

1 Cv. Popište v souřadnicích duální zobrazení
 k zobrazení $A: k^n \rightarrow k^m$, tj. $A \in \text{Mat}_{m \times n} k$

2 Cv. Dokažte: Necht' $U = V \oplus W$. Pak kompozice
 $W \hookrightarrow U \twoheadrightarrow U/V$ je izomorfismus.

3 Cv. Popište implicitně podprostor

$$[(1, -1, 0, 0)^T, (1, 0, -1, 0)^T, (0, 1, 0, -1)^T] \subseteq \mathbb{K}^4$$

4 Cv. Popište všechny roviny procházející přímkou
param/ impl.

$$P: \begin{aligned} x^1 + x^2 - x^3 &= 0 \\ x^1 - x^2 + x^3 &= 0 \end{aligned}$$

5 Cv. Pomocí Motzkinovy eliminace rozhodněte o řešitelnosti

$$x, y, z \geq 0$$

$$4 \geq x + y + z \geq 2$$

$$3 \geq x + y \geq 1$$

$$2 \geq z$$

Cv.

$$\min \{ \underbrace{(-2 \ 3 \ -1)}_{\text{lin. fce}} x \mid \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 4 & -3 & 1 \\ 2 & 1 & -1 \end{pmatrix}}_{\text{lin. nerce}} x \leq \begin{pmatrix} 10 \\ 3 \\ 10 \end{pmatrix}, x \geq 0 \}$$

$x = (0 \ 0 \ 3)^T$
 $Cx = -3$

→ převedeme ve tvar $Ax = b, x \geq 0$
 → přidáme proměnné přebytečné/nedostatečné
 $x^1 + x^2 + x^3 \leq 10$ $s^1 \geq 0$

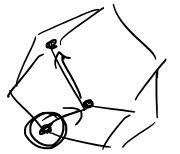
$x^1 = 0$
 $x^2 = 0$
 $x^3 = 0$
 $s^1 = 10$
 $s^2 = 3$
 $s^3 = 10$

→ $x^1 + x^2 + x^3 + s^1 = 10$

$t s^2 \ s^3$

$$\min \{ (-2 \ 3 \ -1 \ 0 \ 0 \ 0) \mid \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & -3 & 1 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} 10 \\ 3 \\ 10 \end{matrix} \mid x^1, x^2, x^3, s^1, s^2, s^3 \geq 0 \}$$

→ tvar, se kterým pracuje simplexová metoda
 "počáteční vodor" x^1, x^2, x^3 potřebujeme



$x^1 \leq 10$
 $x^1 \leq 3/4$
 $x^1 \leq 5$
 nevhodné →

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 10 \\ 4 & -3 & 1 & 0 & 1 & 0 & 3 \\ 2 & 1 & -1 & 0 & 0 & 1 & 10 \\ -2 & 3 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

↑
 použijeme 4
 k eliminaci
 úseč v jejím
 sloupci

$$\begin{pmatrix} 0 & 7 & 3 & 4 & -1 & 0 & 37 \\ 4 & -3 & 1 & 0 & 1 & 0 & 3 \\ 0 & 5 & -3 & 0 & -1 & 2 & 17 \\ 0 & 3/2 & -1/2 & 0 & 1/2 & 0 & 3/2 \end{pmatrix}$$

$3/3$
 3

$(0 \ 0 \ 0 \ 10 \ 3 \ 10) \dots Cx = 0$

$(3/4 \ 0 \ 0 \ 37/4 \ 0 \ 17/2) \quad Cx = -3/2$

$$\sim \begin{pmatrix} -12 & 16 & 0 & 4 & -4 & 0 & 28 \\ 4 & -3 & 1 & 0 & 1 & 0 & 3 \\ 12 & -4 & 0 & 0 & 2 & 2 & 26 \\ 2 & 0 & 0 & 0 & 1 & 0 & 3 \end{pmatrix}$$

$(0 \ 0 \ 3 \ 7 \ 0 \ 13) \dots Cx = -3$

→ minimum ✓

Cv.

$$\min \{ (2 \ 4 \ 7 \ 2 \ 5) x \mid \begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 1 & 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \end{pmatrix} x = \begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix}, x \geq 0 \}$$

$$x = (1 \ 0 \ 1 \ 0 \ 2)^T$$

$$Cx = 19$$

Nalezení počátečního vrcholu ...

$\min t^1 + t^2 + t^3$ $t^1, t^2, t^3 \geq 0$
 počet 0 --- řešení (vrchol)
 v původní hlavě

$$\begin{array}{cccccc|ccc} 1 & 1 & 2 & 1 & 2 & 1 & 0 & 0 & 7 \\ 1 & 2 & 3 & 1 & 1 & 0 & 1 & 0 & 6 \\ 1 & 1 & 1 & 2 & 1 & 0 & 0 & 1 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ \hline -3 & -4 & -6 & -4 & -4 & 0 & 0 & 0 & -17 \end{array}$$

$$\sim \begin{array}{cccccc|ccc} 1 & -1 & 0 & 1 & 4 & 3 & -2 & 0 & 9 \\ 1 & 2 & 3 & 1 & 1 & 0 & 1 & 0 & 6 \\ 2 & 1 & 0 & 5 & 2 & 0 & -1 & 3 & 6 \\ \hline -1 & 0 & 0 & -2 & -2 & 0 & 2 & 0 & -5 \end{array}$$

