Exercises 3

- A morphism $f : A \to B$ is a mono if for all $X \overset{g}{\underset{h}{\rightrightarrows}} A$ we have $fg = fh$ implies $g = h$.

1) Let $E \xrightarrow{e} A \overset{f}{\underset{g}{\rightrightarrows}} B$ be an equaliser diagram. Using the u.p. of the equaliser, prove that $e : E \to A$ is mono.

2) a) Show that in Set, each injective function is monic.

b) & that each monic is injective.

3) Pullbacks & pushouts:

- Pullbacks are limits of shape $\boxed{\begin{tikzcd}[ampersand replacement=\&] \& 0 \arrow[d] \\ 2 \arrow[r] \& 1 \end{tikzcd}}$ whilst pushouts are colimits of shape

$$\boxed{\begin{tikzcd} 1 \arrow[r] \arrow[d] & 0 \\ 2 & \end{tikzcd}} = \boxed{\begin{tikzcd} & 0 \arrow[d] \\ 2 \arrow[r] & 1 \end{tikzcd}}^{op}$$

- In elementary terms, given

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "g"'] & C \\ B & \end{tikzcd}$$

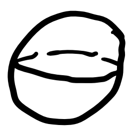
its pushout is an ob. P & comm. square

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "g"'] & C \arrow[d, "i"] \\ B \arrow[r, "j"'] & P \end{tikzcd}$$

which is universal amongst such comm. squares.

a) What does this universality mean precisely?

b) Show that you can construct pushouts from coproducts and coequalisers.

4) Pushouts in topology allow one to glue spaces together.
Try to draw a picture showing how to construct the 2-d sphere  as a pushout of two disks  & a circle .