E2011: Theoretical fundamentals of computer science
Topic 3: Numeral systems - Exercises

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Problem 1

Implement a 2-bit adder using logical gates.

Plan

- which numbers can be represented on 2 bits?

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- what is the range of results?
- how many bits you need for the result?
- write the truth table and derive the fuctions for the outputs
- design the circuit

Solution

Input: $a = [a_1 a_0]$, $b = [b_1 b_0]$.
Output: $s = [c s_1 s_0]$; c: carry

Truth table:

a_1	a_0	b_1	b_0	c	s_1	s_0
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
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1	0	1	1			
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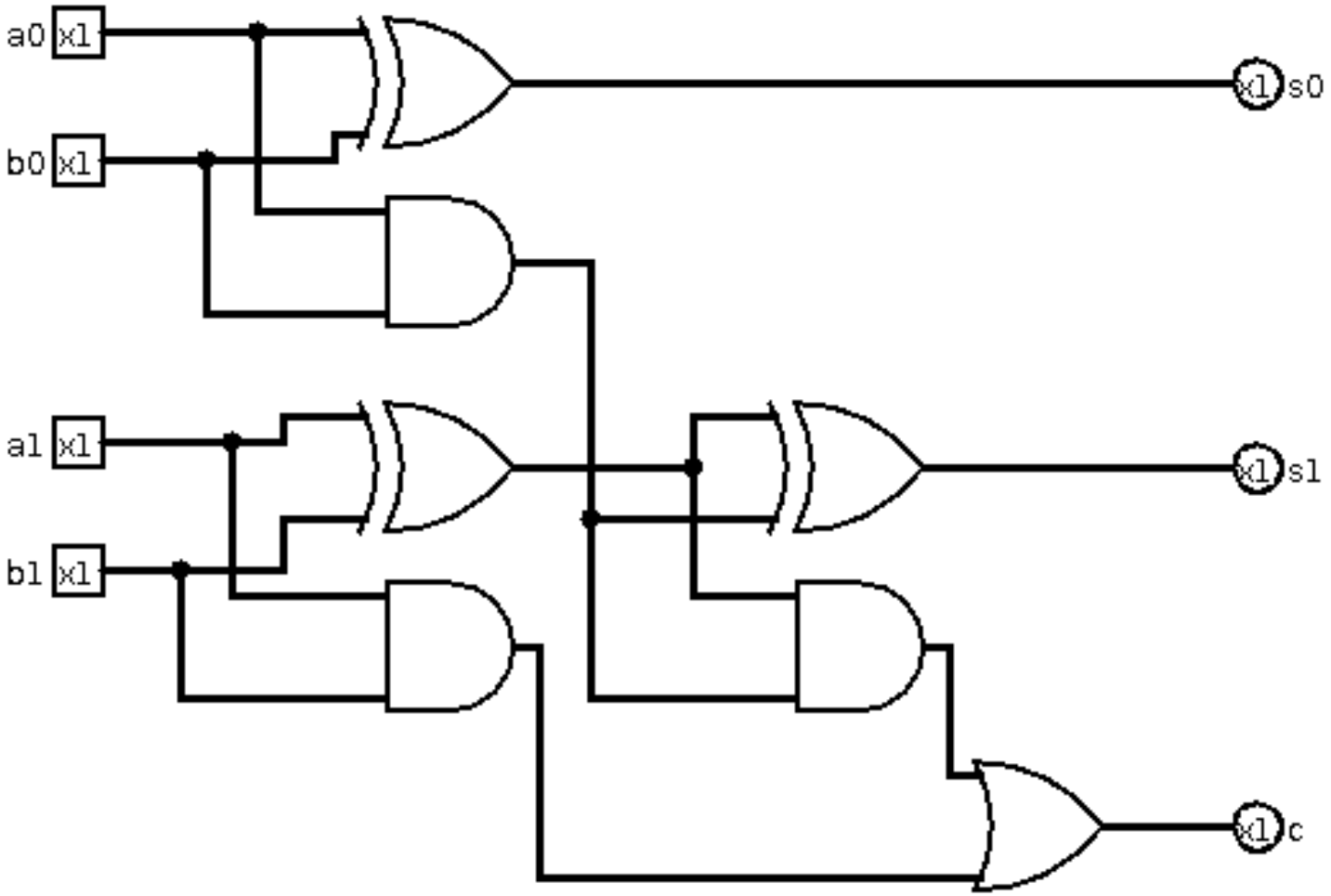
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0


a0
x1
b0
x1
a1
x1
b1
x1
x1
s0
x1
s1
x1
c

Problem 2

Using bitwise operations, extract the R, G, B values from a HTML-like specification (in hexa) of the form "#RRGGBB", where each symbol corresponds to a hexa digit. Example, from "#ABCDEF", you should get R="AB", G="CD", B="EF".

Solution

- let x be the input value (on 24 bits, i.e. 6 bytes)
- $R = x >> 16$ (right shift by 16 bits)
- $G = (x << 4) >> 16$ (left shift followed by right shift)
- $B = x\&FF$ (bitwise AND)
- can you see what happened in each case?
- can you find other solutions?