CVIČENÍ 3

3.1

$f(x) = -(2x-3)^2$

1. $D(f) = \mathbb{R}$

2. $H(f)$: $-(2x-3)^2 = y$

$-(4x^2 - 12x + 9) = y$

$-4x^2 + 12x - 9 - y = 0$

$4x^2 - 12x + 9 + y = 0$

$D = b^2 - 4ac$
$= 144 - 4 \cdot 4(9+y)$
$= 144 - 144 - 16y$
$= -16y$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{12 \pm \sqrt{-16y}}{8} = \frac{3 \pm \sqrt{-y}}{2}$

$\Downarrow$

$-y \geq 0$
$y \leq 0$

$H(f) = (-\infty; 0\rangle$

3. $f(-x) = -(2(-x)-3)^2 = -(-2x-3)^2 = -(2x+3)^2$

$-f(x) = (2x-3)^2$

$f(-x) \neq f(x)$… není sudá
$f(-x) \neq -f(x)$… není lichá

4. neperiodická, neobsahuje periodickou funkci

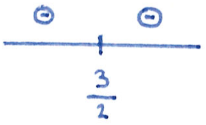
5. BN nemá

6. $-(2x-3)^2 = 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$



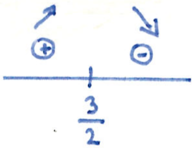
7. $(-(2x-3)^2)' = 0$

$-2(2x-3) \cdot 2 = 0$

$-8x + 12 = 0$

$8x = 12$

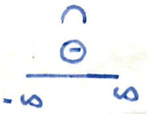
$x = \frac{12}{8} = \frac{3}{2}$



8. $(-(2x-3)^2)'' = 0$

$(-8x + 12)' = 0$

$-8 = 0$



10. $f(0) = -9$

$f(\frac{3}{2}) = 0$

9. ABS: nemá BN → nemá ABS

ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{-(2x-3)^2}{x} =$

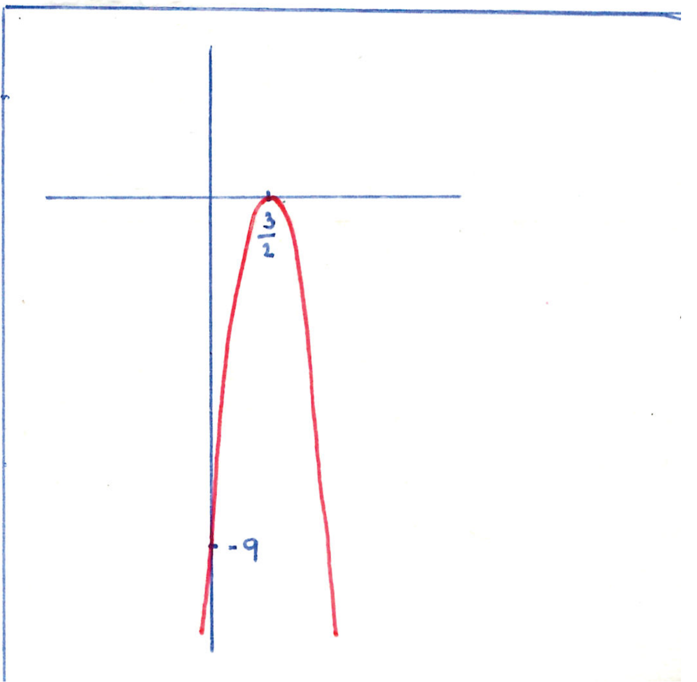
$= \lim\limits_{x \to \infty} \frac{-4x^2 + 12x - 9}{x} = \lim\limits_{x \to \infty} \frac{x(-4x + 12 - \frac{9}{x})}{x}$

$= \lim\limits_{x \to \infty} -4x + 12 - \frac{9}{x} = -\infty$

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{-4x^2 + 12x - 9}{x} =$

$= \lim\limits_{x \to -\infty} -4x + 12 - \frac{9}{x} = \infty$

$\Rightarrow$ ASS neexistuje



3. 2

$f(x) = -x^3 - 1$

1. $D(f) = \mathbb{R}$

2. $H(f):\ -x^3 - 1 = y$

$x^3 = -1 - y$

$x = \sqrt[3]{-1-y}$ $y \in \mathbb{R} \Rightarrow H(f) = \mathbb{R}$

3. $f(-x) = -(-x^3) - 1 = x^3 - 1$ $f(-x) \neq f(x)$... není sudá

$-f(x) = x^3 + 1$ $f(-x) \neq -f(x)$... není lichá

4. neperiodická

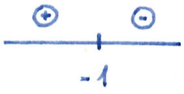
5. BN nemá

6. $-x^3 - 1 = 0$

$x^3 = -1$

$x = \sqrt[3]{-1}$

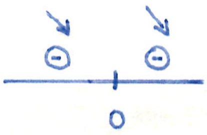
$x = -1$



7. $(-x^3-1)' = 0$

$-3x^2 = 0$

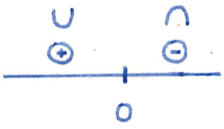
$x^2 = 0$



8. $(-x^3-1)'' = 0$

$(-x^2)' = 0$

$-2x = 0$



9. ABS: nemá BN → nemá ABS

ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{-x^3 - 1}{x} = \lim\limits_{x \to \infty} -x^2 - \frac{1}{x} =$

