discharge dimension equivalently. The simplest similarity law is Paschen's law, which describes the discharge onset (breakdown) voltage Ub as a function of pd, i.e., Ub = f (pd)

Other examples:

Glow discharges: J (current density)/p^2 = const.

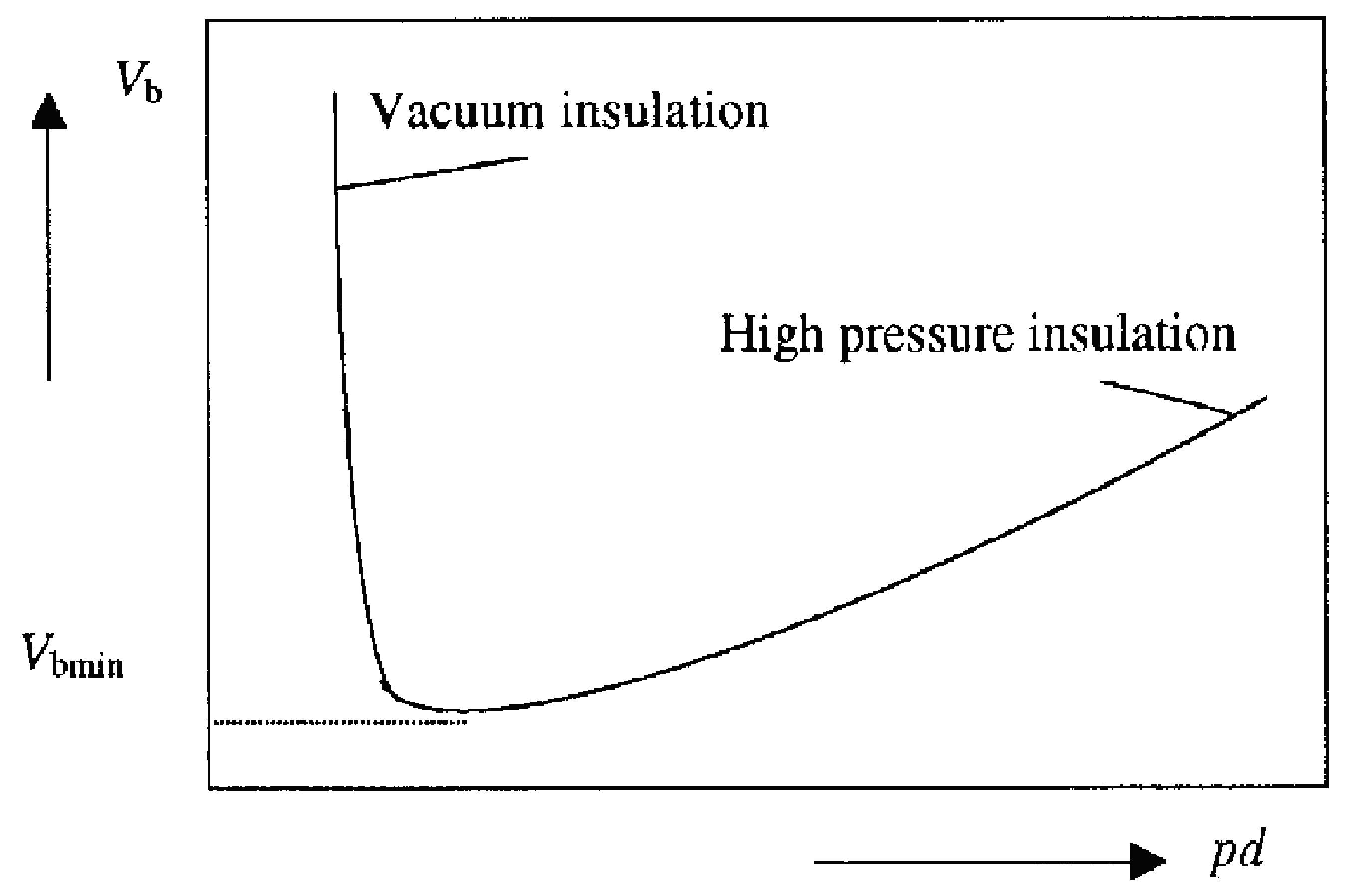
High-frequency discharges: to be similar, pd, and f /p must be the same.

Similarity laws are not valid when, for example, many-body collisions and field emission, become important

Similarity of gas discharges enables us to use the known properties of the discharge at one pressure to extrapolate features of unknown discharges, especially in cases when the length scales may not be feasibly reached by experimental diagnostics, for example:

Sprite streamers are phenomenologically similar to electrical discharges observed in laboratory experiments, and it is natural to compare analyses of streamers observed in laboratory experiments with sprite streamerobservations. This may indicate the extent to which laboratory experiments, typically performed at ground pressure, can be related to sprite streamers in near vacuum at 80 km altitude where, although the many processes involved may be the same, their ranking inimportance may be different.

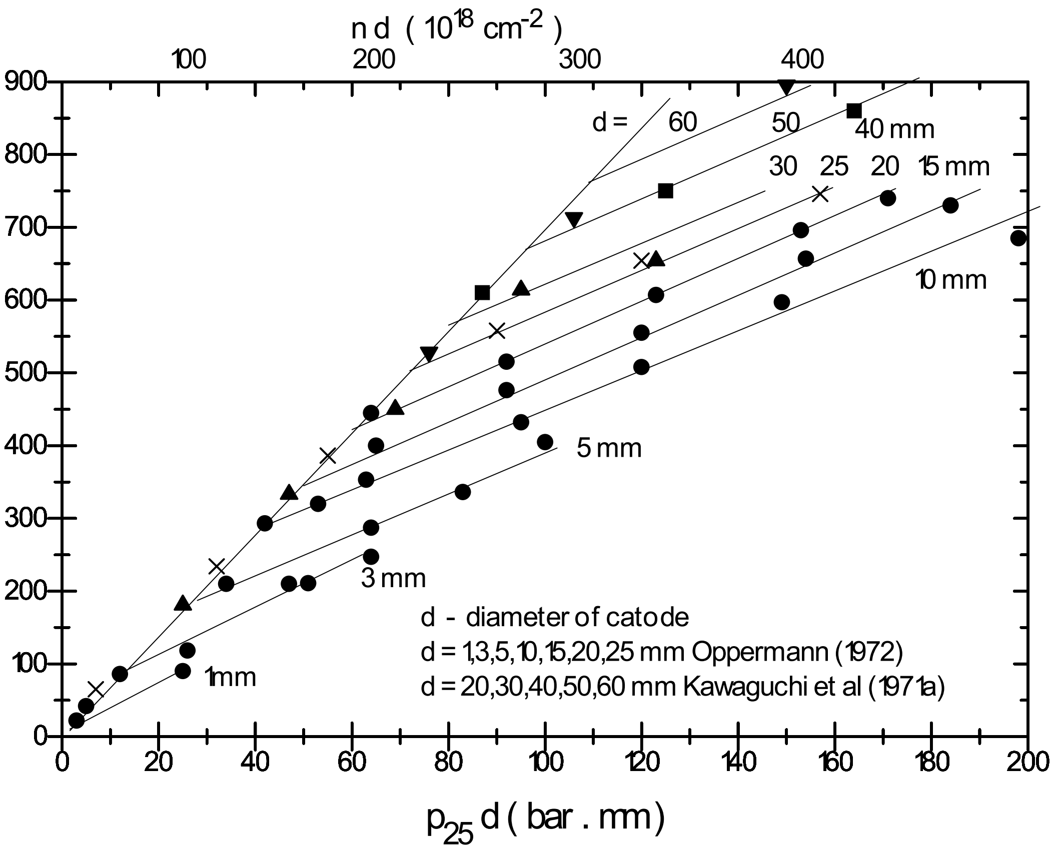
High voltage el. insulation

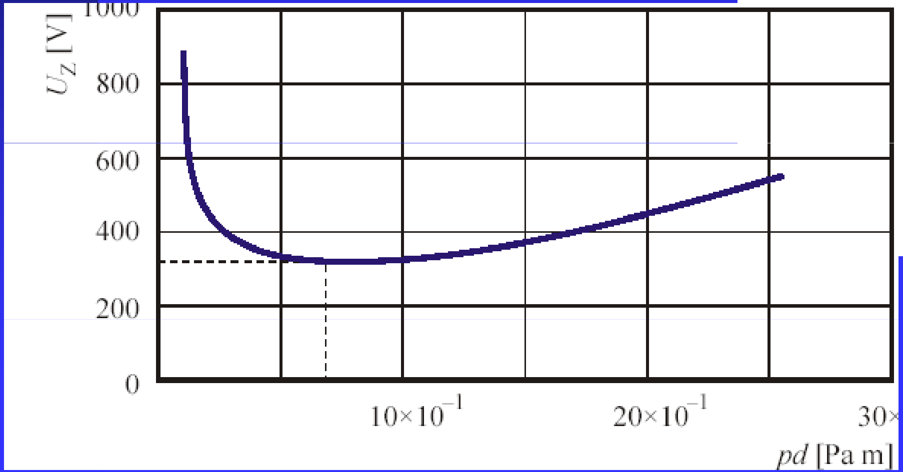


However, field emission does not depend on **E/N** but on **E** !

