CVIČENÍ 1

1.1

1. $m=4$
2. $m=3$

1.2

1. $e+f=(3,0,1,3)+(-1,1,0,-2)=(2,1,1,1)$
2. $2c-2f=2(1,2,1,1)-2(-1,1,0,-2)=(2,4,2,2)-(-2,2,0,-4)=(2,4,2,2)+(2,-2,0,4)=(4,2,2,6)$
3. $a+b-4c=(2,1,2)+(-1,0,1)-4(1,2,1,1)=(1,1,3)+(-4,-8,-4,-4)=$ nejde
4. $4d-2(e-f)=4(1,0,-2,0)-2((3,0,1,3)-(-1,1,0,-2))$
$=(4,0,-8,0)-2(4,-1,1,5)=$
$=(4,0,-8,0)+(-8,2,-2,-10)=(-4,2,-10,-10)$

1.3

1. $7f\times c=7(-1,1,0,-2)\times(1,2,1,1)=(-7,7,0,-14)\times(1,2,1,1)=-7+14+0-14=-7$
2. $b\times a-6e\times d=(-1,0,1)\times(2,1,2)-6(3,0,1,3)\times(1,0,-2,0)$
$=-2+0+2-6(3+0-2+0)=0-6\cdot 1=-6$

1.4

1. $C^T=\begin{pmatrix}1&3&0\\-1&0&-2\end{pmatrix}^T=\begin{pmatrix}1&-1\\3&0\\0&-2\end{pmatrix}$
2. $E^T=\begin{pmatrix}-2&0\\1&-2\end{pmatrix}^T=\begin{pmatrix}-2&1\\0&-2\end{pmatrix}$

1.5

1. $\dim D=\dim\begin{pmatrix}2&-1\\0&2\\-1&3\end{pmatrix}=3\times 2$

2. $\dim(D^T\times F^T)=\dim(D^T_{3\times 2}\times F^T_{3\times 3})=\dim((D^T)_{2\times 3}\times(F^T)_{3\times 3})=2\times 3$

3. $\dim(C\times E)=\dim(C_{2\times 3}\times E_{2\times 2})$ ... nelze

1.6

1. $3F-B^T=3\begin{pmatrix}1&3&2\\-1&2&0\\0&1&2\end{pmatrix}-\begin{pmatrix}-1\\0\\1\end{pmatrix}^T=$ ... nejde

2. $D-2C^T=\begin{pmatrix}2&-1\\0&2\\-1&3\end{pmatrix}-2\begin{pmatrix}1&3&0\\-1&0&-2\end{pmatrix}^T=\begin{pmatrix}2&-1\\0&2\\-1&3\end{pmatrix}-2\begin{pmatrix}1&-1\\3&0\\0&-2\end{pmatrix}=\begin{pmatrix}2&-1\\0&2\\-1&3\end{pmatrix}+\begin{pmatrix}-2&2\\-6&0\\0&4\end{pmatrix}=\begin{pmatrix}0&1\\-6&2\\-1&7\end{pmatrix}$

1.7

1. $D^T \times F^T = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}^T \times \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix}$

2. $A \times D - 2B^T \times C^T = (2,1,2) \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - 2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}^T = (2;6) - 2\,(-1\ 0\ 1) \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$

$= (2,6) - 2\,(-1,-1) = (2,6) + (2,2) = (4,8)$

1.8

1. $\operatorname{diag}(D^T \times F^T) = \operatorname{diag}\left(\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}^T \times \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}^T \right) = \operatorname{diag}\left(\begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix} \right) =$

$= \operatorname{diag}\left(\begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix} \right) = (0,5)$

2. $\operatorname{diag}(A \times D - 2B^T \times C^T) = \operatorname{diag}(4,8) = 4$

1.9

1. $\begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} -2 & -2 & -1 \\ 1 & -1 & -1 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & -4 & 1 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 5 \\ 0 & -4 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 21 \end{pmatrix}$ .... LN

2. $\begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 5/2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ ... LZ

