

Exercises 13

- ① By a projective cover of an obj. A we mean $A' \xrightarrow{p_A} A$ with p_A surj. & A' projective?
Does the assignment $A \mapsto A'$ give a functor?
- ② Show that each vector space is both projective & injective.
- ③ Show \mathbb{Z}_n not an inj \mathbb{Z} -module for $n > 1$.
- ④ Prove that \mathbb{Q} is not a projective \mathbb{Z} -module. (Hint: if it was it would be a submodule of a free module - even a retract. Find a property of \mathbb{Q} not held by free \mathbb{Z} -modules.)
- ⑤ Use Baer criterion to show that for A an abelian group & $a \in A$ $\exists f: A \rightarrow \mathbb{Q}/\mathbb{Z}$ s.t. $f(a) \neq 0$. (We say that \mathbb{Q}/\mathbb{Z} is a cogenerator in the cat. of abelian groups.)

Hint: consider cyclic
group $\langle a \rangle \subseteq A$.