

# CVIČENÍ 0

0.1

- |   |   |
|---|---|
| 1. $4.6 \in \mathbb{R}$   | T |
| 2. $-10 \in \mathbb{N}$   | F |
| 3. $6 \in \mathbb{Z}$   | T |
| 4. $1 \in \mathbb{R} \setminus \{2, 3, 4\}$   | T |
| 5. $3 \in \mathbb{R} \setminus \{2, 3, 4\}$   | F |
| 6. $6.4 \in \mathbb{N}$   | F |
| 7. $\{\frac{1}{4}, 0, -\frac{\sqrt{2}}{2}, -0.524, -50, 10^2, 0.666\} \in \mathbb{R}$ | T |
| 8. $\{-2, 0, 4, 0, -\frac{9}{3}\} \in \mathbb{Z}$                                     | T |
| 9. $\{4, -5, 3, 0, -15\} \in \mathbb{N}$  | F |
| 10. $\{4.0, 2.5, -3.7, -6.0, 4.3\} \in \mathbb{Z}$                                    | F |

0.2

1.  $\{-5, -15\}$
2.  $\{2.5, -3.7, 4.3\}$
3. řádní
4. řádní
5.  $\{\frac{3}{2}, \sqrt{2}, 0.2\}$

0.3

1.  $\langle 4; 5 \rangle$  ... 4 máloví, 5 menáloví
2.  $(-\infty; -3) \cup \langle 6; \infty \rangle$  ... -3 menáloví, 6 máloví
3.  $(0; 1)$  ... 0 menáloví, 1 menáloví

0.4

1.  $\langle 8; 10 \rangle$
2.  $(-\infty; -2) \cup (-2; \infty)$
3.  $\langle \frac{1}{2}; 2 \rangle$
4.  $(-\infty; -3) \cup \langle 6; \infty \rangle$

0.5

$$1. \frac{10}{3} + \frac{1}{4} - \frac{1}{12} = \frac{40+3-1}{12} = \frac{42}{12} = \frac{7}{2} = 3\frac{1}{2}$$

$$2. \frac{8}{5} - \frac{2}{3} + \frac{1}{6} = \frac{48-20+5}{30} = \frac{33}{30} = \frac{11}{10} = 1\frac{1}{10}$$

$$3. \frac{10}{3} - \frac{18}{8} = \frac{80-54}{24} = \frac{26}{24} = \frac{13}{12} = 1\frac{1}{12}$$

$$4. \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$$

$$5. \frac{x^2}{2} - x + \frac{1}{x+1} = \frac{x^2(x+1) - x \cdot 2(x+1) + 2}{2(x+1)} = \frac{x^3 + x^2 - 2x^2 - 2x + 2}{2(x+1)} = \frac{x^3 - x^2 - 2x + 2}{2(x+1)}$$

$$6. \frac{1}{x-1} - \frac{2}{x+2} = \frac{x+2-2(x-1)}{(x-1)(x+2)} = \frac{x+2-2x+2}{(x-1)(x+2)} = \frac{4-x}{(x-1)(x+2)} = -\frac{x-4}{(x-1)(x+2)}$$