KOMBINACE: $\frac{5}{2} \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \\ -6 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

1.10

1. $\operatorname{rank} \begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} = \operatorname{rank} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 21 \end{pmatrix} = 3$

2. $\operatorname{rank} \begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} = \operatorname{rank} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} = 2$

1.11

1. $\left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 1 & 0 & 2 & 5 \\ 6 & 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & -1 & -1 & -4 \\ 0 & 6 & -5 & -20 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & +4 \\ 0 & 0 & -11 & -44 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right)$

$x_1 = -3 \quad x_2 = 0 \quad x_3 = 4$

2. $\left(\begin{array}{ccc|c} -1 & 2 & 5 & 3 \\ -2 & 2 & 7 & 0 \\ 1 & 0 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 2 & 3 & 1 \\ 0 & 2 & 3 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right)$ ... nemá řešení

1.12

1. $\begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix} = -3\cdot(-1) - 2\cdot 2 = +3 - 4 = -1$

2. $\begin{vmatrix} 2 & -1 & 3 & 2 & -1 \\ -1 & 0 & -2 & -1 & 0 \\ 5 & 1 & 4 & 5 & 1 \end{vmatrix} = 0 + 10 - 3 - (0 - 4 + 4) = 7$

1.13

1. $2\begin{vmatrix} 1 & -1 & 0 \\ x & 3 & x \\ 0 & x & 1 \end{vmatrix} + \begin{vmatrix} x & 0 & 1 \\ -x & 0 & 1 \\ -1 & 3 & x \end{vmatrix} = 6$

$2\begin{vmatrix} 1 & -1 & 0 & 1 & -1 \\ x & 3 & x & x & 3 \\ 0 & x & 1 & 0 & x \end{vmatrix} + \begin{vmatrix} x & 0 & 1 & x & 0 \\ -x & 0 & 1 & -x & 0 \\ -1 & 3 & x & -1 & 3 \end{vmatrix} = 6$

$2\cdot(3 + 0 + 0 - (0 + x^2 - x)) + (0 + 0 - 3x - (0 + 3x + 0)) = 6$

$2(3 - x^2 + x) + (-3x - 3x) = 6$

$6 - 2x^2 + 2x - 6x = 6$

$-2x^2 - 4x = 0$

$-x^2 - 2x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0 \implies x_1 = -2$

$x_2 = 0$

CVIČENÍ 2

2.1

1. $D(f(x)) = (-\infty ; \infty)$
2. $H(f(x)) = \langle -1, 1 \rangle$
3. spojitá na celém def. oboru
4. shora ohraničená 1, zdola ohraničená −1
5. periodická s periodou 2π
6. sudá funkce
7. klesající na $\langle 2k\pi ; (2k+1)\pi \rangle$, rostoucí na $\langle k\pi, (k+1)\pi \rangle$; $k \in \mathbb{N}$
8. $\lim_{x \to \infty} f(x)$ neexistuje, $\lim_{x \to -\infty} f(x)$ neexistuje

2.2

1. $D(f(x)) = (-\infty ; \infty)$
2. $H(f(x)) = \langle 0 ; \infty)$
3. spojitá na celém def. oboru
4. zdola ohraničená 0, shora neohraničená
5. neperiodická
6. sudá
7. klesající na intervalu $(-\infty ; 0\rangle$, rostoucí na $\langle 0 ; \infty)$
8. $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = \infty$

2.3

1. $x^2 + x - 6$ $\quad \pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 6$

	1	1	−6
1	1	2	−4
−1	1	0	−6
2	1	3	0 $\longrightarrow x = 2$
2	1	5	
−2	1	−1	
3	1	9	
−3	1	0 $\longrightarrow x = -3$	