In real conditions at fields above 10^5 V/cm (theoretically at 10^7 V/cm)

Paschen´s low is no more valid :





Vacuum breakdown

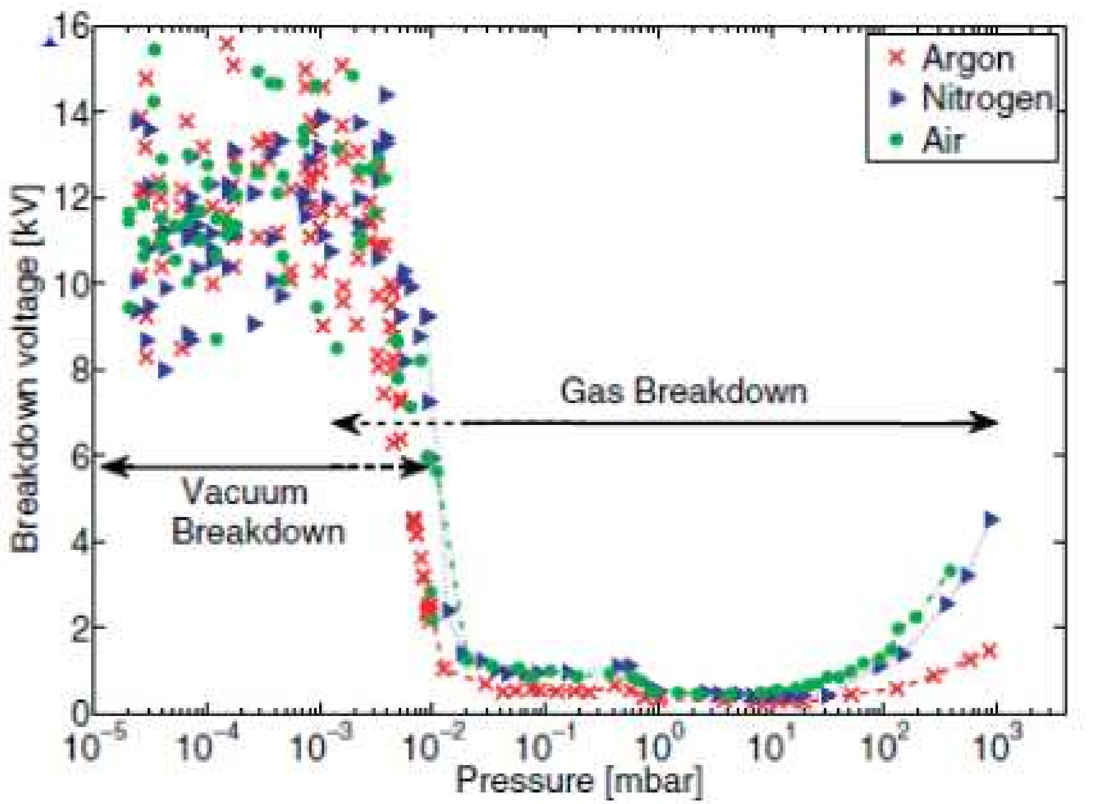
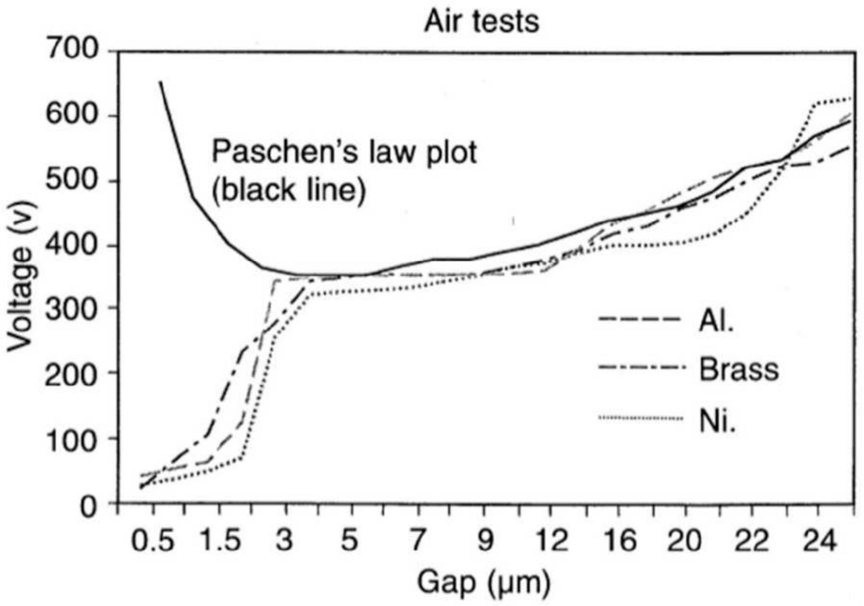
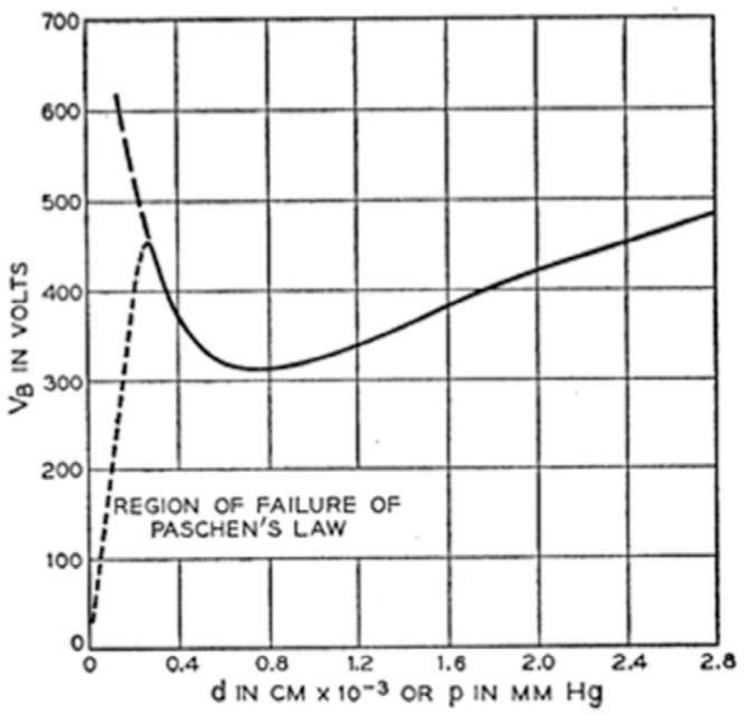


Figure 3. Measured dc breakdown voltages for the ring assembly from 2×10^{-5} to 10^{3} mbar in different gases.

J. Phys. D: Appl. Phys. 46 (2013) 285205

Short ~µm electrode distances

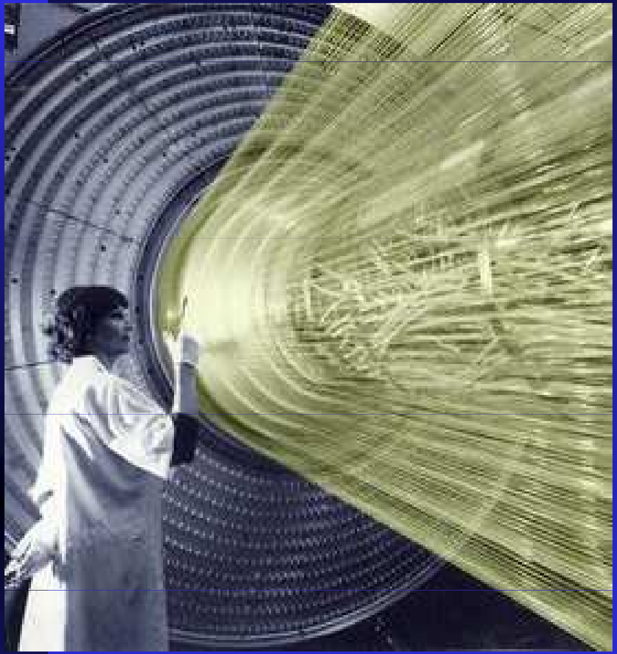




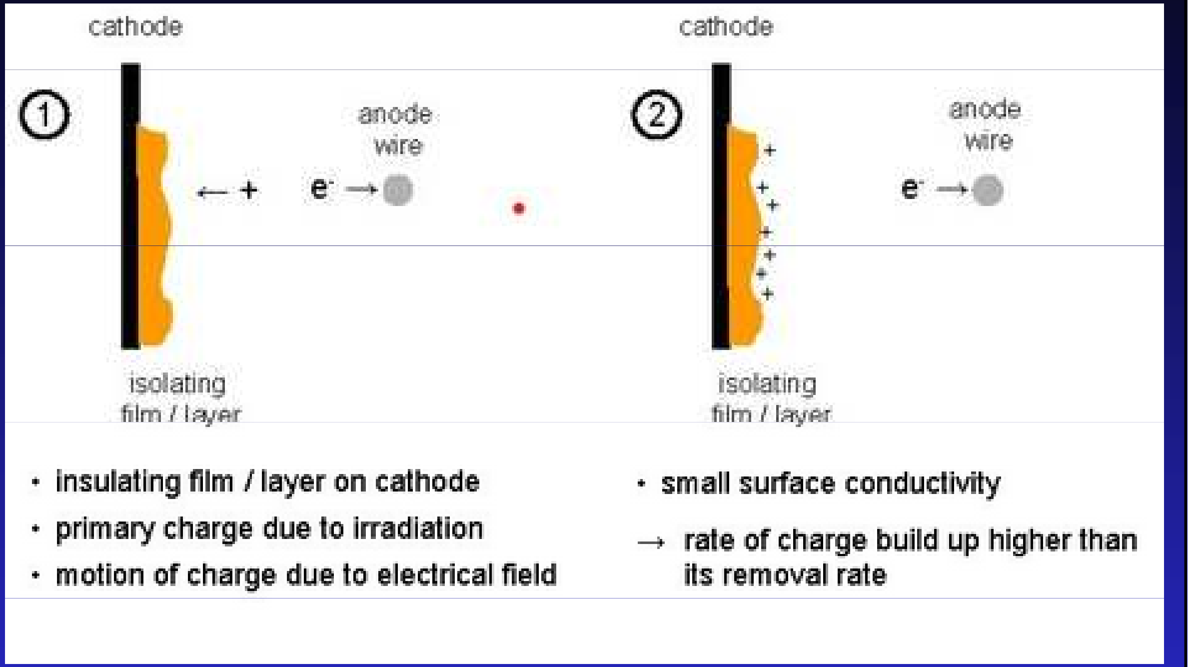
Plot of the breakdown voltage as a function of the electrode gap spacing *d* for ambient air at atmosph
pressure using different cathode materials

D B Go and A Venkattraman 2014 *J. Phys. D: Appl. Phys.* **47**

Field emission theoretically starts at 10^7 V/cm, however for real metal surfaces at 10^5 V/cm^{-1}. Why ?