0.6

$$1. \frac{\oplus}{\oplus} = \oplus$$

$$4. \frac{-4}{2} = -2$$

$$2. \frac{\oplus}{\ominus} = \ominus$$

$$5. \frac{3}{-6} = -\frac{1}{2}$$

$$3. \frac{\ominus}{\ominus} = \oplus$$

$$6. \frac{-6}{-12} = \frac{1}{2}$$

0.7

$$1. (9x-2)^2 = 81x^2 - 36x + 4$$

$$2. (3x^2-1)^2 = 9x^4 - 6x^2 + 1$$

$$3. (6x+1)^2 = 36x^2 + 12x + 1$$

$$4. (x+\sqrt{2})(x-\sqrt{2}) = x^2 - 2$$

$$5. (2x + \frac{1}{2})^2 = 4x^2 + 2x + \frac{1}{4}$$

$$6. (x^2 + \frac{1}{2})(x^2 - \frac{1}{2}) = x^4 - \frac{1}{4}$$

0.8

$$1. (2x-1)(x+3) = 2x^2 + 6x - x - 3 = 2x^2 + 5x - 3$$

$$2. (-2x+1)(x^2-1) = -2x^3 + 2x + x^2 - 1 = -2x^3 + x^2 + 2x - 1$$

$$3. (x-\frac{1}{2})(x+\frac{2}{4}) = x^2 - \frac{1}{4}$$



0.9

$$1. \frac{9x^4 + 12x^3 + 4}{-3x^2 - 2} = \frac{(3x^2 + 2)^2}{-(3x^2 + 2)} = -(3x^2 + 2)$$

$$2. \frac{x^2 - 3}{x + \sqrt{3}} = \frac{(x - \sqrt{3})(x + \sqrt{3})}{x + \sqrt{3}} = x - \sqrt{3}$$

$$3. \frac{x^2 - 6x + 9}{2x - 6} = \frac{(x - 3)^2}{2(x - 3)} = \frac{(x - 3)}{2}$$

$$4. \frac{x^2 + 1}{x + 1} \dots \text{nejde dale upravil}$$

$$5. \frac{x^2 - 10x + 25}{x^2 - 5x} = \frac{(x - 5)^2}{x(x - 5)} = \frac{(x - 5)}{x}$$

$$6. \frac{x^4 - \frac{1}{4}}{x^3 - \frac{x}{2}} = \frac{(x^2 - \frac{1}{2})(x^2 + \frac{1}{2})}{x(x^2 - \frac{1}{2})} = \frac{x^2 + \frac{1}{2}}{x}$$

0.10

$$1. \frac{\sqrt{x^3}}{3} - \frac{1}{y} = \frac{x^{\frac{3}{2}}}{3} - y^{-1}$$

$$2. -\sqrt[4]{y} + \frac{2}{-x^6} = -y^{\frac{1}{4}} - 2x^{-6}$$

$$3. \frac{2}{(-x)^6} + \frac{1}{\sqrt[3]{y^2}} = 2x^{-6} + y^{\frac{2}{3}}$$

$$4. \sqrt[5]{x^5} - \sqrt[5]{x} - \sqrt{x^5} - \frac{1}{\sqrt{x^5}} - \frac{1}{\sqrt[5]{x^2}} = x^{\frac{5}{5}} - x^{\frac{1}{5}} - x^{\frac{5}{2}} - x^{-\frac{5}{2}} - x^{-\frac{2}{5}}$$

0.11

$$1. 4x^{-\frac{4}{2}} + y^{-2} = \frac{4}{x^2} + \frac{1}{y^2}$$

$$2. 3y^{\frac{2}{5}} - x^{\frac{4}{2}} = 3\sqrt[5]{y^2} - x^2$$

$$3. x^{\frac{2}{3}} + y^{\frac{3}{2}} + x^{-\frac{2}{3}} + y^{-\frac{3}{2}} = \sqrt[3]{x^2} + \sqrt{y^3} + \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt{y^3}}$$

$$4. x^{-2} + x^{-1} + x^{-\frac{1}{2}} + x^0 + x^{\frac{1}{2}} + x^1 + x^2 = \frac{1}{x^2} + \frac{1}{x} + \frac{1}{\sqrt{x}} + 1 + \sqrt{x} + x + x^2$$

0.12

$$1. x^4 x^7 - (y^2)^5 = x^{11} - y^{10}$$

$$2. y^4 y^{-3} + (x^2)^2 x^{-4} = y + x^2 x^{-4} = y + 1$$

$$3. (x^3)^{\frac{4}{4}} + y^2 y^{-\frac{1}{2}} = x^3 + y^{\frac{3}{2}} = \sqrt[4]{x^3} + \sqrt{y^3}$$