$\Longrightarrow (x-2)(x+3) = x^2 + 3x - 2x - 6 = x^2 + x - 6$

2. $x^3+7x^2+11x+5 \qquad \pm 1 \ \pm 5$

	1	7	11	5	
1	1	8	19	24	
-1	1	6	5	0	$\rightarrow x_1=-1$
-1	1	5	0		$\rightarrow x_2=-1$
-1	1	4			
5	1	10			
-5	1	0			$\rightarrow x_3=-5$

$$\Rightarrow (x+1)(x+1)(x+5)=(x^2+2x+1)(x+5)$$
$$= x^3+2x^2+x+5x^2+10x+5$$
$$= x^3+7x^2+11x+5$$

2.4

1. $\lim\limits_{x\to 5} \dfrac{x^2-5x+4}{x-1} \overset{\text{dosadím } x=5}{=} \dfrac{25-25+4}{5-1} = \dfrac{4}{4} = 1$

2. $\lim\limits_{x\to 2} \dfrac{3^x+3}{2^x-4^x} = \dfrac{3^2+3}{2^2-4^2} = \dfrac{9+3}{4-16} = \dfrac{12}{-12} = -1$

3. $\lim\limits_{x\to 3} \dfrac{x^3-x^2-5x-3}{x^2-9} = \dfrac{3^3-3^2-5\cdot3-3}{3^2-9} = \dfrac{27-9-15-3}{9-9} = \dfrac{0}{0}$ ... neurčitý výraz → dosazení nestačí → zkusím krátit

$$= \lim_{x\to 3} \frac{(x+1)(x+1)(x-3)}{(x-3)(x+3)}$$
$$= \lim_{x\to 3} \frac{(x+1)(x+1)}{(x+3)}$$
$$= \frac{(3+1)(3+1)}{(3+3)} =$$
$$= \frac{4\cdot4}{6} = \frac{16}{6} = \frac{8}{3}$$

|| $x^3-x^2-5x-3 \qquad \pm 1 \ \pm 3$

	1	-1	-5	-3	
1	1	0	-5	-8	
-1	1	-2	-3	0	$x_1=-1$
-1	1	-3	0		$x_2=-1$
-1	1	-4			
+3	1	0			$x_3=3$

$$\Rightarrow (x+1)(x+1)(x-3)$$

4. $\lim\limits_{x\to 3} 3x^2-7 = 3\cdot3^2-7 = 3\cdot9-7 = 20$

2.5

1. $$\lim_{x \to -\infty} \frac{2x^2 - x}{x} = \lim_{x \to -\infty} \frac{2x-1}{1} = \frac{-\infty}{1} = -\infty$$

2. $$\lim_{x \to -\infty} \frac{4x+2-3x^2-2x^7}{2x^3+x^5-3} = \lim_{x \to -\infty} \frac{-2x^7-3x^2+4x+2}{x^5+2x^3-3} = \lim_{x \to -\infty} \frac{x^5\left(-2x^2 - \frac{3}{x^3} + \frac{4}{x^4} + \frac{2}{x^5}\right)}{x^5\left(1 + \frac{2}{x^2} - \frac{3}{x^5}\right)} =$$

$$= \lim_{x \to -\infty} \frac{-2x^2 - \frac{3}{x^3} + \frac{4}{x^4} + \frac{2}{x^5}}{1 + \frac{2}{x^2} - \frac{3}{x^5}} = \frac{-2(+\infty) - 0 + 0 + 0}{1+0+0} = \frac{-\infty}{1} = -\infty$$

3. $$\lim_{x \to \infty} \frac{3+x^3+x}{3x^5-2x^3-6x+2} = \lim_{x \to \infty} \frac{x^3+x+3}{3x^5-2x^3-6x+2} = \lim_{x \to \infty} \frac{x^3\left(1+\frac{1}{x^2}+\frac{3}{x^3}\right)}{x^3\left(3x^2-2-\frac{6}{x^2}+\frac{2}{x^3}\right)} =$$

$$= \lim_{x \to \infty} \frac{1+\frac{1}{x^2}+\frac{3}{x^3}}{3x^2-2-\frac{6}{x^2}+\frac{2}{x^3}} = \frac{1+0+0}{3\infty-2-0+0} = \frac{1}{\infty} = 0$$

