E2011: Theoretical fundamentals of computer science Topic 2: Boolean algebra - Exercises

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Some useful tools:

- Logic.ly: https://logic.ly/demo/ demo version is enough to try out some simple designs
- LogiSim: http://www.cburch.com/logisim/download.html a free, portable (Java) application with many features
- **Digital**: https://github.com/hneemann/Digital another educational tool for digital circuits

Show that:

$$X(X + Y) = X$$
 (law of absorption)

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Show that:

$$X(X + Y) = X$$
 (law of absorption)

Proof:

$$egin{aligned} X(X+Y) &= X \cdot X + X \cdot Y \ &= X + X \cdot Y \ &= X(1+Y) \ &= X \end{aligned}$$

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Show that:

 $XY + YZ + \overline{X}Z = XY + \overline{X}Z$

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Show that:

 $XY + YZ + \overline{X}Z = XY + \overline{X}Z$

Proof:

$$XY + YZ + \overline{X}Z = XY + (X + \overline{X})YZ + \overline{X}Z$$
$$= XY + XYZ + \overline{X}YZ + \overline{X}Z$$
$$= (XY + XYZ) + (\overline{X}YZ + \overline{X}Z)$$
$$= XY + \overline{X}Z$$

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Prove de Morgan's theorem using truth table.

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Using de Morgan's theorem, expand

$\overline{\overline{X}} + \overline{Y} + \overline{Z}$

and construct the corresponding truth table.

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Identify the boolean function corresponding to the following truth table:

| Х | X Y | Ζ | W | F(X, Y, Z, W) |
|---|-----|---|---|--------------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 |) 1 | 0 | 0 | 0 |
| 0 |) 1 | 0 | 1 | 0 |
| 0 |) 1 | 1 | 0 | 0 |
| 0 |) 1 | 1 | 1 | 0 |
| 1 | . 0 | 0 | 0 | 0 |
| 1 | . 0 | 0 | 1 | 0 |
| 1 | . 0 | 1 | 0 | 0 |
| 1 | . 0 | 1 | 1 | 1 |
| 1 | . 1 | 0 | 0 | 1 |
| 1 | . 1 | 0 | 1 | 1 |
| 1 | . 1 | 1 | 0 | 1 |
| 1 | . 1 | 1 | 1 | 1 ∢∂> ∢≣> ∢≣> ≣ |

Exercise 5 - cont'd

Function identification:

 $F(X, Y, Z, W) = X\overline{Y}ZW + XY\overline{Z}\ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW$

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Simplify previous function.

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Simplify previous function.

$$F(X, Y, Z, W) = X\overline{Y}ZW + XY\overline{Z} \ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW$$

$$= XZW(\overline{Y} + Y) + XY\overline{Z}(\overline{W} + W) + XYZ\overline{W}$$

$$= XZW + XY\overline{Z} + XYZ\overline{W}$$

$$= XY\overline{Z} + XZ(W + Y\overline{W})$$

$$= XY\overline{Z} + XZ(Y + W)$$

$$= XY\overline{Z} + XYZ + XZW$$

$$= XY(Z + \overline{Z}) + XZW$$

$$= XY + XZW$$

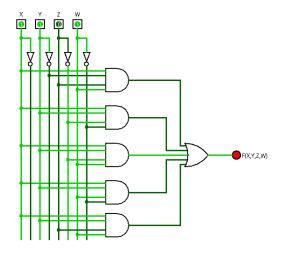
$$= X(Y + ZW)$$

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Design the logic circuits corresponding to the initial and simplified forms of the previous function, respectively.

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