0.13

$$1. 2 \ln(3) - 4 \ln(2) = \ln(3^2) - \ln(2^4) = \ln \frac{9}{16}$$

$$2. \frac{1}{2} \ln(4) + \ln 5 = \ln(\sqrt{4}) + \ln 5 = \ln 2.5 = \ln 10$$

$$3. 2 \ln(9) - 3 \ln(3) = \ln(9^2) - \ln(3^3) = \ln \frac{81}{27} = \ln 3$$

$$4. 3 \ln(4) + \ln(3) - 4 \ln(2) = \ln 4^3 + \ln(3) - \ln 2^4 = \ln 64 + \ln 3 - \ln 8 = \ln \frac{64 \cdot 3}{8} = \ln 4.3$$

$$5. 4 \ln(1) - \frac{1}{2} \ln(9) + \ln(3) = 4 \cdot 0 - \ln(\sqrt{9}) + \ln 3 = 0 - \ln 3 + \ln 3 = 0 = \ln 1$$

0.14

$$1. D\left(\frac{1}{x^2+1}\right) = \mathbb{R} \quad x \in \mathbb{R}$$

$$2. D\left(\frac{2}{x^2-4}\right) = |x^2-4 \neq 0 \Rightarrow x^2 \neq 4 \Rightarrow x \neq \pm 2| \\ = \mathbb{R} \setminus \{-2, 2\} \quad x \neq \pm 2$$

$$3. D\left(\frac{1}{\sqrt[3]{x-5}}\right) = \mathbb{R} \setminus \{5\} \quad x \neq 5$$

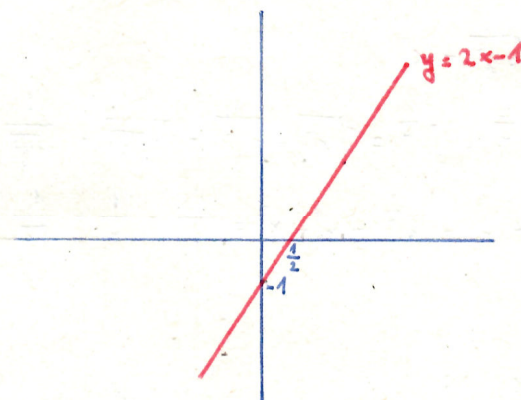
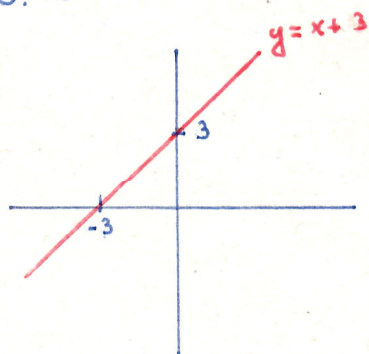
$$4. D(\sqrt{x-5}) = |x-5 \geq 0 \rightarrow x \geq 5| \\ = \langle 5; \infty \rangle \quad x \geq 5$$

$$5. D(\ln|x+3|) = \mathbb{R} \setminus \{-3\} \quad x \neq -3$$

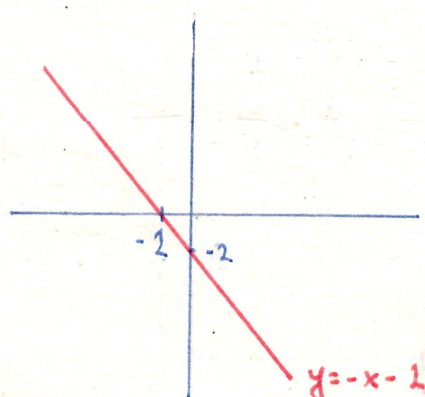
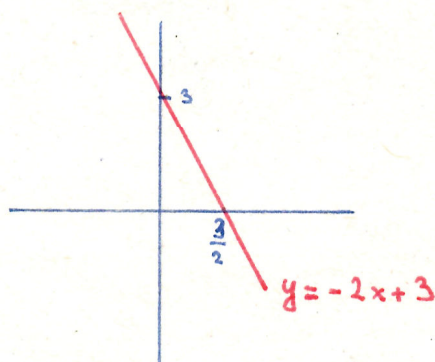
$$6. D\left(\frac{1}{\ln(-x-1)}\right) = |\ln(-x-1) \neq 0 \rightarrow (-x-1) > 0 \rightarrow x < -1| \\ = (-\infty; -1) \quad x < -1$$