4. $$\lim_{x \to -\infty} \frac{7^x-5^x}{5^x} = \lim_{x \to -\infty} \frac{5^x(2^x-1)}{5^x} = \lim_{x \to -\infty} \frac{2^x-1}{1} = \lim_{x \to -\infty} 2^x - 1 = 2^{-\infty} - 1 = 0 - 1 = -1$$

5. $$\lim_{x \to \infty} \frac{2+\frac{1}{4^x}}{2-\frac{1}{5^x}} = \lim_{x \to \infty} \frac{2+4^{-x}}{2-5^{-x}} = \frac{2+0}{2-0} = \frac{2}{2} = 1$$

6. $$\lim_{x \to -\infty} \frac{6^x-2^x}{3^x} = \lim_{x \to -\infty} \underbrace{\frac{6^x}{3^x}}_{= \left(\frac{6}{3}\right)^x = 2^x} + \lim_{x \to -\infty} \frac{-2^x}{3^x} = \lim_{x \to \infty} \frac{3^x \cdot 2^x}{3^x} + \lim_{x \to -\infty} \frac{-2^x}{3^x} =$$

$$= \lim_{x \to -\infty} 2^x - \lim_{x \to -\infty} \left(\frac{2}{3}\right)^x = 3^{-\infty} - \left(\frac{2}{3}\right)^{-\infty} = 0 - \left(\frac{3}{2}\right)^{\infty} = -\infty$$

7. $$\lim_{x \to \infty} \frac{1+2x-x^3+4x^5}{x^4+2x^5-x^3} = \lim_{x \to \infty} \frac{4x^5-x^3+2x+1}{2x^5+x^4-x^3} = \lim_{x \to \infty} \frac{x^5\left(4-\frac{1}{x^2}+\frac{2}{x^4}+\frac{1}{x^5}\right)}{x^5\left(2+\frac{1}{x}-\frac{1}{x^2}\right)} =$$

$$= \lim_{x \to \infty} \frac{4-\frac{1}{x^2}+\frac{2}{x^4}+\frac{1}{x^5}}{2+\frac{1}{x}-\frac{1}{x^2}} = \frac{4-0+0+0}{2+0-0} = 2$$

8. $$\lim_{x \to \infty} \frac{4^x-3^x}{5^x} = \lim_{x \to \infty} \frac{4^x}{5^x} - \lim_{x \to \infty} \frac{3^x}{5^x} = \lim_{x \to \infty} \left(\frac{4}{5}\right)^x - \lim_{x \to \infty} \left(\frac{3}{5}\right)^x = 0$$

2.6

1. $(x^4 + x^{-4} + x^0 - \tan(x) + e^x)' = 4x^3 - 4x^{-5} + 0 - \frac{1}{\cos^2 x} + e^x$

2. $\left(\frac{e^x + x^2 - 4x}{\ln(x)}\right)' = \left((e^x + x^2 - 4x) \cdot \left(\frac{1}{\ln(x)}\right)\right)' = (e^x + 2x - 4)\frac{1}{\ln x} + (e^x + x^2 - 4x) \cdot \frac{-\frac{1}{x}}{\ln^2 x}$

$= \frac{e^x + 2x - 4}{\ln x} - \frac{e^x + x^2 - 4x}{x \ln^2 x}$

3. $((x + x^4)\ln x - 4x\sin(x))' = (1 + 4x^3)\ln x + (x + x^4) \cdot \frac{1}{x} - 4\sin x - 4x\cos x$

$= (1 + 4x^3)\ln x + 1 + x^3 - 4\sin x - 4x\cos x$

4. $(\ln(\cos x) + \ln(\ln x))' = \frac{1}{\cos x}(-\sin x) + \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{-\sin x}{\cos x} + \frac{1}{x\ln x} = -\tan x + \frac{1}{x\ln x}$

5. $(3\ln x \tan x + \sin x \cos x)' = \frac{3}{x}\tan x + \frac{3\ln x}{\cos^2 x} + \cos^2 x - \sin^2 x$

