2.) $S = 1 + 2 + 3 + 4 + \ldots$

$$\left.\begin{aligned} G &= \sum_{n=0}^{\infty} (-1)^n = 1 - 1 + 1 - 1 + 1 - \ldots \\ G &= \phantom{1 - {}} 1 - 1 + 1 - 1 + 1 - \ldots \end{aligned}\right\} +$$

$$2 \cdot G = 1 \Rightarrow G = \frac{1}{2}$$

$$\begin{aligned} S &= 1+2+3+4+5+6+7+8+9+10+\dots \\ -R &= -1+2-3+4-5+6-7+8-9+10-\dots \end{aligned} \quad \oplus$$

$$\begin{aligned} S-R &= 0+4+0+8+0+12+0+16+0+20+\dots \\ &= 4\cdot(1+2+3+4+5+\dots) = 4\cdot S \end{aligned}$$

Pr.:

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$\left.\begin{aligned} S_n &= \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n} \\ \frac{S_n}{2} &= \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \dots + \frac{n}{2^{n+1}} \end{aligned}\right\} \ominus$$

Př.: $$\sum_{1}^{\infty} \frac{2n-1}{2^n} = \sum_{1}^{\infty} \frac{2n}{2^n} - \sum_{1}^{\infty} \frac{1}{2^n} =$$

$$= 2 \cdot 2 - 1 = \underline{\underline{3}}$$

Př.: $\ln x + \ln\sqrt{x} + \ln\sqrt[4]{x} + \ldots = 2$, $x = ?$

$$\ln x + \frac{1}{2}\ln x + \frac{1}{4}\ln x + \ldots = 2$$

$$\ln x \cdot \left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\right) = 2$$

$$\ln x \cdot \sum_{n=0}^{\infty} \frac{1}{2^n} = 2$$

$$\ln x \left(\frac{1}{1-\frac{1}{2}}\right) = 2$$

$$x = e$$

b) $\sum_{1}^{\infty} \frac{n^2}{n^3-4n-5} \geq \sum_{1}^{\infty} \frac{n^2}{n^3} = \sum_{1}^{\infty} \frac{1}{n} = \infty$

DIV.

c) $\sum_{1}^{\infty} \frac{\ln n}{n}$ ... $\sum_{3}^{\infty} \frac{\ln n}{n} \geq \sum_{3}^{\infty} \frac{1}{n} = \infty$

DIV.

$$= \infty - \ln(\ln 2) = \infty \Rightarrow \text{Div.}$$

$$= 2 \cdot \lim \left(\frac{n}{n+1}\right)^{n} = 2 \cdot \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1} = \frac{2}{e} < 1$$

K

b) $\sum_{1}^{\infty} \left(\arccos\frac{1}{n}\right)^{n^2}$, $\lim_{n\to\infty} \sqrt[n]{a_n} =$

$= \lim_{n\to\infty} \left(\arccos\frac{1}{n}\right)^{n} = \left(\frac{\pi}{2}\right)^{\infty} = \infty > 1$ D

j) $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{3n-1}$ ALT. Ř. & $\frac{1}{3n-1} \searrow 0$

$\Rightarrow$ K

LEIB. KRIT.

m) $\sum_{1}^{\infty} \frac{(-2)^n}{\left(5+(-1)^{n+1}\right)^n}$ | ADS. K. ? ANO

$$\sum_{1}^{\infty} \frac{2^n}{\left(5\pm(-1)^{n+1}\right)^n} \leq \sum_{1}^{\infty} \frac{2^n}{4^n} = \sum_{1}^{\infty} \left(\frac{1}{2}\right)^n = 1$$

$$\Rightarrow R = \frac{1}{2} \Rightarrow \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$x = -\frac{1}{2} \Rightarrow \sum_{1}^{\infty} \frac{\cancel{2}^n}{n^2} \cdot \left(-\frac{1}{\cancel{2}}\right)^n = \sum_{1}^{\infty} \frac{(-1)^n}{n^2}$$

ABS. K.

$$x = \frac{1}{2} \Rightarrow \sum_{1}^{\infty} \frac{1}{n^2}$$

ABS. K.

ABS. K

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

$x=-3$ | $\sum_{1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n+\sqrt{n}} = \sum_{1}^{\infty} \frac{1}{n+\sqrt{n}} \geq \sum \frac{1}{n+n}$

$\| $

$\frac{1}{2} \sum \frac{1}{n} = \infty$

DIV.

$x=-1$ | $\sum \frac{(-1)^n}{n+\sqrt{n}}$

DLE LEIB. K. KONV.

ABS. NE

$$= \lim_{n\to\infty} \frac{n\cdot(n+1)}{(n+2)\cdot(n-1)} = \lim \frac{n^2+n}{n^2+n-2} = 1$$

$$\Rightarrow R = \frac{1}{1} = 1 \quad \Rightarrow \quad (-1,1) \ni \frac{e-1}{e}$$

OK

$$\sum_{0}^{\infty} \frac{x^{n-1}}{n+1} = \frac{-\ln|1-x|}{x^2} \quad \Big| \frac{d}{dx}$$

$$\sum_{0}^{\infty} \frac{n-1}{n+1} \cdot x^{n-2} = \frac{1}{x^2 \cdot (1-x)} + \frac{2 \cdot \ln(1-x)}{x^3} \quad \Big| \cdot x^2$$

$$\sum_{0}^{\infty} \frac{n-1}{n+1} \cdot x^{n} = \frac{1}{1-x} + \frac{2 \cdot \ln(1-x)}{x}$$

DÚ

$$\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 5^n}$$

$$= \ln\left|\frac{5}{5-x}\right| - \frac{x^2}{50} - \frac{x}{5}$$

$$x \in [-5, 5)$$

$$x = -5$$

$$\Downarrow$$

$$\sum_{1}^{\infty} \frac{(-1)^n}{n} = -\ln 2$$