0.15

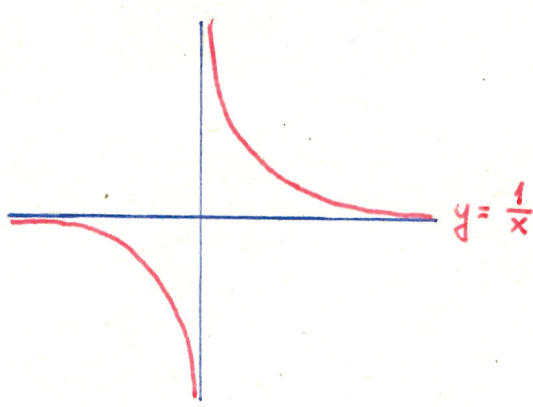
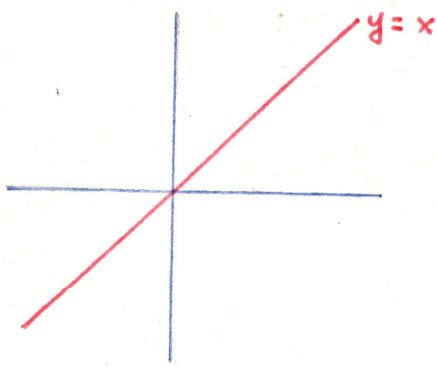
1.



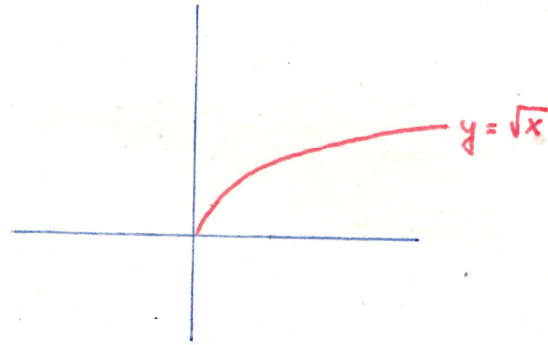
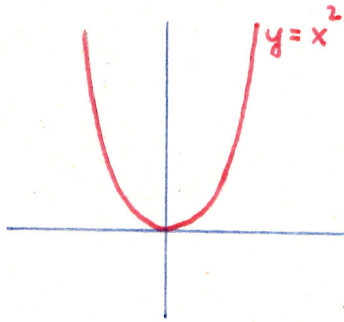
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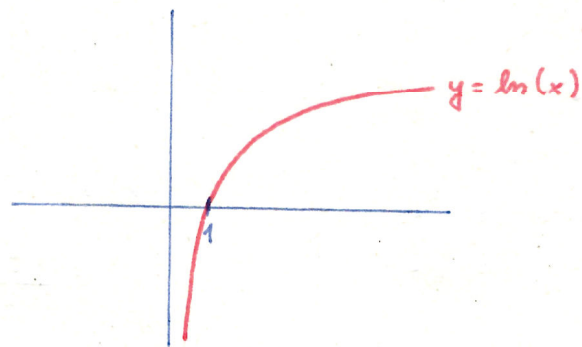
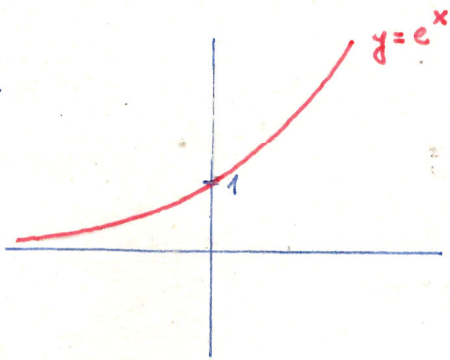
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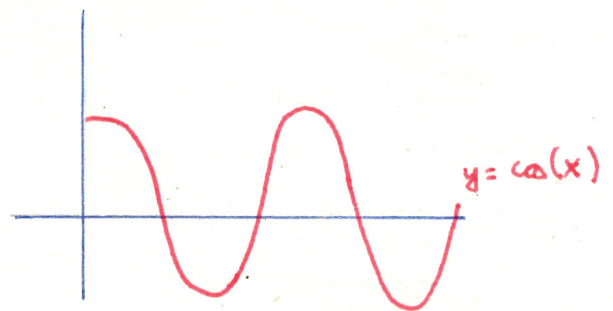
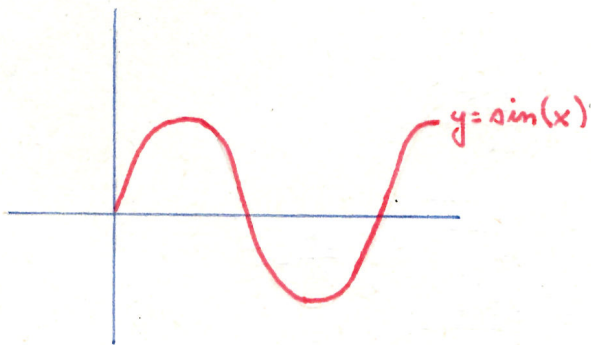
4.