$= -\infty - 0 = -\infty$

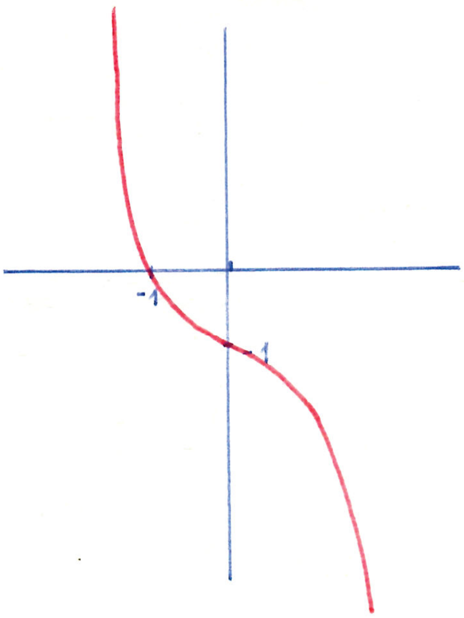
$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{-x^3 - 1}{x} = \lim\limits_{x \to -\infty} -x^2 - \frac{1}{x} =$

$= -\infty - 0 = -\infty$

→ ASS neexistuje

10. $f(0) = -1$

$f(-1) = 0$



3.3

$f(x) = -\frac{1}{3x^3} + 9$

1. $D(f) = \mathbb{R} \setminus \{0\}$

2. $H(f)$: $-\frac{1}{3x^3} + 9 = y$

$-\frac{1}{3x^3} = y - 9$

$-1 = (y-9)3x^3$

$3x^3 = \frac{-1}{y-9}$

$3x^3 = \frac{1}{9-y}$

$x^3 = \frac{1}{27-3y}$

$x = \sqrt[3]{\frac{1}{27-3y}}$ => $27 - 3y \neq 0$

$3y \neq 27$

$y \neq 9$

Zk: $9 = -\frac{1}{3x^3} + 9$

$0 = -\frac{1}{3x^3} \Rightarrow y \neq 9$

$H(f) = \mathbb{R} \setminus \{9\}$

3. $f(-x) = -\frac{1}{3(-x)^3} + 9 = \frac{1}{3x^3} + 9$

$-f(x) = \frac{1}{3x^3} - 9$

$f(-x) \neq f(x) \rightarrow$ není sudá

$f(-x) \neq -f(x) \rightarrow$ není lichá

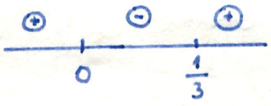
4. neperiodická, neobsahuje periodickou funkci

5. BN: $x = 0$

6. $-\frac{1}{3x^3} + 9 = 0$

$\frac{1}{3x^3} = 9$

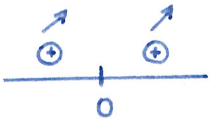
$x^3 = \frac{1}{27} \rightarrow x = \frac{1}{3}$



7. $(-\frac{1}{3x^3} + 9)' = 0$

$(-\frac{1}{3}x^{-3} + 9)' = 0$

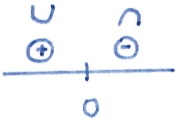
$x^{-4} = 0$



8. $(-\frac{1}{3x^3} + 9)'' = 0$

$(x^{-4})' = 0$

$-4x^{-5} = 0$



9. ABS: $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} -\frac{1}{3x^3} + 9 = -\infty$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} -\frac{1}{3x^3} + 9 = \infty$

10. $f(0)$... neexistuje

$f(\frac{1}{3}) = 0$

ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} -\frac{1}{3x^4} + \underset{0}{\frac{9}{x}} = \lim\limits_{x \to \infty} \frac{-1 + 3x^3}{3x^4}$

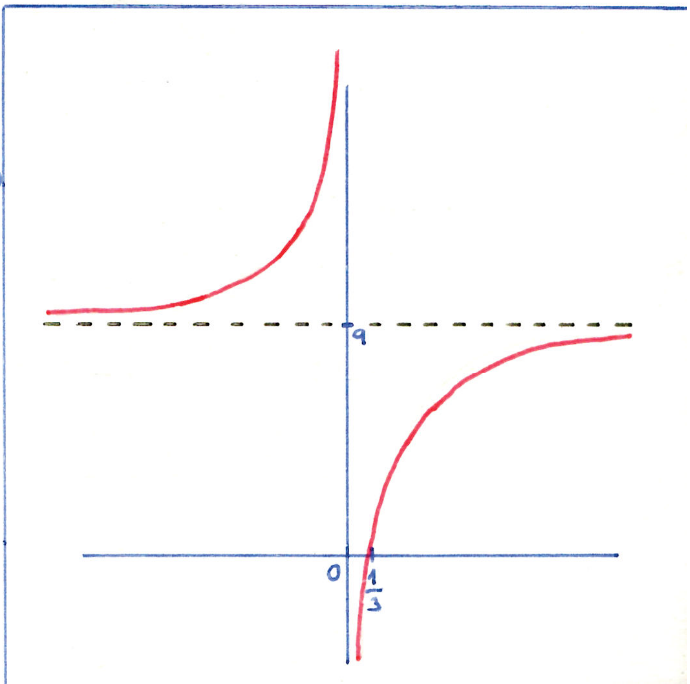
$= \lim\limits_{x \to \infty} \frac{x^3(-\frac{1}{x^3} + 3)}{x^3 3x} = \lim\limits_{x \to \infty} \frac{-\frac{1}{x^3} + 3}{3x} = 0$... $a = 0$

