DIR. KR.:

$$\exists k : \left| \int_a^b f(x)\,dx \right| \le k \in \mathbb{R} \quad (\forall b > a)$$

$$\& \quad g \to 0 \ (\text{monot.}) \implies \int_a^\infty f \cdot g \, dx$$

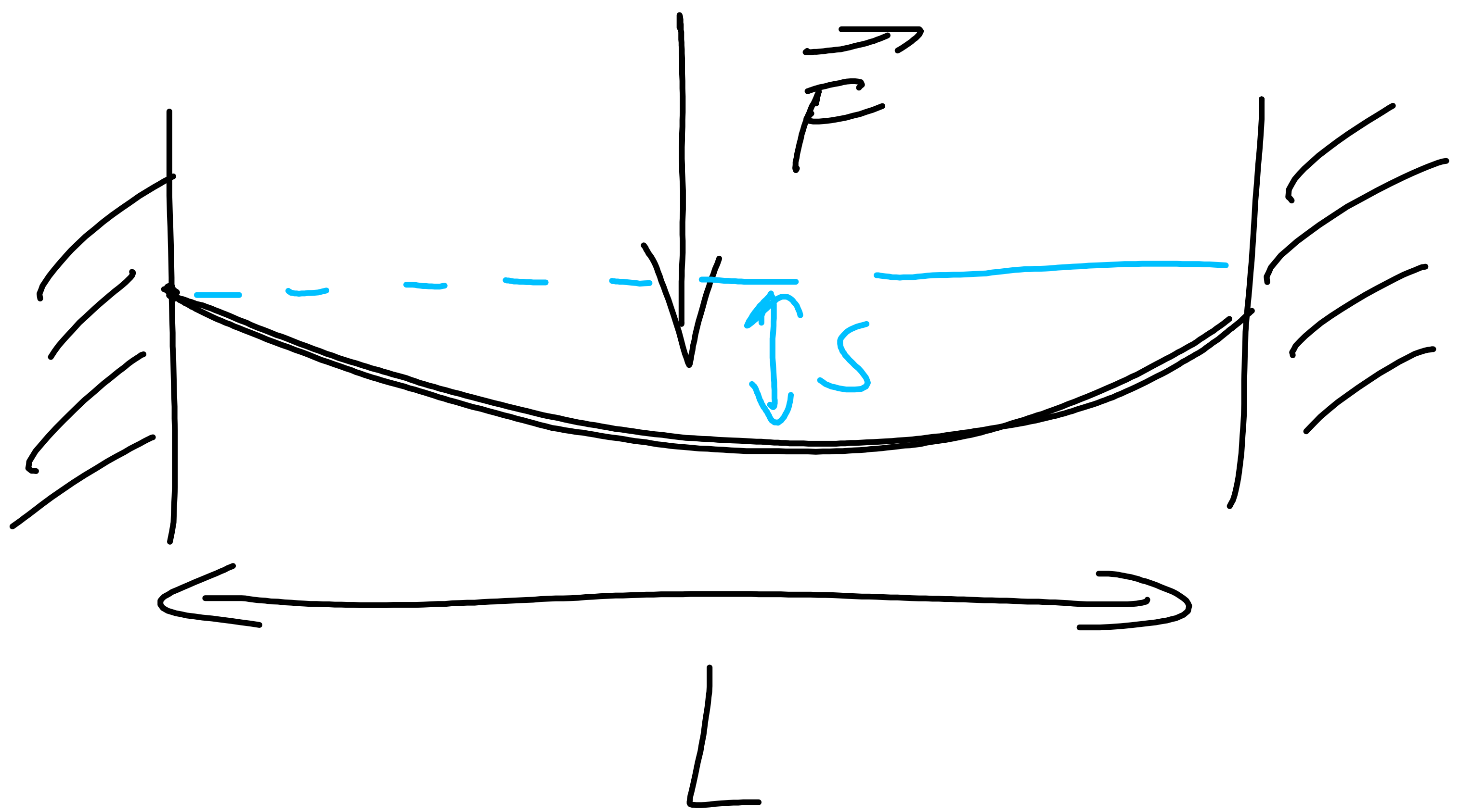
KONV.