5.



6.



0.16

$$1. \frac{\sin^2(x) + \cos^2(x)}{2} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$2. \tan(x) \cdot \cot(x) - 1 = \frac{\sin(x)}{\cos(x)} \cdot \frac{\cos(x)}{\sin(x)} - 1 = 1 - 1 = 0$$

$$3. \tan(x) + \sin(x) = \frac{\sin(x) + \sin(x)\cos(x)}{\cos(x)} = \frac{\sin(x)(1 + \cos(x))}{\cos(x)}$$

$$4. \sin(0) + \cos(\pi) + 1 - \sin^2\left(\frac{\pi}{2}\right) + \cos(0) = 0 - 1 + 1 - (1)^2 + 1 = 0$$

$$5. -\sin\left(\frac{3\pi}{2}\right) + 3\cos(\pi) - \cos^2(0) + \sin\left(\frac{\pi}{2}\right) + 4(\sin(0)) = -(-1) - 3 - 1^2 + 1 + 0 = 1 - 3 - 1 + 1 = -2$$



0.17

$$1. \quad \frac{2x-1}{7} = \frac{4x-5}{2}$$

$$2(2x-1) = 7(4x-5)$$

$$4x-2 = 28x-35$$

$$4x-28x = -35+2$$

$$-24x = -33$$

$$24x = 33$$

$$x = \frac{33}{24}$$

$$x = \frac{11}{8}$$

$$2. \quad \frac{3x-2}{5} = \frac{2x-1}{4}$$

$$12x-8 = 10x-5$$

$$12x-10x = 8-5$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$3. \quad x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$\begin{aligned} D: b^2 - 4ac &= 2^2 - 4 \cdot 1 \cdot (-8) \\ &= 4 + 32 \\ &= 36 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{2 \pm 6}{2} = \begin{cases} \frac{8}{2} = 4 \\ -\frac{4}{2} = -2 \end{cases} \end{aligned}$$

$$x_1 = -2, x_2 = 4$$

$$4. \quad 3x^2 - 10 = 13x$$

$$3x^2 - 13x - 10 = 0$$

$$\begin{aligned} D = b^2 - 4ac &= 13^2 + 4 \cdot 3 \cdot 10 \\ &= 169 + 120 \\ &= 289 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{13 \pm \sqrt{289}}{6} \\ &= \frac{13 \pm 17}{6} = \begin{cases} \frac{30}{6} = 5 \\ -\frac{4}{6} = -\frac{2}{3} \end{cases} \end{aligned}$$

$$x_1 = -\frac{2}{3}, x_2 = 5$$

$$5. \quad 9x - x^2 = 20$$

$$-x^2 + 9x - 20 = 0$$

$$x^2 - 9x + 20 = 0$$

$$\begin{aligned} D: b^2 - 4ac &= 81 - 4 \cdot 20 \\ &= 81 - 80 \\ &= 1 \end{aligned}$$