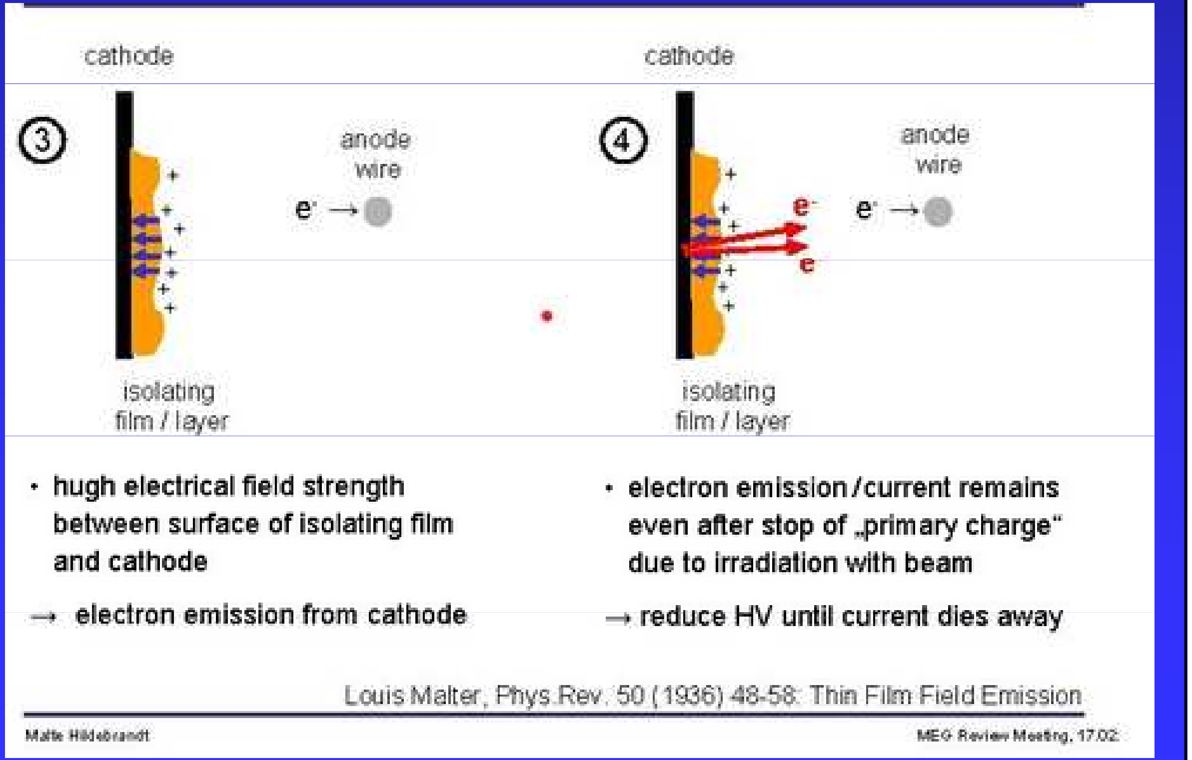
In (multivire) proportional chembers argon-methane gas mixture is used to prevent photoionization (fast deexcitation of Ar*) – Problem with „plasmachemical“ polymer deposition at the cathodes



- insulating film / layer on cathode
- primary charge due to irradiation
- motion of charge due to electrical field

- small surface conductivity

→ rate of charge build up higher than its removal rate



- hugh electrical field strength between surface of isolating film and cathode

→ electron emission from cathode

- electron emission / current remains even after stop of „primary charge“ due to irradiation with beam

→ reduce HV until current dies away

Louis Malter, Phys.Rev. 50 (1936) 48-58: Thin Film Field Emission

Malte Hildebrandt

MEG Review Meeting, 17.02

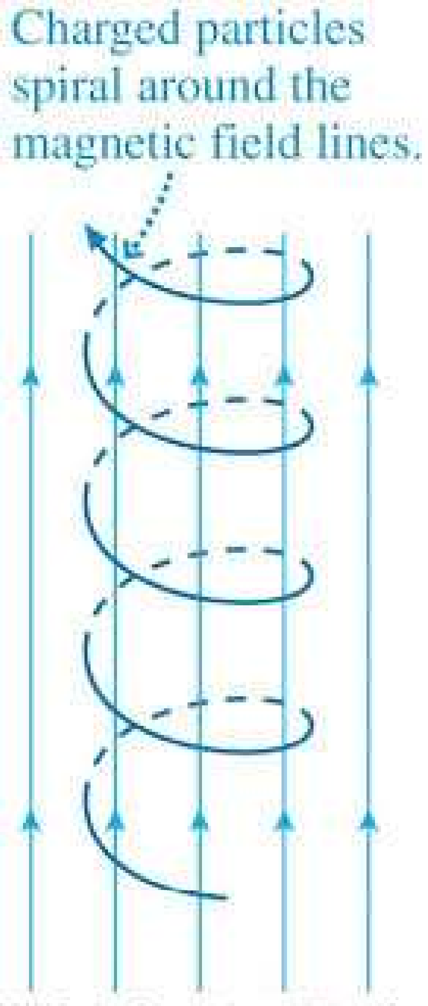
Malterov´s emissin – field emission from oxide coated surfaces autoemisia occuring at relatively low fields

OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS
Vol. 6, No. 3-4, March - April 2012, p. 416 - 421

Investigation of field electron emission from ITO/glass interfaces
JADWIGA OLESIK

- „In 1936 Louis Malter studied the phenomenon of secondary emission from poorly conducting oxides and discovered some anomalies. The anomalous secondary emission was caused by charging of the emitter surface and production of an internal electric field in investigated samples. Uncontrolled behavior of this emission made impossible practical application of its properties like e.g. some high values of the secondary emission coefficient. If it was possible to produce a given value internal field in a sample, then the secondary emission would be controllable. In this work such an attempt has been taken.“

Paschen law – effect of magnetic field



Charged particles spiral around the magnetic field lines.

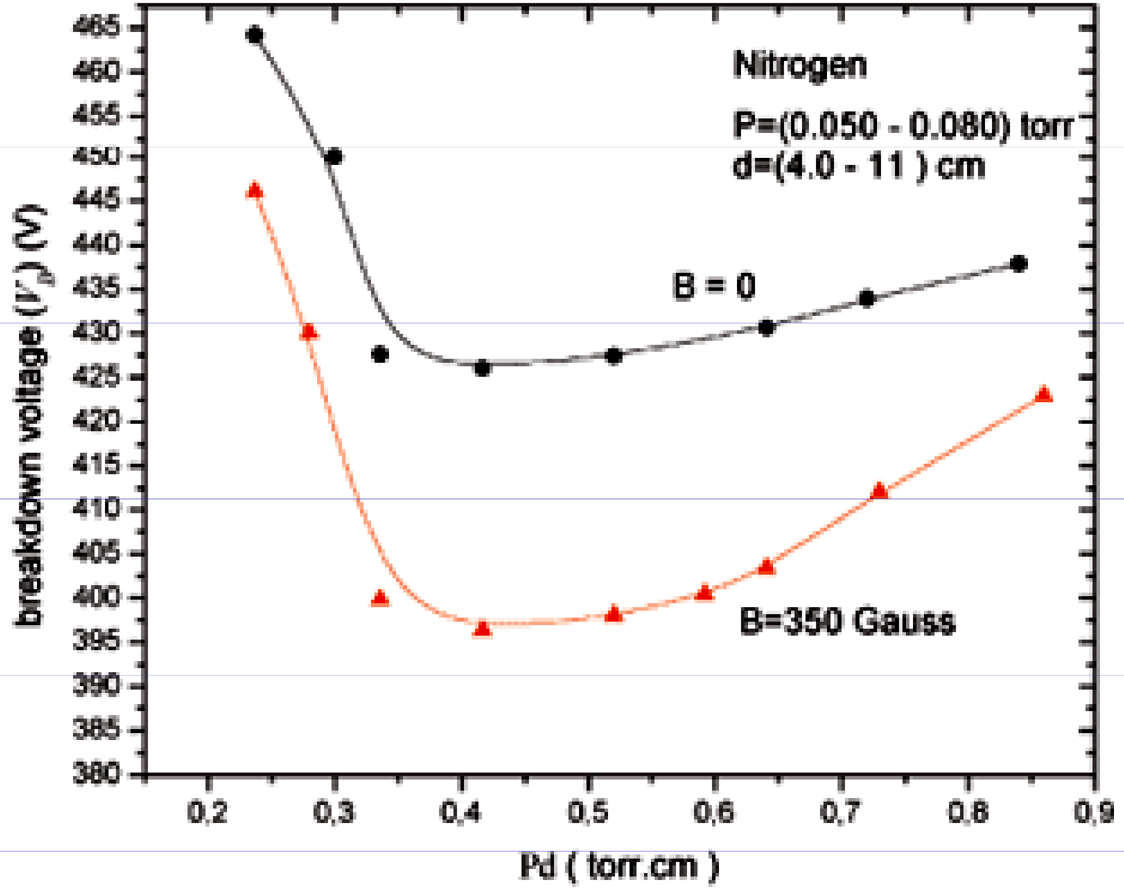
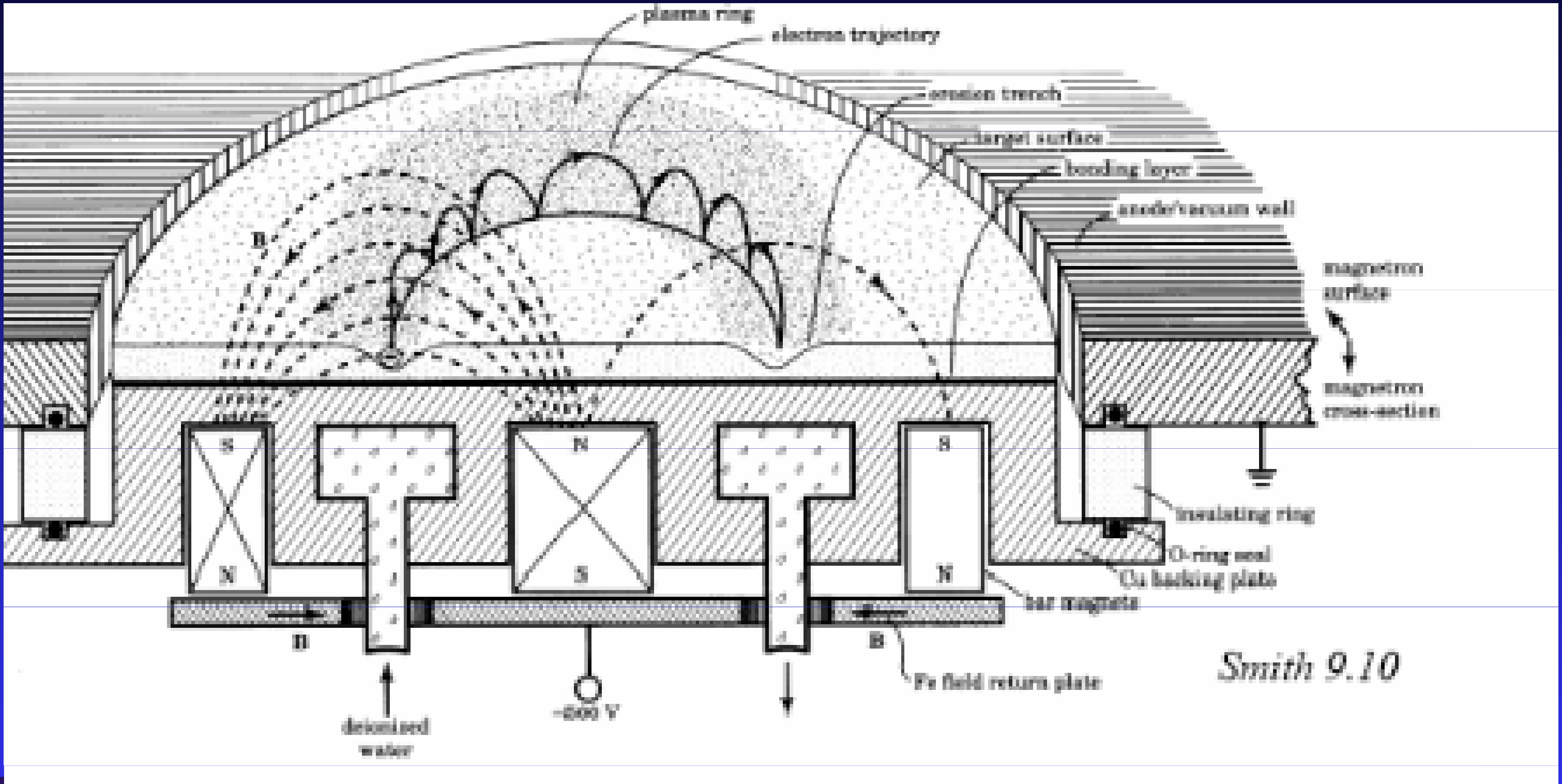


