

Exercises 7

Ex 1 a) Describe a signature Ω st \mathbb{R} -vector spaces are certain Ω -algebras.

b) What about G -sets for G a group?

Ex 2) Can you describe the initial object in Ω -Alg for a general signature Ω ?

Ex 3) - Let Ω be a unary signature: all operations have arity 1.

Show Ω -Alg $\cong [J, \text{Set}]$ for some small category J .

Defⁿ) A functor $U: A \rightarrow B$ is

- faithful if given $x \begin{matrix} \xrightarrow{f} \\ \xrightarrow{g} \end{matrix} y \in A$ then $Uf = Ug$ implies $f = g$;
- full if given $Ux \xrightarrow{F} Uy \in B$

there exists $\bar{F}: x \rightarrow y$ st $u\bar{F} = g$;

- fully faithful if it is Full and faithful.
- a full subcategory if U is inj on obs and fully faithful.

Ex 4) Exercise 1, is \mathbb{R} -Vect a full subcategory of Ω -Alg?

Ex 5) Describe some more examples of faithful & full functors in algebra.