$\lim\limits_{x \to \infty} f(x) - ax = \lim\limits_{x \to \infty} -\frac{1}{3x^3} + 9 = 9$... $b = 9$

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} -\frac{1}{3x^4} + \frac{9}{x} = -0 + 0 = 0$... $a = 0$

$\lim\limits_{x \to -\infty} f(x) - ax = \lim\limits_{x \to -\infty} -\frac{1}{3x^3} + 9$... $b = 9$

ASS: $y = 9$



3.4

$f(x) = -\frac{5}{x+2}$

1. $D(f) = \mathbb{R} \setminus \{-2\}$

2. $H(f)$: $-\frac{5}{x+2} = y$

$-5 = y(x+2)$

$-\frac{5}{y} - 2 = x$

$\frac{-5-2y}{y} = x$

$x = \frac{-(5+2y)}{y}$ ... $y \neq 0$

Zk: $0 = -\frac{5}{x+2}$

$0 = -5 \Rightarrow y \neq 0$

$H(f) = \mathbb{R} \setminus \{0\}$

3. $f(-x) = -\frac{5}{-x+2} = \frac{5}{x-2}$

$-f(x) = \frac{5}{x+2}$

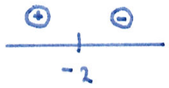
$f(-x) \neq f(x)$ ... není sudá

$f(-x) \neq -f(x)$ ... není lichá

4. neperiodická, neobsahuje periodickou funkci

5. BN: $x = -2$

6. $-\frac{5}{x+2} = 0$ $\quad x \neq -2$



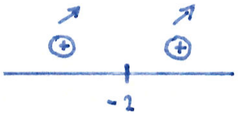
$-5 = 0$ ... nemá nulové body kromě BN

7. $\left(-\frac{5}{x+2}\right)' = 0$

$\left(-5(x+2)^{-1}\right)' = 0$

$-5(-1)(x+2)^{-2} = 0$

$\frac{5}{(x+2)^2} = 0$ ... $x \neq -2$

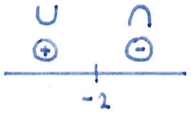


8. $\left(-\frac{5}{x+2}\right)'' = 0$

$\left(5(x+2)^{-2}\right)' = 0$

$5(-2)(x+2)^{-3} = 0$

$\frac{-10}{(x+2)^3} = 0$ ... $x \neq -2$



9. ABS: $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} -\frac{5}{x+2} = -\infty$

$\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} -\frac{5}{x+2} = \infty$

ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} -\frac{5}{x^2+2x} = 0$ ... $a = 0$

$\lim\limits_{x \to \infty} f(x) - ax = \lim\limits_{x \to \infty} -\frac{5}{x+2} = 0$ ... $b = 0$

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} -\frac{5}{x^2+2x} = 0$ ... $a = 0$

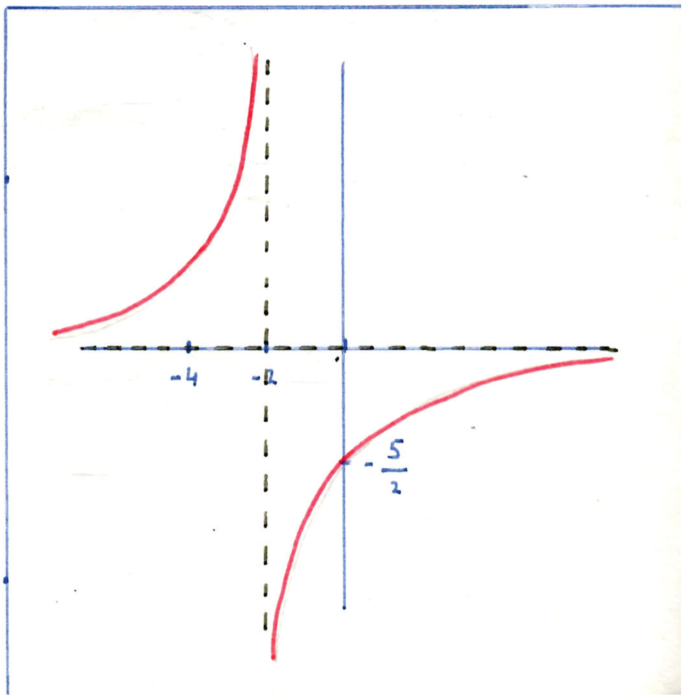
$\lim\limits_{x \to -\infty} f(x) - ax = \lim\limits_{x \to -\infty} -\frac{5}{x+2} = 0$ ... $b = 0$