Figure 3. Breakdown voltage (V_B) for nitrogen as a function of *Pd* (Paschen curves) for two values of magnetic field.

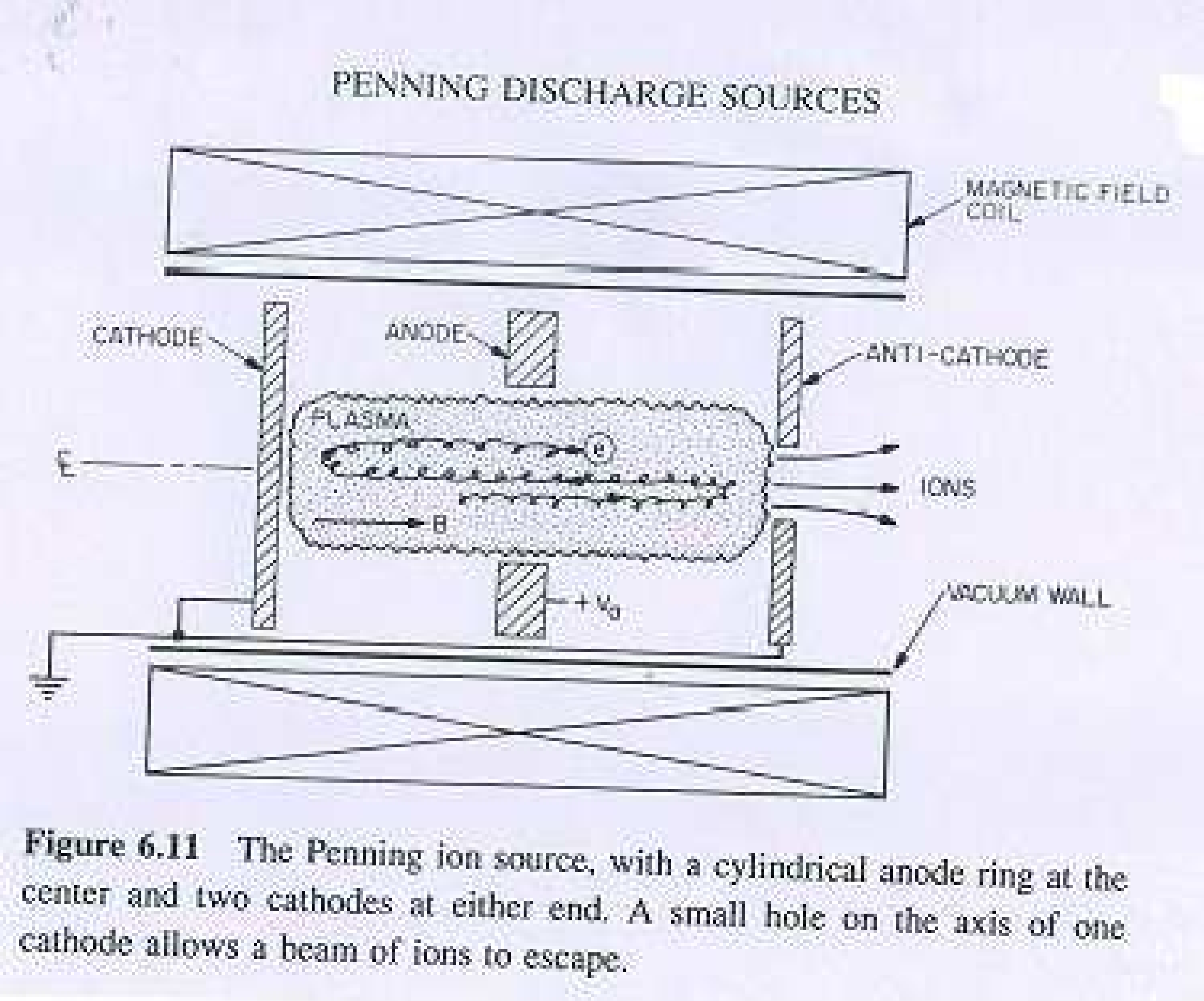
Magnetron sputtering



Smith 9.10

Penning source of ions

Synergic effect of mag. field and „hollow cathode“

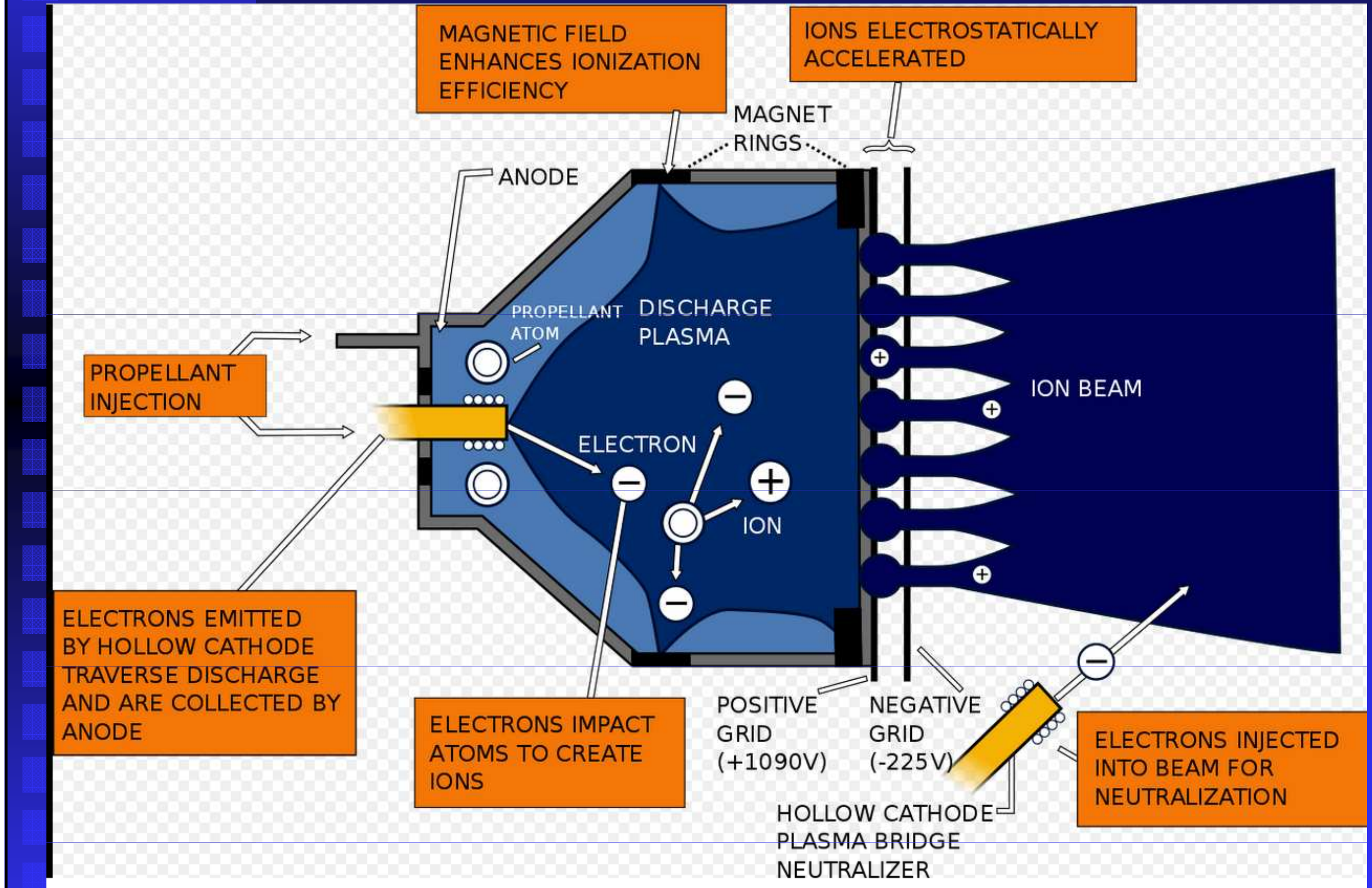


PENNING DISCHARGE SOURCES

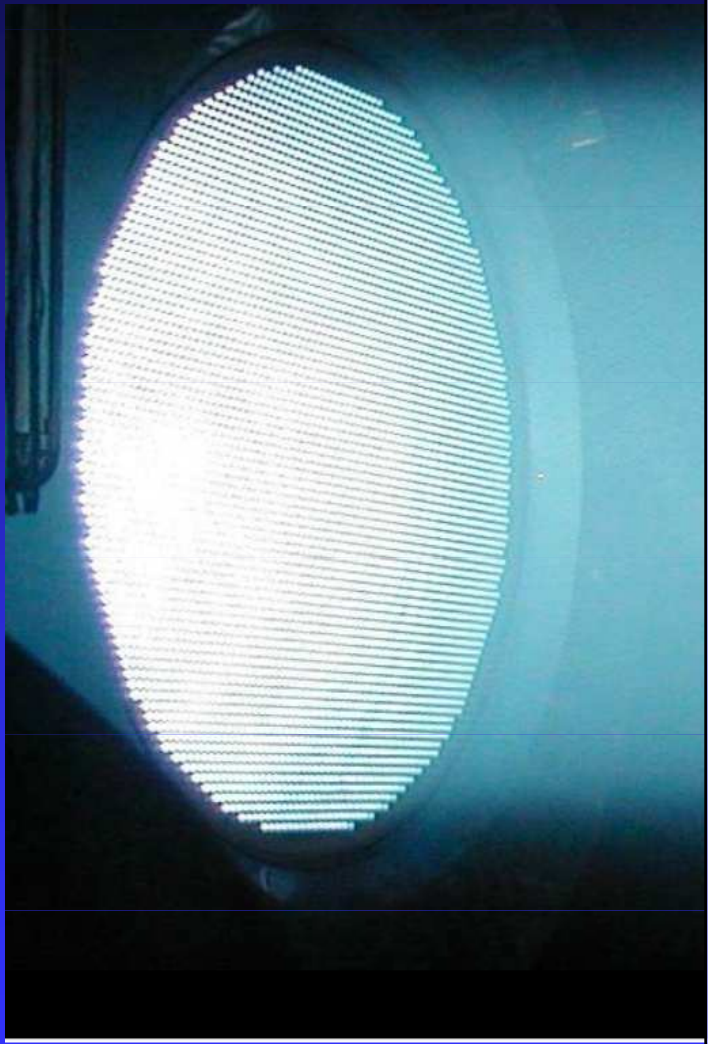
Figure 6.11 The Penning ion source, with a cylindrical anode ring at the center and two cathodes at either