$$\int_1^\infty \left| \frac{\sin x}{x} \right| dx = \int_1^\infty \frac{|\sin x|}{x}\, dx = \left| 0 \le \sin^2 x \le |\sin x| \right|$$

$$\ge \int_1^\infty \frac{\sin^2 x}{x}\, dx = \int_1^\infty \frac{1-\cos 2x}{2x}\, dx$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} \left(\int_y^{\sqrt{\frac{\pi}{2}}} y^2 \cdot \sin x^2 \, dx \right) dy = \int_0^{\sqrt{\frac{\pi}{2}}} \int_0^x y^2 \cdot \sin x^2 \, dy \, dx$$

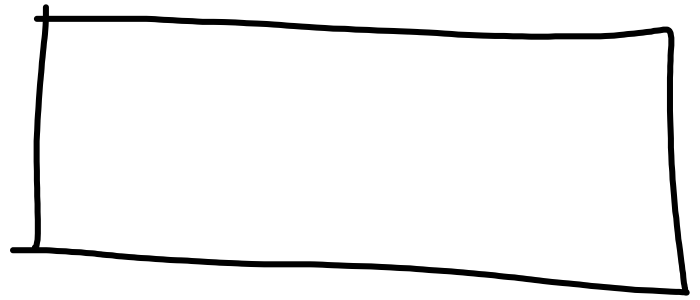
$$= \int_0^{\sqrt{\frac{\pi}{2}}} \sin x^2 \cdot \left[\frac{y^3}{3} \right]_0^x dx = \frac{1}{3} \cdot \int_0^{\sqrt{\frac{\pi}{2}}} x^3 \cdot \sin x^2 \, dx =$$



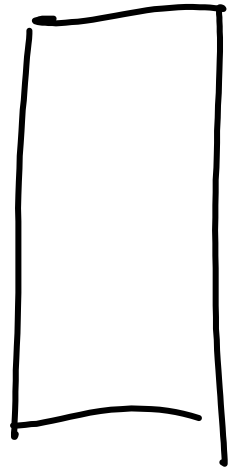
$$s = \frac{F \cdot L^3}{48 \cdot E \cdot I}$$

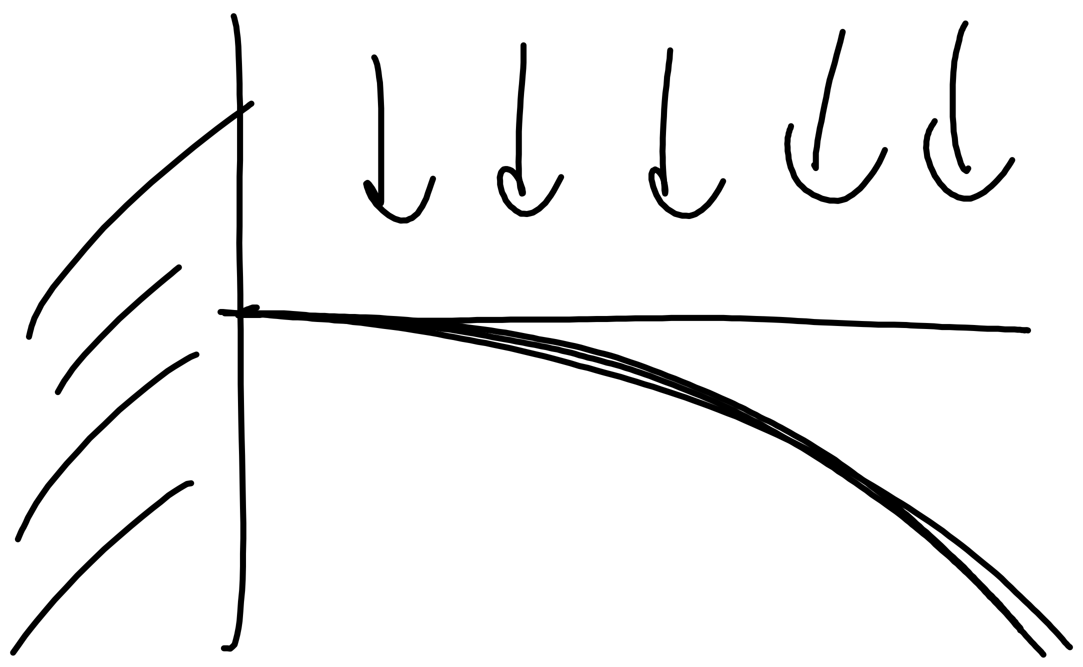
MAT. KONST.