ASS: $y = 0$

10. $f(0) = -\frac{5}{2}$

$f(-2)$ ... neexistuje

$f(-4) = \frac{5}{2}$



3.5

$$f(x) = \frac{(x+1)^2}{2x}$$

1. $D(f) = \mathbb{R} \setminus \{0\}$

2. $H(f): \quad \frac{(x+1)^2}{2x} = y$

$$x^2 + 2x + 1 = 2xy$$

$$x^2 + x(2-2y) + 1 = 0$$

$D: b^2 - 4ac$

$(2-2y)^2 - 4$

$4 - 8y + 4y^2 - 4$

$4y^2 - 8y$

$4y(y-2)$

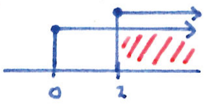
$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{2y-2 \pm \sqrt{4y(y-2)}}{2}$$

$$= y - 1 \pm \sqrt{\underbrace{4y(y-2)}_{\geq 0}}$$

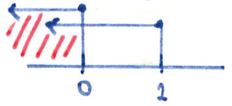
1. $y \geq 0 \wedge (y-2) \geq 0$

$y \geq 0 \wedge y \geq 2$



2. $y \leq 0 \wedge (y-2) \leq 0$

$y \leq 0 \wedge y \leq 2$



$H(f) = (-\infty; 0\rangle \cup \langle 2; \infty)$

3. $f(-x) = \frac{(-x+1)^2}{-2x} = \frac{(1-x)^2}{-2x}$

$-f(x) = -\frac{(x+1)^2}{2x}$

$f(-x) \neq f(x)$ … není sudá

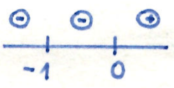
$f(-x) \neq -f(x)$ … není lichá

4. neperiodická, neobsahuje periodickou funkci

5. BN: $x = 0$

6. $\frac{(x+1)^2}{2x} = 0$ … $x = 0$

$x = -1$



7. $\left(\frac{(x+1)^2}{2x}\right)' = 0$

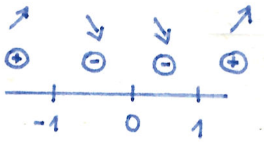
$$\left(\frac{2(x+1)2x - (x+1)^2 2}{4x^2} = 0\right)$$

$$\left(\frac{(x+1)(4x-2x-2)}{4x^2} = 0\right)$$

$$\frac{(x+1)(2x-2)}{4x^2} = 0$$

$\frac{x^2-1}{2x^2} = 0$ … $x \neq 0$

$x \neq \pm 1$

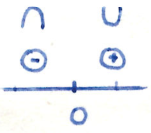


8. $\left(\frac{(x+1)^2}{2x}\right)'' = ($

$$\left(\frac{1}{2} - \frac{1}{2}x^{-2}\right)' = 0$$

$$-\frac{1}{2}(-2)x^{-3} = 0$$

$\frac{1}{x^3} = 0$ … $x \neq 0$



10. $f(0)$ … neexistuje

$f(1) = 2$

$f(-1) = 0$

9. ABS: $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \frac{(x+1)^2}{2x} = \lim\limits_{x \to 0^+} \frac{x^2 + 2x + 1}{2x}$

$$= \lim_{x \to 0^+} \frac{x}{2} + 1 + \frac{1}{2x} = 0 + 1 + \infty = \infty$$

$$\lim_{x \to 0^-} f(x) = \lim \frac{x}{2} + 1 + \frac{2}{x} = 0 + 1 - \infty = -\infty$$

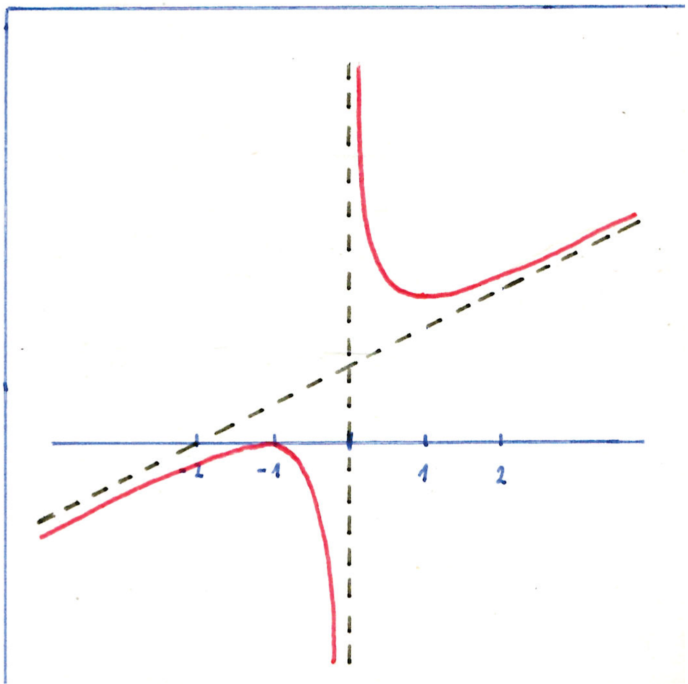
ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{x^2+2x+1}{2x^2} = \lim\limits_{x \to \infty} \frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2} = \frac{1}{2}$

$$\lim_{x \to \infty} f(x) - ax = \lim_{x \to \infty} \frac{x^2+2x+1-x^2}{2x} = \lim_{x \to \infty} 1 + \frac{1}{2x} = 1$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x^2+2x+1}{2x^2} = \lim_{x \to -\infty} \frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2} = \frac{1}{2}$$

$$\lim_{x \to -\infty} f(x) - ax = \lim_{x \to -\infty} \frac{x^2+2x+1-x^2}{2x} = \lim_{x \to -\infty} 1 - \frac{1}{2x} = 1$$