end. A small hole on the axis of one cathode allows a beam of ions to escape.

Ion plasma engine (ion thruster) for spacecrafts

https://en.wikipedia.org/wiki/Ion_thruster

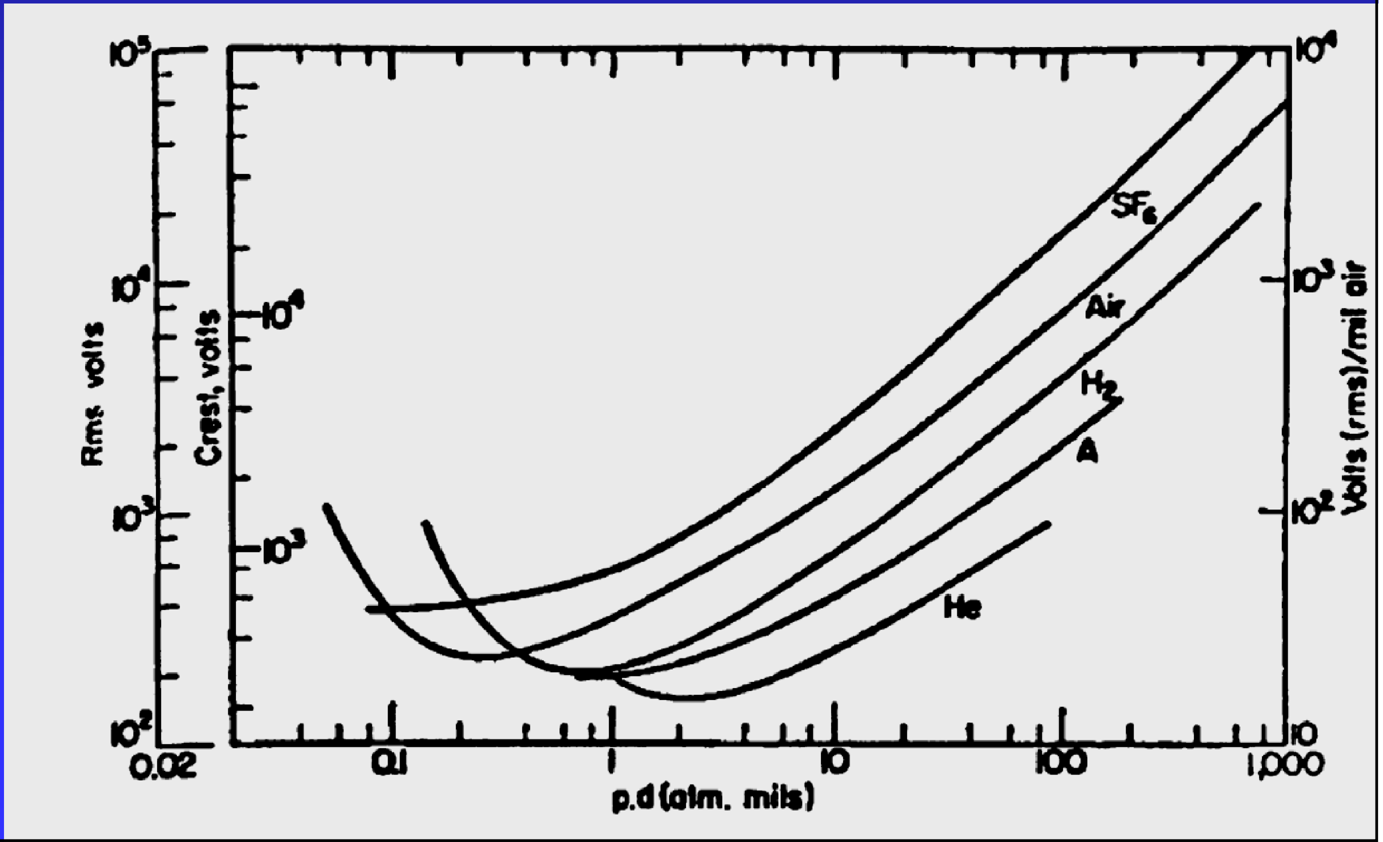


gridded electrostatic ion engine (multipole magnetic cusp type) works

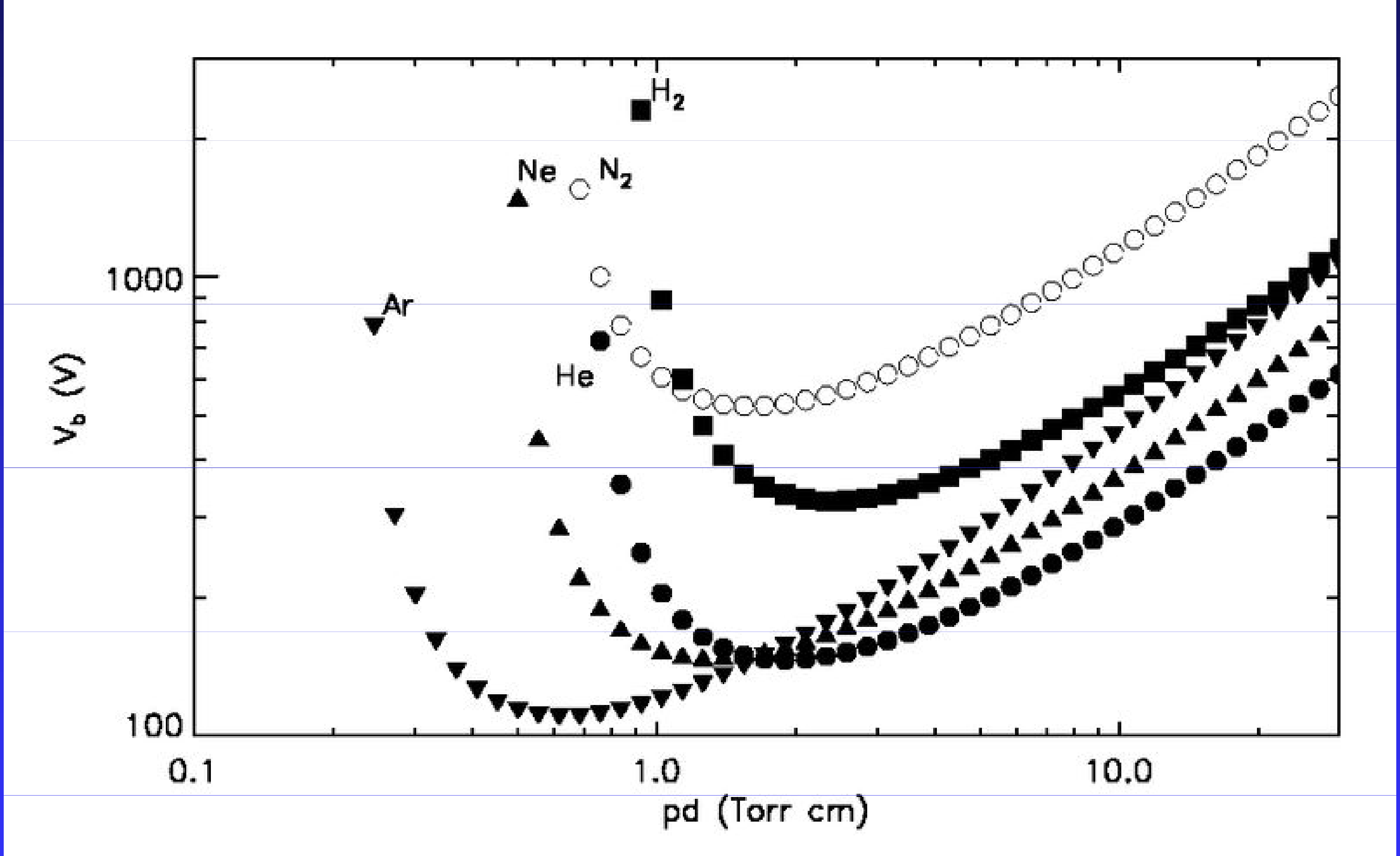


Penningova ionizácia

- *Ionizačné energie*:
- He (24,59 eV)
- Ar (15,4 eV)
- SF_6 (15,3)



Lieberman and Lichtenberg, Principles of Plasma Discharges, Wiley 2005.