a) $a=2,\ b=1 \Rightarrow I = \frac{1}{6}$

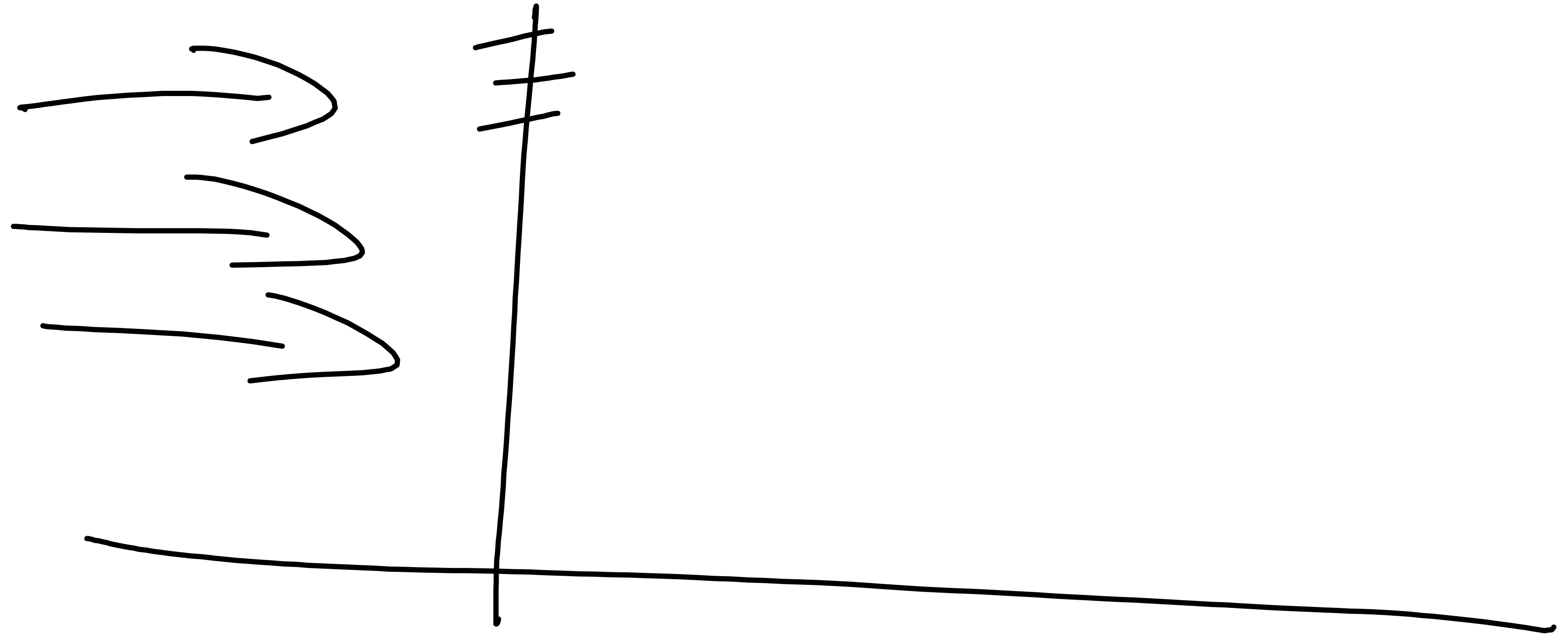


b) $a=1,\ b=2 \Rightarrow I = \frac{4}{6}$





$$\delta = \frac{q \cdot L^4}{8 \cdot E \cdot \boxed{I}}$$



$$\boxed{\int_0^\infty e^{-x^2}\,dx} \cdot \int_0^\infty e^{-y^2}\,dy =$$

$$= \int_0^\infty \left(\int_0^\infty e^{-x^2}\,dx \right) e^{-y^2}\,dy = \int_0^\infty \int_0^\infty e^{-x^2-y^2}\,dx\,dy =$$

$$= \left| \begin{array}{l} t = -\rho^2 \\ 1\,dt = -2\rho\, d\rho \\ \rho\, d\rho = -\frac{1}{2} dt \end{array} \right| \begin{array}{l} \rho = \infty \Rightarrow t = -\infty \\ \\ \rho = 0 \Rightarrow t = 0 \end{array} \left| = \int_0^{\frac{\pi}{2}} \int_0^{-\infty} e^t \cdot \left(-\frac{1}{2}\right) dt\, d\varphi = \right.$$

