E2011: Theoretical fundamentals of computer science
Topic 2: Boolean algebra - Exercises

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Some useful tools:

- **Logic.ly**: `https://logic.ly/demo/` - demo version is enough to try out some simple designs
- **LogiSim**: `http://www.cburch.com/logisim/download.html` - a free, portable (Java) application with many features
- **Digital**: `https://github.com/hneemann/Digital` - another educational tool for digital circuits

Exercise 1

Show that:

$$X(X + Y) = X \quad \text{(law of absorption)}$$

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Proof:

$$\begin{aligned} X(X + Y) &= X \cdot X + X \cdot Y \\ &= X + X \cdot Y \\ &= X(1 + Y) \\ &= X \end{aligned}$$

Exercise 2

Show that:

$$XY + YZ + \overline{X}Z = XY + \overline{X}Z$$

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Proof:

$$\begin{aligned} XY + YZ + \overline{X}Z &= XY + (X + \overline{X})YZ + \overline{X}Z \\ &= XY + XYZ + \overline{X}YZ + \overline{X}Z \\ &= (XY + XYZ) + (\overline{X}YZ + \overline{X}Z) \\ &= XY + \overline{X}Z \end{aligned}$$

Exercise 3

Prove de Morgan's theorem using truth table.

Exercise 4

Using de Morgan's theorem, expand

$$\overline{\overline{X} + \overline{Y} + \overline{Z}}$$

and construct the corresponding truth table.

Exercise 5

Identify the boolean function corresponding to the following truth table:

X	Y	Z	W	$F(X, Y, Z, W)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Exercise 5 - cont'd

Function identification:

$$F(X, Y, Z, W) = X\overline{Y}ZW + XY\overline{Z}\ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW$$

Exercise 6

Simplify previous function.

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$$
\begin{aligned}
F(X,Y,Z,W) &= X\overline{Y}ZW + XY\overline{Z}\ \overline{W} + XY\overline{Z}W + XYZ\overline{W} + XYZW \\
&= XZW(\overline{Y} + Y) + XY\overline{Z}(\overline{W} + W) + XYZ\overline{W} \\
&= XZW + XY\overline{Z} + XYZ\overline{W} \\
&= XY\overline{Z} + XZ(W + Y\overline{W}) \\
&= XY\overline{Z} + XZ(Y + W) \\
&= XY\overline{Z} + XYZ + XZW \\
&= XY(Z + \overline{Z}) + XZW \\
&= XY + XZW \\
&= X(Y + ZW)
\end{aligned}
$$

Exercise 7

Design the logic circuits corresponding to the initial and simplified forms of the previous function, respectively.

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