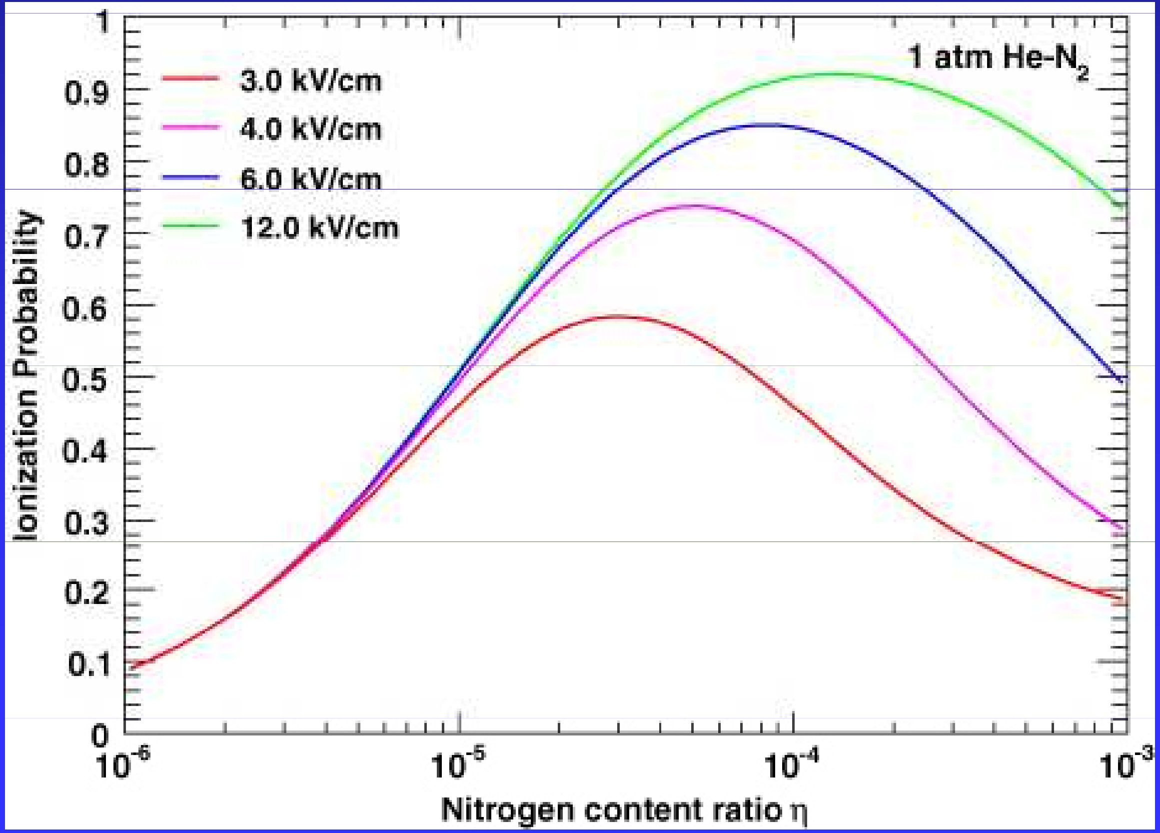
$$He + e \rightarrow He^*$$

$$He^* + 2\,He \rightarrow He^*_2 + He$$

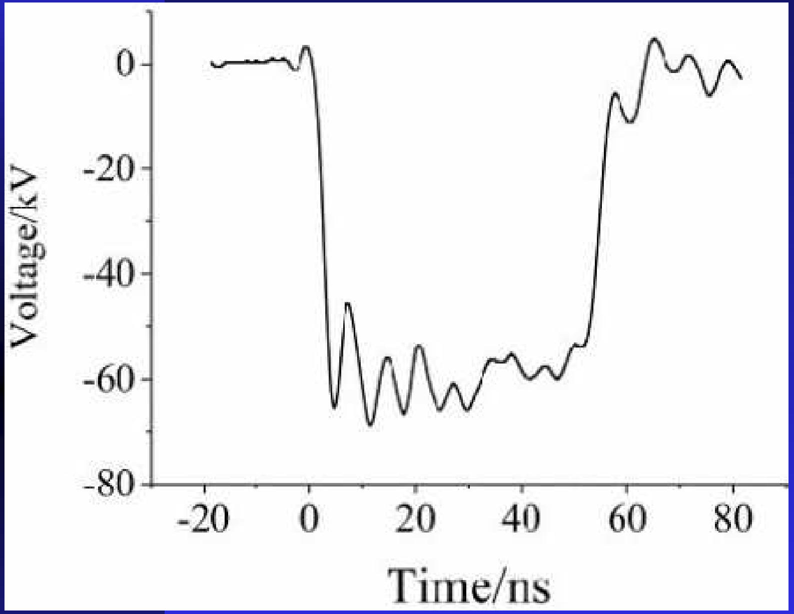
$$He^*_2 + X \rightarrow 2He + X$$

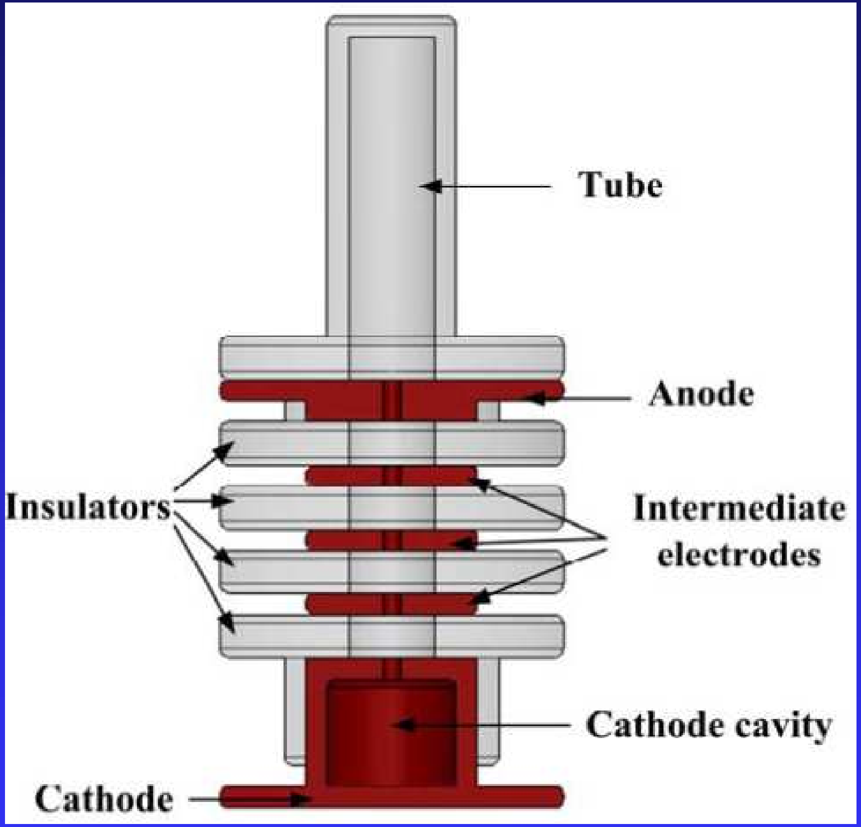
$$He^* + N_2 \rightarrow N^+_2 + He + e$$

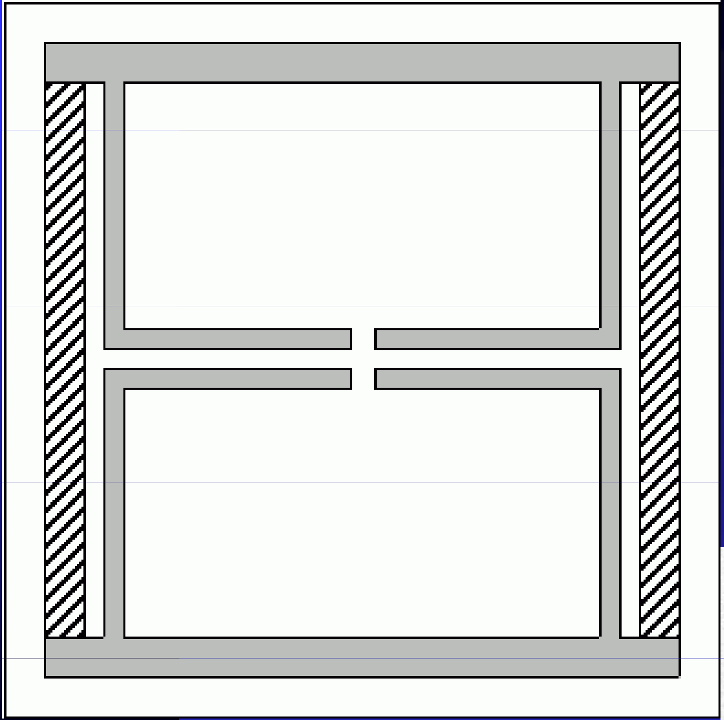
$$He^*_2 + N_2 \rightarrow N^+_2 + 2\,He + e$$

