E2011: Theoretical fundamentals of computer science Topic 2: Boolean algebra - Exercises

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Some useful tools:

- Logic.ly: https://logic.ly/demo/ demo version is enough to try out some simple designs
- LogiSim: http://www.cburch.com/logisim/download.html a free, portable (Java) application with many features
- **Digital**: https://github.com/hneemann/Digital another educational tool for digital circuits

Show that:

$$X(X + Y) = X$$
 (law of absorption)

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Show that:

$$X(X + Y) = X$$
 (law of absorption)

Proof:

$$egin{aligned} X(X+Y) &= X \cdot X + X \cdot Y \ &= X + X \cdot Y \ &= X(1+Y) \ &= X \end{aligned}$$

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Show that:

 $XY + YZ + \overline{X}Z = XY + \overline{X}Z$ 

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Show that:

 $XY + YZ + \overline{X}Z = XY + \overline{X}Z$ 

Proof:

$$XY + YZ + \overline{X}Z = XY + (X + \overline{X})YZ + \overline{X}Z$$
$$= XY + XYZ + \overline{X}YZ + \overline{X}Z$$
$$= (XY + XYZ) + (\overline{X}YZ + \overline{X}Z)$$
$$= XY + \overline{X}Z$$

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Prove de Morgan's theorem using truth table.

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Using de Morgan's theorem, expand

# $\overline{\overline{X}} + \overline{Y} + \overline{Z}$

and construct the corresponding truth table.

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# Identify the boolean function corresponding to the following truth table:

Х	X Y	Ζ	W	F(X, Y, Z, W)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	) 1	0	0	0
0	) 1	0	1	0
0	) 1	1	0	0
0	) 1	1	1	0
1	. 0	0	0	0
1	. 0	0	1	0
1	. 0	1	0	0
1	. 0	1	1	1
1	. 1	0	0	1
1	. 1	0	1	1
1	. 1	1	0	1
1	. 1	1	1	1 ∢∂> ∢≣> ∢≣> ≣

#### Exercise 5 - cont'd

Function identification:

 $F(X, Y, Z, W) = X\overline{Y}ZW + XY\overline{Z}\ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW$ 

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Simplify previous function.

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Simplify previous function.

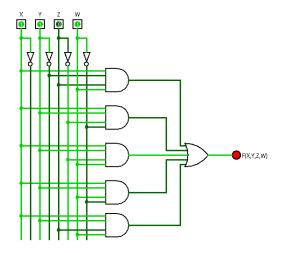
$$F(X, Y, Z, W) = X\overline{Y}ZW + XY\overline{Z} \ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW$$
  
$$= XZW(\overline{Y} + Y) + XY\overline{Z}(\overline{W} + W) + XYZ\overline{W}$$
  
$$= XZW + XY\overline{Z} + XYZ\overline{W}$$
  
$$= XY\overline{Z} + XZ(W + Y\overline{W})$$
  
$$= XY\overline{Z} + XZ(Y + W)$$
  
$$= XY\overline{Z} + XYZ + XZW$$
  
$$= XY(Z + \overline{Z}) + XZW$$
  
$$= XY + XZW$$
  
$$= X(Y + ZW)$$

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Design the logic circuits corresponding to the initial and simplified forms of the previous function, respectively.

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