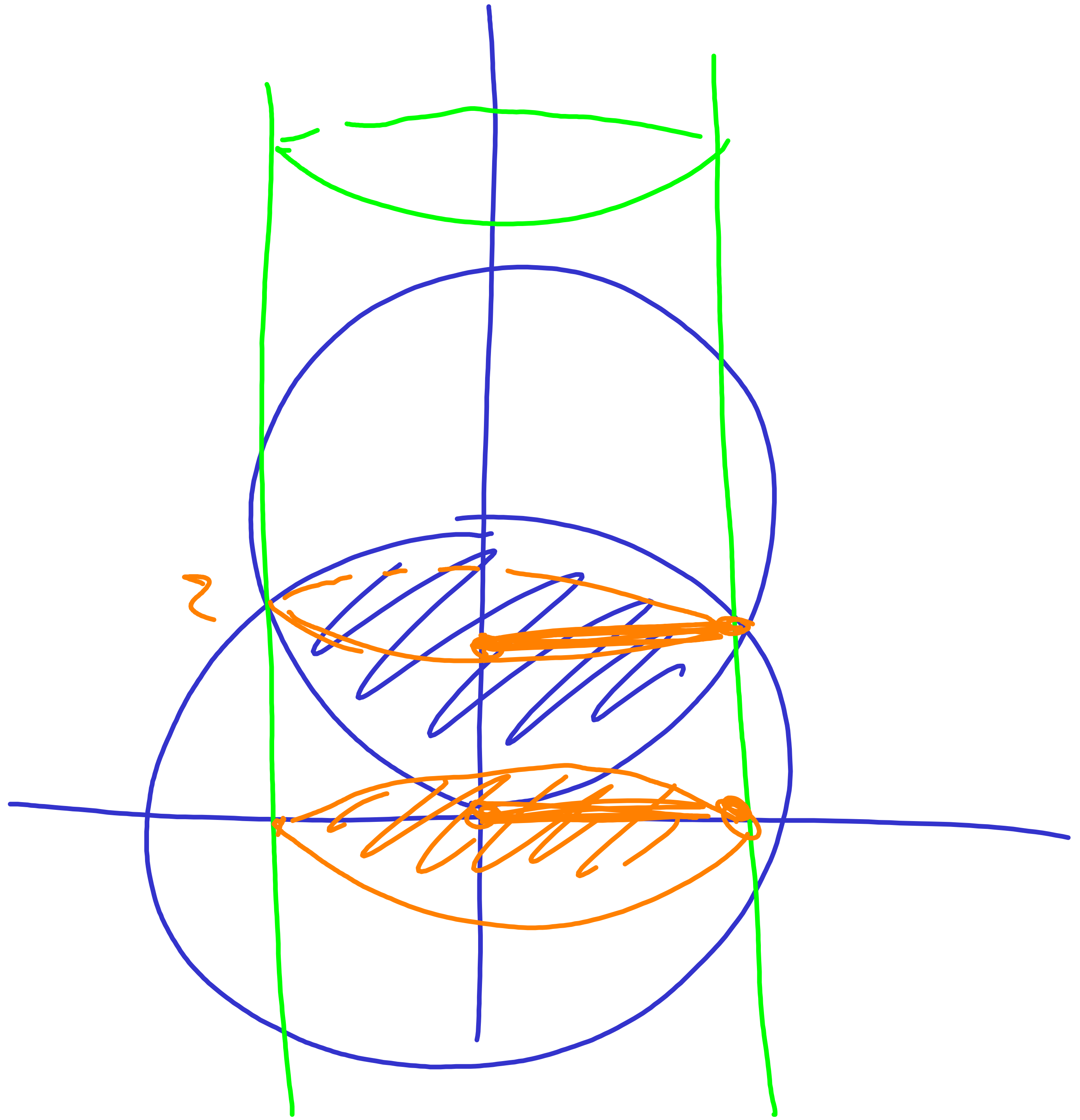
$$= +\frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \int_{-\infty}^{0} e^t \, dt\, d\varphi = \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \cdot \left[e^t \right]_{-\infty}^{0} d\varphi =$$