6. $(2x^6 - x^4 + 3x^3 + 5x)' = 12x^5 - 4x^3 + 9x^2 + 5$

7. $\left(\frac{x}{(1-x)^2}\right)' = \frac{(1-x)^2 - x\,2(1-x)(-1)}{(1-x)^4} = \frac{(1-x)(1-x+2x)}{(1-x)^4} = \frac{1+x}{(1-x)^3}$

8. $\left(\frac{-2}{\sin(2x+3)}\right)' = \left(\frac{+2\cos(2x+3)(2)}{\sin^2(2x+3)}\right) = \frac{4\cos(2x+3)}{\sin^2(2x+3)}$

2.7

1. $(x^{-1}\ln x)'' = (\frac{1}{x}\ln x)'' = \left(-x^{-2}\ln x + x^{-1}\frac{1}{x}\right)' = \left(x^{-2}(1-\ln x)\right)' = -2x^{-3}(1-\ln x) - x^{-2}\frac{1}{x} =$

$= -2x^{-3}(1-\ln x) - x^{-3} = \frac{-2 + 2\ln x - 1}{x^3} = \frac{2\ln x - 3}{x^3}$

2. $(\cos x^2 + \sin 2x)'' = (-2x\sin x^2 + 2\cos 2x)' = -2\sin x^2 - 2x \cdot \cos x^2\, 2x - 2\sin 2x \cdot 2 =$

$= -2\sin x^2 - 4x^2\cos x^2 - 4\sin 2x$

3. $(\sin x \ln x)'' = \left(\cos x \ln x + \frac{\sin x}{x}\right)' = -\sin x \ln x + \frac{\cos x}{x} + \frac{x\cos x - \sin x}{x^2} =$

$= -\sin x \ln x + \frac{\cos x}{x} + \frac{\cos x}{x} - \frac{\sin x}{x^2} = 2\frac{\cos x}{x} - \sin x\left(\ln x + \frac{1}{x^2}\right)$

4. $(2x^6 - x^4 + 3x^3 + 4x^2 - 5)'' = (12x^5 - 4x^3 + 9x^2 + 8x)' = 60x^4 - 12x^2 + 18x + 8$

$= 2(30x^4 - 6x^2 + 9x + 4)$

2.8

1. $\lim_{x \to 3} \frac{x^3 - x^2 + 5x - 3}{x^2 - 9} = \frac{3^3 - 3^2 + 5 \cdot 3 - 3}{3^2 - 9} = \frac{27 - 9 + 15 - 3}{9 - 9} = \frac{29}{0}$ ... L'Hosp. pr. nelze použít

2. $\lim_{x \to 3} \frac{x^3 - x^2 - 5x - 3}{x^2 - 9} = \frac{3^3 - 3^2 - 15 - 3}{9 - 9} = \frac{27 - 9 - 15 - 3}{0} = \frac{0}{0} \Rightarrow$ L'Hosp. pr. ANO

$= \lim_{x \to 3} \frac{(x^3 - x^2 - 5x - 3)'}{(x^2 - 9)'} = \lim_{x \to 3} \frac{3x^2 - 2x - 5}{2x} = \frac{3 \cdot 3^2 - 6 - 5}{6}$

$= \frac{3 \cdot 9 - 11}{6} = \frac{16}{6} = \frac{8}{3}$

3. $\lim_{x \to -1} \frac{3x^3 - 7x^2 - 2x + 8}{4x^2 + x - 3} = \frac{3(-1)^3 - 7(-1)^2 - 2(-1) + 8}{4(-1)^2 + (-1) - 3} = \frac{-3 - 7 + 2 + 8}{4 - 4} = \frac{0}{0}$ ... L'Hosp. ANO

$= \lim_{x \to -1} \frac{9x^2 - 14x - 2}{8x + 1} = \frac{9(-1)^2 - 14(-1) - 2}{8(-1) + 1} = \frac{9 + 14 - 2}{-7} = \frac{21}{-7} = -3$