

Exercise 1: What is the initial object in Grp ? In Rng ?

Exercise 2: Coproducts & products are dual concepts. Thinking about Set , can you see a sense in which addition and multiplication of numbers are dual?

Exercise 3: What are coproducts of vector spaces, abelian groups or top. space?

we will look at coproducts in this setting when we study modules.

Exercise 4: Suppose that \mathcal{C} admits a terminal object 1 .

a) Show that there are isomorphisms

$$A \times 1 \cong A \cong 1 \times A$$

b) Define the product $A \times B \times C$ of three objects using a universal property & show it is unique up to iso.

c) Show that $(A \times B) \times C \cong A \times (B \times C)$

by showing both have the univ.
property defining $A \times B \times C$.

d) Given maps $f: A \rightarrow B$ &
 $g: C \rightarrow D$ use the univ. prop.
of the product to construct a map
 $f \times g: A \times C \rightarrow B \times D$.