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{9 \pm 1}{2} = \begin{cases} \frac{10}{2} = 5 \\ \frac{8}{2} = 4 \end{cases} \end{aligned}$$

$$x_1 = 4, x_2 = 5$$

0.18

$$1. \frac{3x-7}{6} > \frac{12}{9}$$

$$9(3x-7) > 12 \cdot 6$$

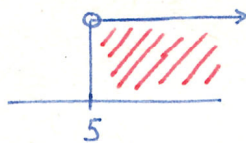
$$27x - 63 > 72$$

$$27x > 72 + 63$$

$$27x > 135$$

$$x > \frac{135}{27}$$

$$x > 5 \rightarrow x \in (5; \infty)$$



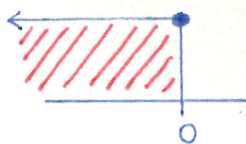
$$2. \frac{5x-6}{2} \geq 4x-3$$

$$5x-6 \geq 8x-6$$

$$0 \geq 3x$$

$$3x \leq 0$$

$$x \leq 0 \rightarrow x \in (-\infty; 0]$$

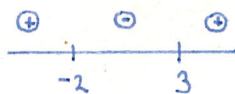
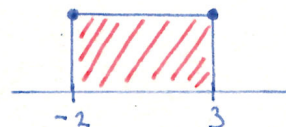


$$3. x^2 - x - 6 \leq 0$$

$$D: b^2 - 4ac = 1 + 4 \cdot 6$$

$$= 25$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm 5}{2} = \begin{cases} 3 \\ -2 \end{cases}$$



$$\rightarrow x \geq -2 \wedge x \leq 3$$

$$x \in [-2, 3]$$

$$4. -7x + 2 \geq -5x^2$$

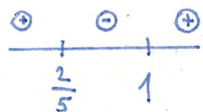
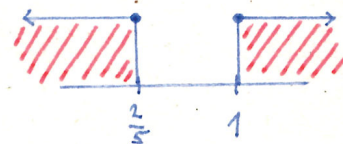
$$5x^2 - 7x + 2 \geq 0$$

$$D: b^2 - 4ac = 49 - 4 \cdot 5 \cdot 2$$

$$= 49 - 40$$

$$= 9$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{7 \pm 3}{10} = \begin{cases} \frac{10}{10} = 1 \\ \frac{4}{10} = \frac{2}{5} \end{cases}$$



$$\rightarrow x \leq \frac{2}{5} \vee x \geq 1$$

$$x \in (-\infty; \frac{2}{5}] \cup [1; \infty)$$

$$5. 8x^2 < 6x + 5$$

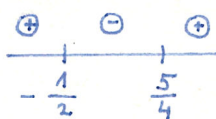
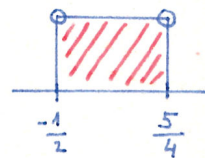
$$8x^2 - 6x - 5 < 0$$

$$D: b^2 - 4ac = 36 + 4 \cdot 5 \cdot 8$$

$$= 36 + 160$$

$$= 196$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm 14}{16} = \begin{cases} \frac{20}{16} = \frac{5}{4} \\ -\frac{8}{16} = -\frac{1}{2} \end{cases}$$



$$\rightarrow x > -\frac{1}{2} \wedge x < \frac{5}{4}$$

$$x \in (-\frac{1}{2}, \frac{5}{4})$$