1.) $S = 1 + 10 + 100 + \ldots \quad | \cdot 10$

$10 \cdot S = 10 + 100 + 1000 + \ldots$

$-$

$9 \cdot S = -1 + 0 + 0 + \ldots = -1$

$$S = -\frac{1}{9}$$

$$R = 1 - 2 + 3 - 4 + 5 - \ldots$$
$$R = \quad 1 - 2 + 3 - 4 + 5 - \ldots$$
$\oplus$

$$2 \cdot R = 1 - 1 + 1 - 1 + 1 - \ldots = \frac{1}{2} \Rightarrow R = \frac{1}{4}$$

$$S - 12 = 4S$$

$$-12 = 3S$$

$$-\frac{1}{4} = 3 \cdot S \quad \Rightarrow \quad S = -\frac{1}{12}$$

Pr.:

$$\sum_{n=0}^{\infty} \frac{3}{5^n} = 3 \cdot \frac{1}{1-\frac{1}{5}} = \frac{15}{4}$$

Pr.:

$$\sum_{n=4}^{\infty} \frac{3}{5^n} = 3 \cdot \frac{\frac{1}{5^4}}{1-\frac{1}{5}}$$

$$\left[\frac{15}{4} - \left(3 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3}\right)\right.$$

$$S_n - \frac{S_n}{2} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n} - \frac{n}{2^{n+1}}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 2\cdot\left(\text{---}\,''\,\text{---} \right) = 2\cdot\left(\frac{\frac{1}{2}}{1-\frac{1}{2}} - \lim_{n\to\infty} \frac{n}{2^{n+1}} \right) =$$

$$= 2(1-0) = 2\cdot 1 = \underline{\underline{2}}$$

Př.: $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\dots+\frac{n-1}{n^2}\right)=$

$=\lim\limits_{n\to\infty}\frac{1}{n^2}\cdot\left(1+2+3+\dots+(n-1)\right)=$

$=\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\cdot\frac{n\cdot(n-1)}{2}\right]=\lim\limits_{n\to\infty}\frac{n^2-n}{2n^2}=\underline{\underline{\frac{1}{2}}}$

Pr.: KONV.?

a) $0 \leq \sum_{1}^{\infty} \frac{1}{n \cdot (n+1)} \leq \sum_{1}^{\infty} \frac{1}{n \cdot n}$ KONV.

(KL. PRVKT)

KONV.

d) $\sum_{2}^{\infty} \frac{1}{\sqrt[n]{\ln n}}$

$$1 \le \ln n \le n \quad / \sqrt[n]{(\cdot)}$$

$n \ge 3$

$$1 \le \sqrt[n]{\ln n} \le \sqrt[n]{n}$$

$$\lim \frac{1}{\sqrt[n]{\ln n}} = \frac{1}{1} = 1$$

NESPL. NUTNOU PODM.

e) $\sum_{2}^{\infty} \frac{1}{n \cdot \ln n}$, $f(x) = \frac{1}{x \cdot \ln x}$

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pro $x \geq 2$

$$\int_{2}^{\infty} \frac{1}{x \cdot \ln x} dx = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x} dx \end{array} \right| = \int_{\ln 2}^{\infty} \frac{1}{t} dt = \left[\ln t \right]_{\ln 2}^{\infty} =$$

5) $\sum_{1}^{\infty} \frac{2^n \cdot n!}{n^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}\cdot(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\cdot n!}{n^n}} = \lim_{n\to\infty} \frac{2\cdot\cancel{(n+1)}\cdot n^n}{(n+1)^{n\cancel{+1}}}$$

g) $\sum_{1}^{\infty} \frac{n}{\left(3+\frac{1}{n}\right)^n}$, $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{3+\frac{1}{n}} =$

$= \frac{1}{3} < 1$ K

i) $\sum_{1}^{\infty} \operatorname{arctg}^n \frac{1}{n}$, $\lim \sqrt[n]{a_n} =$

$= \lim_{n \to \infty} \operatorname{arctg} \frac{1}{n} = 0 < 1$ K

b) $\sum_{1}^{\infty} \frac{(-1)^n \cdot n}{5n-2}$

$\frac{n}{5n-2} \to \frac{1}{5} \neq 0$

NEKONV.

l) $\sum_{1}^{\infty} (-1)^n \cdot \frac{1}{\ln^n(2n+4)}$

ABS. K. $\Rightarrow$ $\sum \frac{1}{\ln^n(2n+4)}$ $\qquad \lim \sqrt[n]{a_n} =$

$= \lim \frac{1}{\ln(2n+4)} = 0 < 1$ $\qquad$ ABS. K $\Rightarrow$ K

$\underbrace{\text{POLOM. \& OBOR KONV.} = ?}$

a) $\sum_{1}^{\infty} \frac{2^n}{n^2} \cdot x^n \quad , \quad x_0 = 0 \quad , \quad a_n = \frac{2^n}{n^2}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{2^{\cancel{n}+1}}{(n+1)^2} \cdot \frac{n^2}{\cancel{2^n}} = 2 \cdot \lim \frac{n^2}{n^2+2n+1}$$

$$\boxed{= 2}$$

b) $\sum_{1}^{\infty} \frac{(-1)^n \cdot (x+2)^n}{n+\sqrt{n}}$, $x_0 = -2$, $a_n = \frac{(-1)^n}{n+\sqrt{n}}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{n+\sqrt{n}}{(n+1)+\sqrt{n+1}} = 1 \Rightarrow R = \frac{1}{1} = 1$$

$(-3, -1)$

$$\text{ABS. K.} \rightarrow \left(-3,-1\right)$$

$$\text{K.} \rightarrow \left(-3,-1\right]$$

Pr.: $\sum_{n=1}^{\infty} \frac{(n-1)\cdot(e-1)^n}{n\cdot e^n + e^n} = \sum_{n=1}^{\infty} \frac{n-1}{n+1} \cdot \left(\frac{e-1}{e}\right)^n$

$\sum_{n=1}^{\infty} \frac{n-1}{n+1} x^n \quad , \quad x = \frac{e-1}{e}$

INT. KONV.: $x_0 = 0, \quad \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{\frac{n}{n+2}}{\frac{n-1}{n+1}} =$

$$\sum_{0}^{\infty} x^n = \frac{1}{1-x} \quad \Big/ \int (\cdot)\, dx$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln|1-x| + C \quad \Big/ \cdot \frac{1}{x^2}$$

$\boxed{x=0}$ $\quad 0 = -\ln|1-0| + C \Rightarrow C=0$

$$\sum_{1}^{\infty} \frac{n-1}{n+1} x^n = \frac{1}{1-x} + \frac{2 \cdot \ln(1-x)}{x} - (-1)$$

$$\sum_{n=1}^{\infty} \frac{n-1}{n+1} \cdot \left(\frac{e-1}{e}\right)^n = \frac{1}{1-\frac{e-1}{e}} + \frac{2 \cdot \ln\left(1-\frac{e-1}{e}\right)}{\frac{e-1}{e}} + 1 =$$

$$\boxed{= e - \frac{2e}{e-1} + 1}$$