ASS: $y = \frac{1}{2}x + 1$



3.6

$f(x) = -\frac{(2x-1)^2}{x^2}$

1. $D(f) = \mathbb{R} \setminus \{0\}$

2. $H(f)$: $-\frac{(2x-1)^2}{x^2} = y$

$-(4x^2-4x+1) = yx^2$

$-4x^2 - yx^2 + 4x - 1 = 0$

$x^2(-4-y) + 4 - 1 = 0$

$D: b^2 - 4ac =$

$= 16 - 4(-4-y)(-1)$

$= 16 - 16 - 4y$

$= -4y$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-4 \pm \sqrt{-4y}}{-2(4-y)}$

$= \frac{-4 \pm \sqrt{-4y}}{-8+2y} > 0$ $\neq 0$

• $-4y \geq 0$

$-y \geq 0$

$y \leq 0$

• $-8 + 2y \neq 0$

$2y \neq 8$

$y \neq 4$

$H(f): (-\infty; 0)$

3. $f(-x) = -\frac{(2(-x)-1)^2}{(-x)^2} = -\frac{(-2x-1)^2}{x^2}$

$-f(x) = \frac{(2x-1)^2}{x^2}$

$f(-x) \neq f(x)$... není sudá

$f(-x) \neq -f(x)$... není lichá

4. neperiodická, neobsahuje periodickou funkci

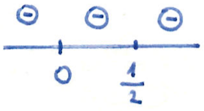
5. BN: $x = 0$

6. $-\frac{(2x-1)^2}{x^2} = 0 \quad x \neq 0$

$-(2x-1)^2 = 0$

$2x = 1$

$x = \frac{1}{2}$



7. $\left(-\frac{(2x-1)^2}{x^2}\right)' = 0$

$\left(-\frac{4x^2-4x+1}{x^2}\right)' = 0$

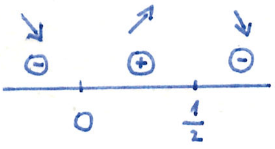
$\left(-4 + \frac{4}{x} - \frac{1}{x^2}\right)' = 0$

$\frac{4}{x^2} + \frac{2}{x^3} = 0$

$\frac{-4x+2}{x^3} = 0 \quad x \neq 0$

$4x = 2$

$x = \frac{1}{2}$



8. $\left(-\frac{(2x-1)^2}{x^2}\right)'' = 0$

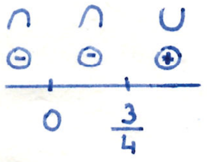
$\left(-\frac{4}{x^2} + \frac{2}{x^3}\right)' = 0$

$\frac{8}{x^3} - \frac{6}{x^4} = 0$

$\frac{8x-6}{x^4} = 0 \quad x \neq 0$

$8x = 6$

$x = \frac{3}{4}$



9. ABS: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -\frac{4x^2-4x+1}{x^2} = \lim_{x\to 0^+} -4 + \frac{4}{x} - \frac{1}{x^2} = -\infty$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -\frac{4x^2-4x+1}{x^2} = \lim_{x\to 0^-} -4 + \frac{4}{x} - \frac{1}{x^2} = -\infty$

ASS: $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} -\frac{4x^2-4x+1}{x^3} = \lim_{x\to\infty} -\frac{4}{x} - \frac{4}{x^2} - \frac{1}{x^3} = 0$ $a = 0$

$\lim_{x\to\infty} f(x) - ax = \lim_{x\to\infty} -\frac{4x^2-4x+1}{x^2} = \lim_{x\to\infty} -4 + \frac{4}{x} - \frac{1}{x^2} = -4$ $b = -4$

$\lim_{x\to -\infty} \frac{f(x)}{x} = \lim_{x\to -\infty} -\frac{4}{x} + \frac{4}{x^2} - \frac{1}{x^3} = 0$ ... $a = 0$

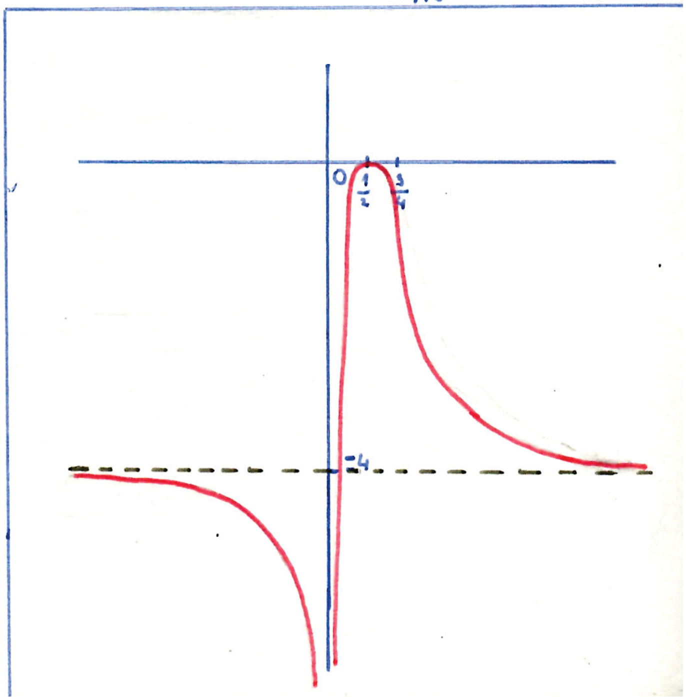
$\lim_{x\to -\infty} f(x) - ax = \lim_{x\to -\infty} -4 + \frac{4}{x} - \frac{1}{x^2} = -4$ $b = -4$