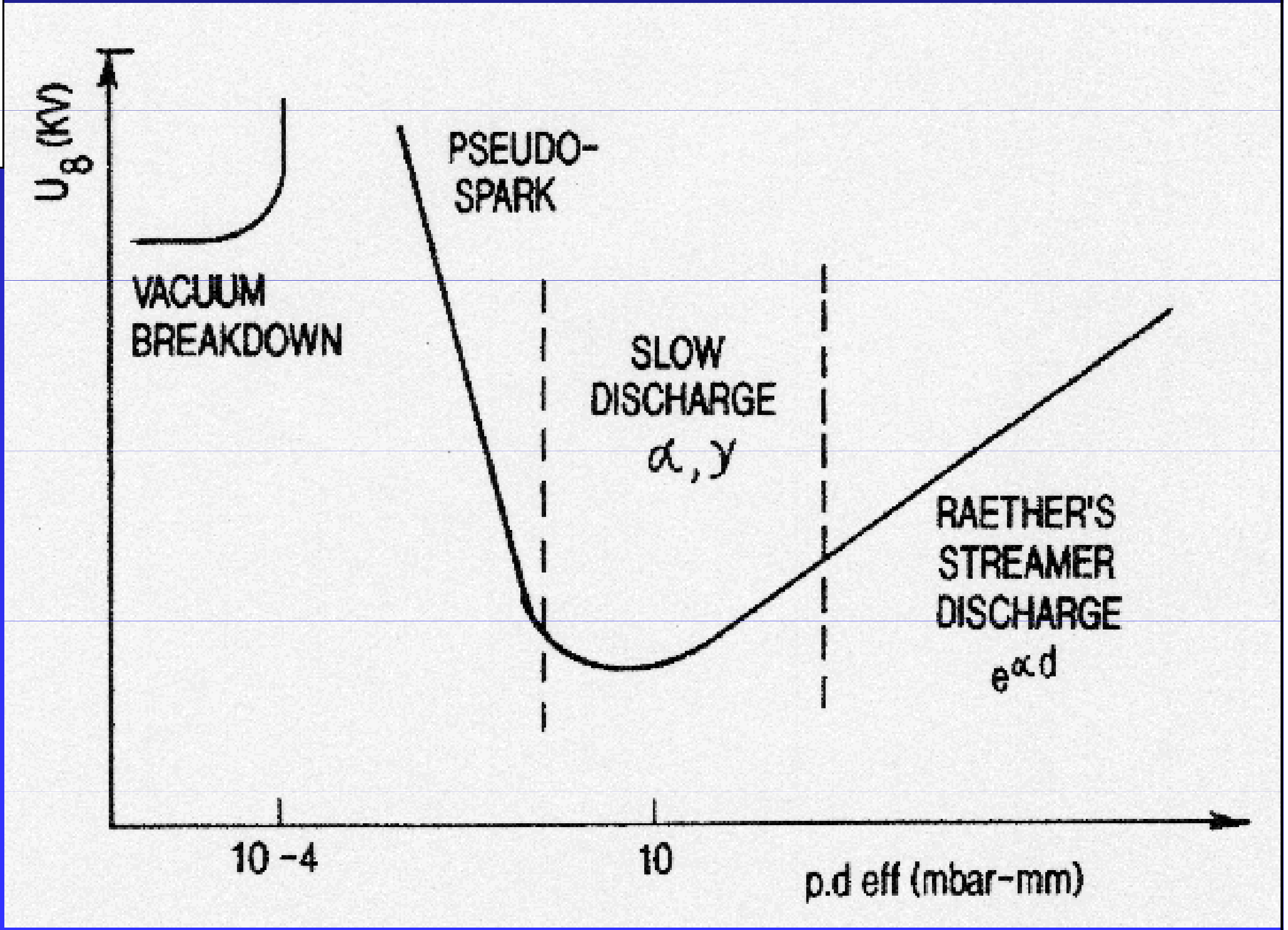


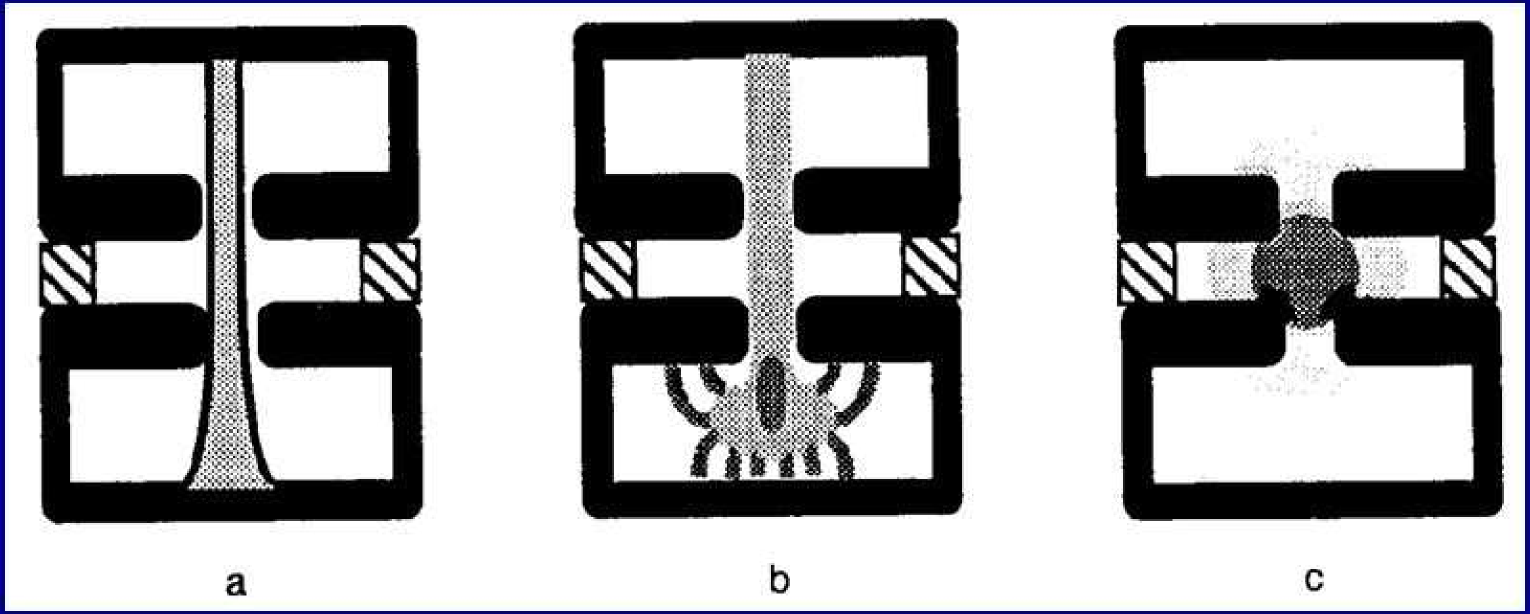
„Pseudospark“ HV switchers





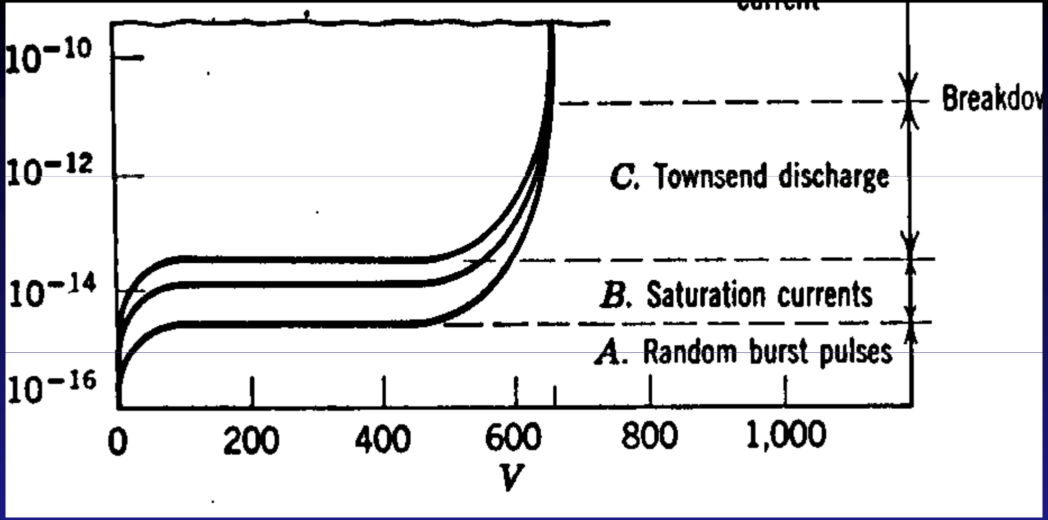



U_B (KV)
VACUUM BREAKDOWN
PSEUDO-SPARK
SLOW DISCHARGE
α, γ
RAETHER'S STREAMER DISCHARGE
$e^{\alpha d}$
10 -4
10
p.d eff (mbar-mm)



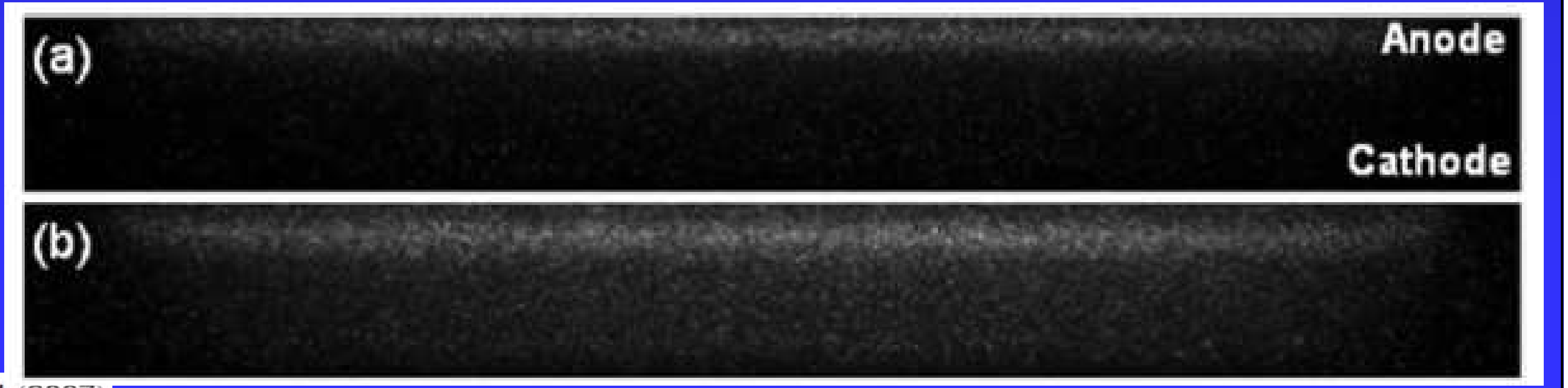
- 3 stages during a pseudospark discharge:
  - a) Townsend discharge
  - b) Hollow cathode discharge
  - c) Superdense glow discharge (conductive phase)