$s^1 \leq 2$
 $s^2 \leq 3$

$$\sim \begin{array}{cccccc|ccc} 0 & -3 & 0 & -3 & 6 & 6 & -3 & -3 & 12 \\ 0 & 3 & 6 & -3 & 0 & 0 & 3 & -3 & 6 \\ 2 & 1 & 0 & 5 & 2 & 0 & -1 & 3 & 6 \\ \hline 0 & 1/2 & 0 & 1/2 & -1 & 0 & 3/2 & 3/2 & -2 \end{array}$$

$$\sim \begin{array}{cccccc|ccc} 0 & -3 & 0 & -3 & 6 & 6 & -3 & -3 & 12 \\ 0 & 3 & 6 & -3 & 0 & 0 & 3 & -3 & 6 \\ 6 & 6 & 0 & 18 & 0 & -6 & 0 & 12 & 6 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

$t^1 = 0$
 $t^2 = 0$
 $t^3 = 0$
 $(1 \ 0 \ 1 \ 0 \ 2)$ vrchol
 pův. pol.

$$\begin{array}{cccccc|ccc} 0 & -3 & 0 & -3 & 6 & 6 & -3 & -3 & 12 \\ 0 & 3 & 6 & -3 & 0 & 0 & 3 & -3 & 6 \\ 6 & 6 & 0 & 18 & 0 & -6 & 0 & 12 & 6 \\ \hline 2 & 4 & 7 & 2 & 5 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc|ccc} 0 & 2 & 7 & -4 & 5 & 2 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccc|ccc} 0 & 1/2 & 0 & 1/2 & 5 & 4 & 0 & 0 & 0 \end{array}$$

$$\rightarrow \begin{array}{cccccc|ccc} 0 & 1 & 0 & 2 & 0 & -19 & 0 & 0 & 0 \end{array}$$

minimum 19

Cv.

$$\min \{ (0 \ 15 \ 30 \ 0 \ 15) x \mid \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 & 3 \end{pmatrix} x \geq \begin{pmatrix} 120 \\ 80 \\ 110 \end{pmatrix}, x \geq 0 \}$$

$x = (120 \ 0 \ 0 \ 55 \ 0)^T$
 $Cx = 0$

$$\begin{array}{c} \left(\begin{array}{ccccccccc|c} 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & \textcircled{1} & 120 \\ \textcircled{1} & 0 & \textcircled{2} & 1 & 0 & 0 & -1 & 0 & 0 & 80 \\ 0 & 1 & 0 & 2 & \textcircled{3} & 0 & 0 & -1 & 0 & 110 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -120 \\ \hline \end{array} \right) \\ \hline \end{array}$$

$$\begin{array}{c} \left(\begin{array}{ccccccccc|c} 0 & \textcircled{1} & -2 & -1 & 0 & -1 & 1 & 0 & \textcircled{1} & 40 \\ \textcircled{1} & 0 & 2 & 1 & 0 & 0 & -1 & 0 & 0 & 80 \\ 0 & 1 & 0 & 2 & \textcircled{3} & 0 & 0 & -1 & 0 & 110 \\ \hline 0 & -1 & 2 & 1 & 0 & 1 & -1 & 0 & 0 & -40 \\ \hline \end{array} \right) \\ \hline \end{array}$$

$$\left(\begin{array}{ccccccccc|c} 0 & \textcircled{1} & -2 & -1 & 0 & -1 & 1 & 0 & 1 & 40 \\ \textcircled{1} & 0 & 2 & 1 & 0 & 0 & -1 & 0 & 0 & 80 \\ 0 & 0 & 2 & 3 & \textcircled{3} & 1 & -1 & -1 & -1 & 70 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline \end{array} \right)$$

$$\begin{array}{c} \left(\begin{array}{ccccccccc|c} 0 & \textcircled{1} & -2 & -1 & 0 & -1 & \textcircled{1} & 0 & 0 & 40 \\ \textcircled{1} & 0 & 2 & 1 & 0 & 0 & -1 & 0 & 0 & 80 \\ 0 & 0 & 2 & 3 & \textcircled{3} & 1 & -1 & -1 & 0 & 70 \\ \hline 0 & 15 & 30 & 0 & 15 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 60 & 15 & 15 & 15 & -15 & 0 & 0 & -600 \\ \hline 0 & 0 & 50 & 0 & 0 & 10 & -10 & 5 & 0 & -950 \\ \hline \end{array} \right) \\ \hline \end{array}$$

$$\sim \left(\begin{array}{ccccccccc|c} 0 & 1 & -2 & -1 & 0 & -1 & \textcircled{1} & 0 & 0 & 40 \\ \textcircled{1} & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 120 \\ 0 & 1 & 0 & \textcircled{2} & \textcircled{3} & 0 & 0 & -1 & 0 & 110 \\ \hline \end{array} \right)$$

$$1) \begin{array}{cccc|c} (1) & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & (2) & (3) \\ \hline 0 & 10 & 30 & -10 & 0 & 0 & 0 & 5 & -550 \end{array}$$

$$2) \begin{array}{cccc|c} 0 & 3 & -4 & 0 & 3 & -2 & (2) & -1 & 190 \\ (1) & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 120 \\ 0 & 1 & 0 & (2) & 3 & 0 & 0 & -1 & 110 \\ \hline 0 & 15 & 30 & 0 & 15 & 0 & 0 & 0 & 0 \end{array}$$

vrchol (120 0 0 55 0) s^1 s^2 s^3
 0 95 0

LP min 0