ASS: $y = -4$

10. $f(0)$ neexistuje

$f(1/2) = 0$

$f(\frac{1}{4}) = -\frac{1/4}{1/16} = -4$



CVIČENÍ 4

4.1

1. $\frac{\partial}{\partial x}(xy - \ln x + \sin y) = y - \frac{1}{x}$

$\frac{\partial}{\partial y}(xy - \ln x + \sin y) = x + \cos y$

2. $\frac{\partial}{\partial x}(y^2 \cos x) = -y^2 \sin x$

$\frac{\partial}{\partial y}(y^2 \cos x) = 2y \cos x$

3. $\frac{\partial}{\partial x}\left(\ln\left(\frac{x+y}{2}\right)\right) = \frac{1}{\frac{x+y}{2}} \cdot \frac{1}{2} = \frac{1}{x+y}$

$\frac{\partial}{\partial y}\left(\ln\left(\frac{x+y}{2}\right)\right) = \frac{1}{\frac{x+y}{2}} \cdot \frac{1}{2} = \frac{1}{x+y}$

4. $\frac{\partial}{\partial x}(y^2 \sin^2 x) = y^2 2 \sin x \cos x = 2y^2 \sin x \cos x$

$\frac{\partial}{\partial y}(y^2 \sin^2 x) = 2y \sin^2 x$

5. $\frac{\partial}{\partial x}(y^2 e^{2x}) = y^2 e^{2x} 2 = 2y^2 e^{2x}$

$\frac{\partial}{\partial y}(y^2 e^{2x}) = 2y e^{2x}$

6. $\frac{\partial}{\partial x}(-\cos(x+y)) = \sin(x+y)$

$\frac{\partial}{\partial y}(-\cos(x+y)) = \sin(x+y)$

7. $\frac{\partial}{\partial x}(e^{x^2y^2}) = e^{x^2y^2} \cdot 2xy^2 = 2xy^2 e^{x^2y^2}$

$\frac{\partial}{\partial y}(e^{x^2y^2}) = e^{x^2y^2} 2yx^2 = 2x^2y\, e^{x^2y^2}$

4.2

1. $f(x,y) = xy - \ln x + \sin y$ $\quad \frac{\partial}{\partial x} = y - \frac{1}{x}$ $\quad \frac{\partial}{\partial y} = x + \cos y$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(y - \frac{1}{x}\right) = \frac{1}{x^2}$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}(x + \cos y) = -\sin y$$

$$\frac{\partial^2}{xy} f(x,y) = \frac{\partial}{\partial y}\left(y - \frac{1}{x}\right) = 1$$

$$\frac{\partial^2}{yx} f(x,y) = \frac{\partial}{\partial x}(x + \cos y) = 1$$

2. $f(x,y) = y^2 \cos x$ $\quad \frac{\partial}{\partial x} = -y^2 \sin x$ $\quad \frac{\partial}{\partial y} = 2y\cos(x)$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(-y^2 \sin x\right) = -y^2 \cos x$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(2y\cos x\right) = 2\cos x$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(-y^2 \sin x\right) = -2y\sin x$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(2y\cos x\right) = -2y\sin x$$

3. $f(x,y) = \ln\left(\frac{x+y}{2}\right)$ $\quad \frac{\partial}{\partial x} = \frac{1}{x+y}$ $\quad \frac{\partial}{\partial y} = \frac{1}{x+y}$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{x+y}\right) = -\frac{1}{(x+y)^2}$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(\frac{1}{x+y}\right) = -\frac{1}{(x+y)^2}$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(\frac{1}{x+y}\right) = -\frac{1}{(x+y)^2}$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{x+y}\right) = -\frac{1}{(x+y)^2}$$

$4.\ f(x,y) = y^2 \sin^2 x$ $\qquad \frac{\partial}{\partial x} = 2y^2 \sin x \cos x$ $\qquad \frac{\partial}{\partial y} = 2y \sin^2 x$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(2y^2 \sin x \cos x\right) = 2y^2 \cos^2 x - 2y^2 \sin^2 x = 2y^2\left(\cos^2 x - \sin^2 x\right)$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(2y \sin^2 x\right) = 2 \sin^2 x$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(2y^2 \sin x \cos x\right) = 4y \sin x \cos x$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(2y \sin^2 x\right) = 2y \cdot 2 \sin x \cos x = 4y \sin x \cos x$$