M. Stetter, P. Felsner, J. Christiansen, K. Frank, A. Gortler, G. Hunts *et al, IEEE Trans Plasma Sci.,* vol. 23, no. 3, *Special Issue on Pseudospark Physics and Applications,* pp283-293, 2004



Townsend discharge

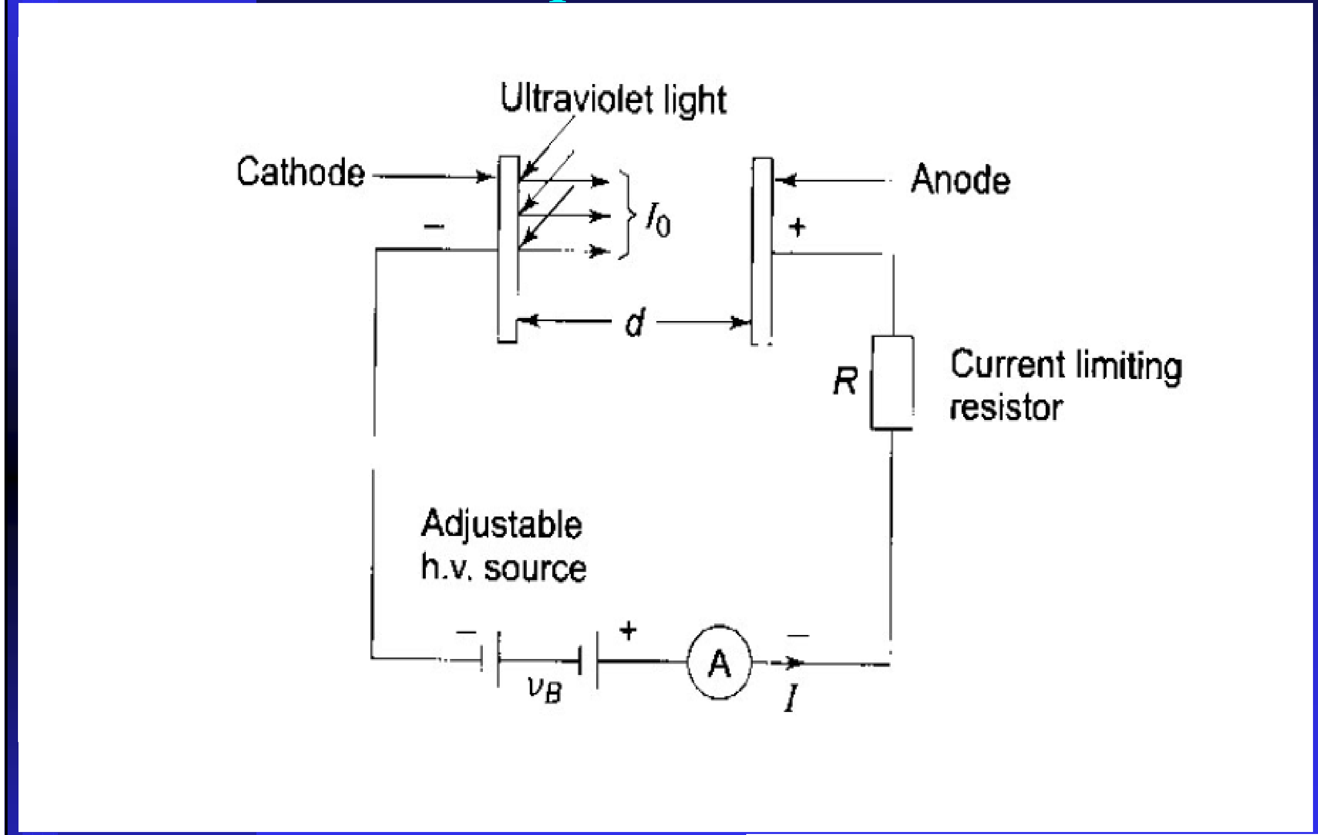
- self-sustained /non-selfsustained (discharge onset statistics!)
- low current densities → low positive ions densities → Laplacian field (the el. field with no effect of the space charge)
- The strongest light emissionis at the anode vicinity. This is because the electron density is the highers there.

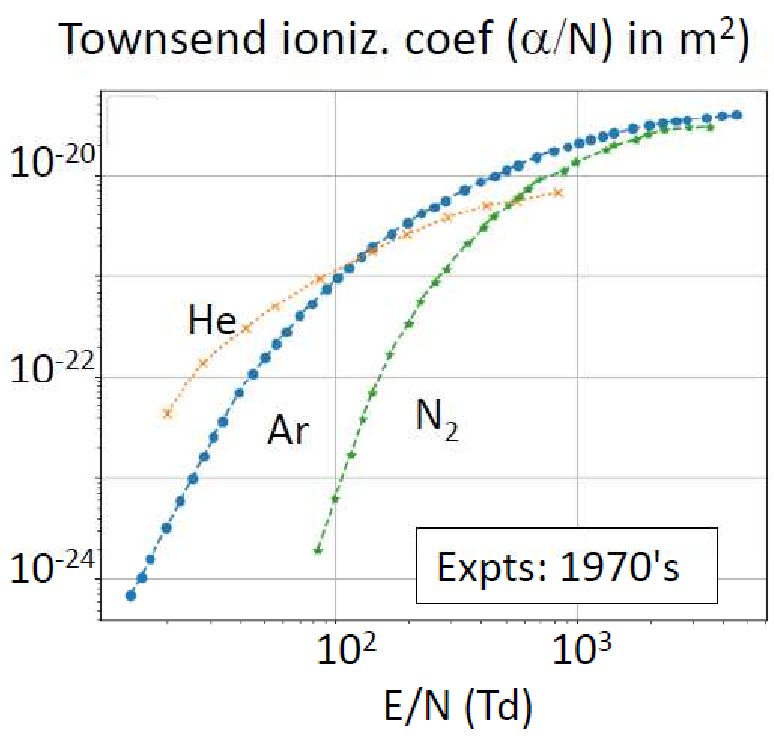


Appl. Phys. Lett. **91**, 221504 (2007)

Measurent of α a γ using *stationary Townsend discharge*:

at pressures on the order of 100 Pa, voltages onthe order of kV and low currents 0,01 pA - 10 nA




Townsend ioniz. coef (α/N) in m²)

For $\alpha d \gg 1$ it is valid that

$$I = \frac{I_0 e^{\alpha d}}{1-\gamma\,(e^{\alpha d}-1)} \approx I_0 e^{\alpha d}$$

so that

$$\ln I = \alpha d + \ln I_0$$

measuring I a I_o we can determine **α** as a function of **E**, i.e. **α/N = f(E/N)**

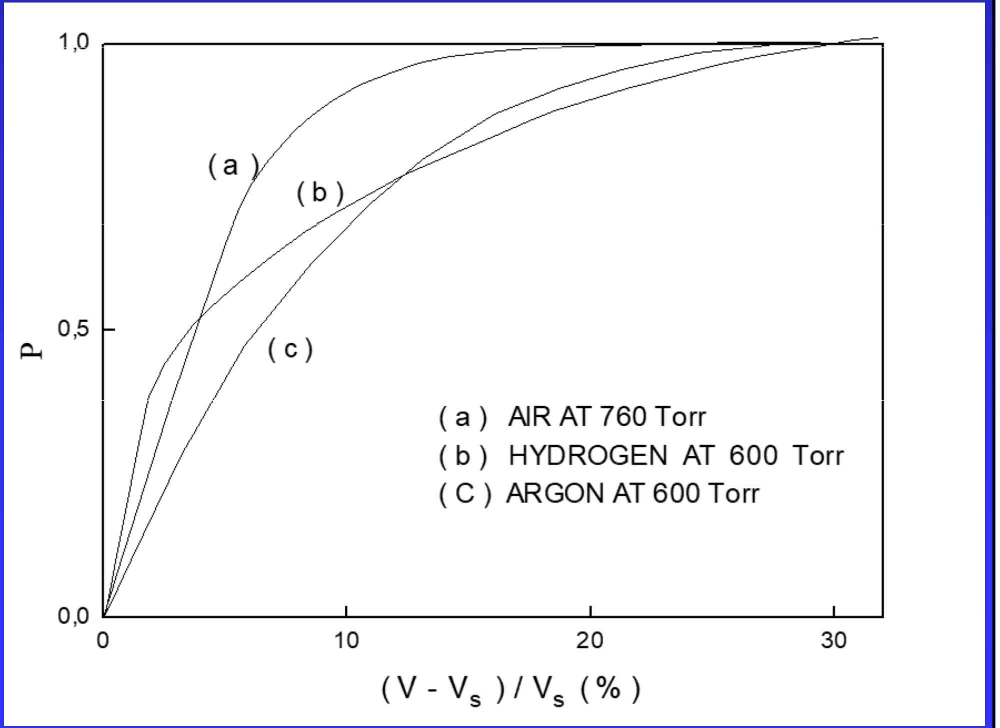
Some notes to the statistics of the discharge onset:

Two processes (α and γ) are required to ignite and to sustain the discharge. Both are highly stochastic !!!!

To ensure the establishment a of an uninterrupted flow of a self sustained discharge current initiated by a single electron a large overvoltage must be applied. This is shown in Figure 1 for the case of discharge between parallel aluminium electrodes set 0.3 mm apart in air, hydrogen , and argon at pressures of 100 kPa and 78 kPa. It can be seen that at least 25% overvoltage in needed before the probability of the discharge initiaition by a single electron P approaches close to unity.

If $\gamma\ (e^{\alpha x} - 1) = 1$, then $P \rightarrow 0$

Alternatively, you can initiate the self sustained discharge by incrising the number of the discharge initiaiting electrons, for example, by UV light irradiation.



Due to an increasing space charge of positive ions t higher current densities Townsend discharge is transferred into the glow discharge