Př.: $V = ?$, $\underbrace{x^2+y^2+z^2=16}$, $\underbrace{x^2+y^2+z^2=8z}$

$$x^2+y^2+\underbrace{z^2-8z}=0$$

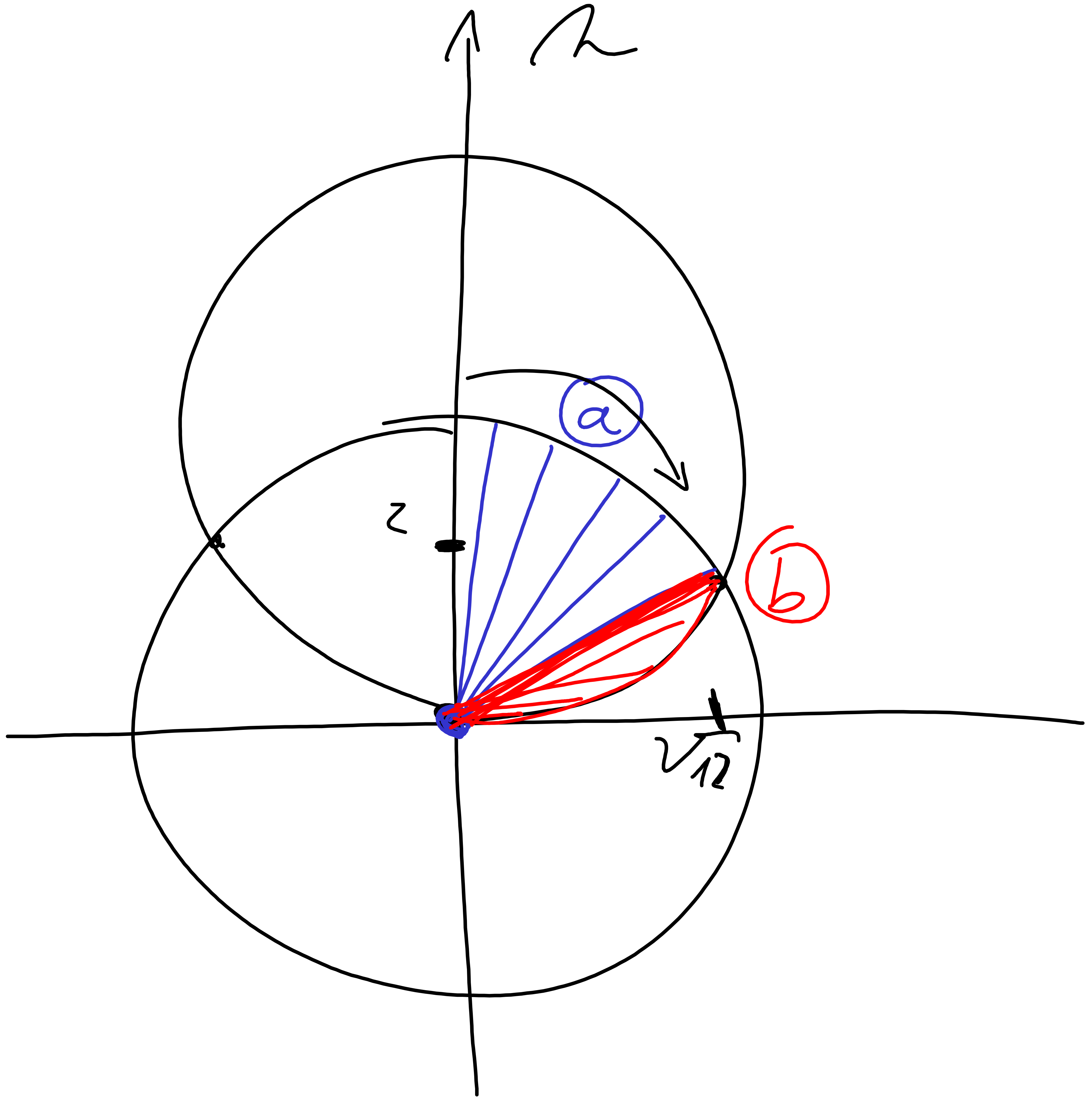
$$x^2+y^2+(z-4)^2-16=0$$

$$x^2+y^2+(z-4)^2=16$$



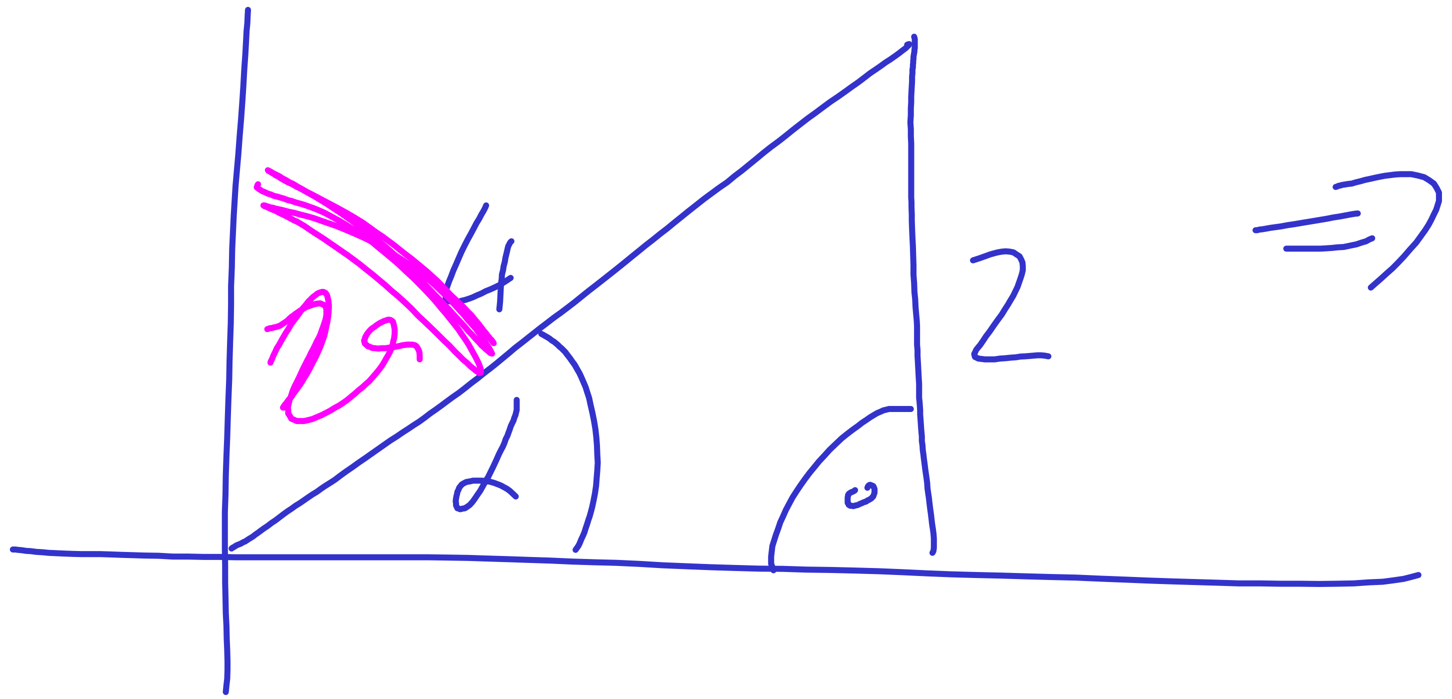
$$V = \iiint_{\Pi} 1 \, dx\, dy\, dz = \int_0^{\sqrt{12}} \left(\int_0^{2\pi} \left(\int_{4-\sqrt{16-\rho^2}}^{\sqrt{16-\rho^2}} 1 \cdot \rho \quad dz \right) d\varphi \right) d\rho =$$

$$= \left| \; 16-\rho^2 = t \; \right| = \ldots = \boxed{\frac{80}{3}\pi}$$



a $\varphi \in [0, 2\pi]$, $\rho \in [0, 4]$

$\vartheta \in [0, ?]$ $\frac{\pi}{3}$



$\sin \alpha = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \alpha = \frac{\pi}{6}$

$\vartheta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

b) $\varphi \in [0, 2\pi]$, $\vartheta \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

$\rho \in [0, ?]$

$$\underbrace{x^2+y^2+z^2}_{\rho^2} = 8z$$

$$\rho^2 = 8 \cdot \rho \cdot \cos\vartheta$$

$$\rho = 8 \cdot \cos\vartheta$$