$5.\ f(x,y) = y^2 e^{2x}$ $\qquad \frac{\partial}{\partial x} = 2y^2 e^{2x}$ $\qquad \frac{\partial}{\partial y} = 2y e^{2x}$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(2y^2 e^{2x}\right) = 2y^2 e^{2x} \cdot 2 = 4y^2 e^{2x}$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(2y e^{2x}\right) = 2e^{2x}$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(2y^2 e^{2x}\right) = 4y e^{2x}$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(2y e^{2x}\right) = 2y e^{2x} \cdot 2 = 4y e^{2x}$$

$6.\ f(x,y) = -\cos(x+y)$ $\qquad \frac{\partial}{\partial x} = \sin(x+y)$ $\qquad \frac{\partial}{\partial y} = \sin(x+y)$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\sin(x+y)\right) = \cos(x+y)$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(\sin(x+y)\right) = \cos(x+y)$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(\sin(x+y)\right) = \cos(x+y)$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(\sin(x+y)\right) = \cos(x+y)$$

7. $f(x,y) = e^{x^2y^2}$ $\qquad \frac{\partial}{\partial x} = 2xy^2e^{x^2y^2}$ $\qquad \frac{\partial}{\partial y} = 2x^2ye^{x^2y^2}$

$$\frac{\partial}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(2xy^2e^{x^2y^2}\right) = 2y^2e^{x^2y^2} + 2xy^2e^{x^2y^2}\,2xy^2 = 2y^2e^{x^2y^2} + 4x^2y^4e^{x^2y^2} = 2y^2e^{x^2y^2}(1+2x^2y^2)$$

$$\frac{\partial}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(2x^2ye^{x^2y^2}\right) = 2x^2e^{x^2y^2} + 2x^2ye^{x^2y^2}\,2x^2y = 2x^2e^{x^2y^2} + 4x^4y^2e^{x^2y^2} = 2x^2e^{x^2y^2}(1+2x^2y^2)$$

$$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(2xy^2e^{x^2y^2}\right) = 4xye^{x^2y^2} + 2xy^2e^{x^2y^2}\,x^2 2y = 4xye^{x^2y^2} + 4x^3y^3e^{x^2y^2} = 4xye^{x^2y^2}(1+x^2y^2)$$

$$\frac{\partial}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(2x^2ye^{x^2y^2}\right) = 4xye^{x^2y^2} + 2x^2ye^{x^2y^2}\,2xy^2 = 4xye^{x^2y^2} + 4x^3y^3e^{x^2y^2} = 4xye^{x^2y^2}(1+x^2y^2)$$

4.3

1. $f(x,y) = 1 + 6y - y^2 - xy - x^2$

$\frac{\partial}{\partial x} f(x,y) = -y - 2x = 0$

$y = -2x \longrightarrow \frac{\partial}{\partial y} f(x,y) = 6 - 2y - x$

$6 - 2(-2x) - x = 0$

$6 + 3x = 0$

$3x = -6$

$x = -2 \implies SB = [-2, 4]$

$y = -2(-2)$

$y = 4$

$\frac{\partial^2}{\partial x^2} = -2 \qquad \frac{\partial^2}{\partial y^2} = -2 \qquad \frac{\partial^2}{\partial xy} = -1 \qquad \frac{\partial^2}{\partial yx} = -1 \implies$ Hessova matice: $\begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$

$\left|\begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}\right| = 4 - 1 = 3 \implies \overset{\text{Hessián}}{\overset{\downarrow}{H}} = 3 > 0 \implies [-2, 4]$ je lokální extrém

$\ominus \rightarrow \cap$

$\frac{\partial^2}{\partial x^2} f(x,y)_{|[-2,4]} = -2 < 0 \implies$ konkávní tvar $f(x,y) \implies M[-2, 4]$

2. $f(x,y) = x^3 + y^3 - 3xy$

$\frac{\partial}{\partial x} f(x,y) = 3x^2 - 3y \longleftarrow$

$\frac{\partial}{\partial y} f(x,y) = 3y^2 - 3x = 0$

$3x = 3y^2$

$x = y^2$

$3y^4 - 3y = 0$

$y^4 - y = 0$

$y(y^3 - 1) = 0 \rightarrow y = 0 \longrightarrow x = 0^2 = 0 \quad SB[0,0]$

↙ $\qquad y = 1 \longrightarrow x = 1^2 = 1 \quad SB[1,1]$

$\frac{\partial^2}{\partial x^2} = 6x \qquad \frac{\partial^2}{\partial y^2} = 6y \qquad \frac{\partial^2}{\partial xy} = -3 \qquad \frac{\partial^2}{\partial yx} = -3 \implies$ Hessova matice: $\begin{pmatrix} 6x & -3 \\ -3 & 6y \end{pmatrix}$

$[0,0]: \left|\begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}\right| = 0-9 = -9 \Rightarrow H=-9<0 \Rightarrow S[0,0]$

$[1,1]: \left|\begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix}\right| = 36-9 = 27 \Rightarrow H = 27 > 0 \Rightarrow [1,1]$ je lokální extrém

$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[1,1]} = \overset{\oplus}{6} > \overset{\cup}{0} \Rightarrow$ konvexní tvar $f(x,y) \rightarrow m[1,1]$

3. $f(x,y) = x^2 - y^2 + 2x - 2y$

$\frac{\partial}{\partial x} = 2x + 2 = 0$

$2x = -2$

$x = -1$

$\frac{\partial}{\partial y} = -2y - 2 = 0$

$2y = -2$

$y = -1 \quad \Rightarrow SB[-1,-1]$

$\frac{\partial^2}{\partial x^2} = 2 \qquad \frac{\partial}{\partial y^2} = -2 \qquad \frac{\partial}{\partial xy} = 0 \qquad \frac{\partial}{\partial yx} = 0 \Rightarrow$ Hessova matice $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$

$\left|\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\right| = -4-0 = -4 < 0 \rightarrow H<0 \rightarrow S[-1,-1]$

4. $f(x,y) = x(x-1) + y(y-1) - xy + 2$

$\frac{\partial}{\partial x} f(x,y) = x - 1 + x - y = 0$

$2x - 1 - y = 0$

$2(2y-1) - 1 - y = 0$

$4y - 2 - 1 - y = 0$

$3y = 3$

$y = 1$

$\frac{\partial}{\partial y} f(x,y) = y - 1 + y - x = 0$

$x = 2y - 1$

$x = 2 \cdot 1 - 1$

$x = 1 \quad \Rightarrow SB[1,1]$

$\frac{\partial^2}{\partial x^2} = 2 \qquad \frac{\partial}{\partial y^2} = 2 \qquad \frac{\partial}{\partial xy} = -1 \qquad \frac{\partial}{\partial yx} = -1 \rightarrow$ Hessova matice: $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$\left|\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\right| = 4-1 = 3 > 0 \rightarrow H>0 \Rightarrow [1,1]$ je lokální extrém.

$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[1,1]} = \overset{\oplus}{2} > \overset{\cup}{0} \Rightarrow$ konvexní tvar $f(x,y) \rightarrow m[1,1]$

5. $f(x,y) = 4-(x-2)^2-(y+3)^2$

$$\frac{\partial}{\partial x} f(x,y) = -2(x-2)=0$$
$$-2x+4=0$$
$$2x=4$$
$$x=2$$

$$\frac{\partial}{\partial y} f(x,y) = -2(y+3)=0$$
$$-2y-6=0$$
$$2y=-6$$
$$y=-3 \to SB[2,-3]$$

$$\frac{\partial^2}{\partial x^2} = -2 \qquad \frac{\partial^2}{\partial y^2} = -2 \qquad \frac{\partial^2}{\partial xy} = 0 \qquad \frac{\partial^2}{\partial yx} = 0 \to \text{Hessova matice: } \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$$

$$\left|\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}\right| = 4-0 = 4>0 \to H>0 \Rightarrow [2,-3] \text{ je lokální extrém}$$

$$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[2,-3]} = \overset{\ominus}{-2} < \overset{\cap}{0} \to \text{konkávní tvar } f(x,y) \to M[2,-3]$$

6. $f(x,y) = 2x^3-xy^2+5x^2+y^2$

$$\frac{\partial}{\partial x} f(x,y) = 6x^2-y^2+10x=0$$

$$\frac{\partial}{\partial y} f(x,y) = -2xy+2y=0$$
$$-2y(x-1)=0 \ldots y=0$$
$$x=1$$

- $y=0$: $6x^2+10x=0$
  $x(6x+10)=0$
  $x(3x+5)=0 \ldots x=0 \to SB[0,0]$
  $3x+5=0$
  $3x=-5$
  $x=-\frac{5}{3} \to SB[-\frac{5}{3},0]$

- $x=1$: $6-y^2+10=0$
  $y^2=16$
  $y=\pm\sqrt{16} \ldots y=4 \to SB[1,4]$
  $y=-4 \to SB[1,-4]$

$$\frac{\partial^2}{\partial x^2} = 12x+10 \qquad \frac{\partial^2}{\partial y^2} = -2x+2 \qquad \frac{\partial^2}{\partial xy} = -2y \qquad \frac{\partial^2}{\partial yx} = -2y \to HM: \begin{pmatrix}12x+10 & -2y\\ -2y & -2x+2\end{pmatrix}$$

$$[0,0]: \left|\begin{pmatrix}10 & 0\\ 0 & 2\end{pmatrix}\right| = 20-0 = 20>0 \longrightarrow H>0 \longrightarrow [0,0] \text{ je lokální extrém}$$

$$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[0,0]} = \overset{\oplus}{10} > \overset{\cup}{0} \to \text{konvexní tvar } f(x,y) \to m[0,0]$$

$$[-\tfrac{5}{3},0]: \left|\begin{pmatrix}-10 & 0\\ 0 & \frac{16}{3}\end{pmatrix}\right| = -\frac{160}{3}-0 = -\frac{160}{3} < 0 \Rightarrow S[-\tfrac{5}{3},0]$$

$$[1,4]: \left|\begin{pmatrix}22 & -8\\ -8 & 0\end{pmatrix}\right| = 0-64 = -64 < 0 \Rightarrow S[1,4]$$

$$[1,-4]: \left|\begin{pmatrix}22 & 8\\ 8 & 0\end{pmatrix}\right| = 0-64 = -64 < 0 \Rightarrow S[1,-4]$$