Basics of SAXS in structural analysis (of biomolecules)

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#### Motivation



#### Terminology

- 1) X-ray diffraction / scattering 2) XS - X-ray scattering 3) SAXS/WAXS - Small/Wide Angle X-ray Scattering 4) SANS - --------"----------- Neutron ---"---
- A) Otto Kratky (1902, Vienna-1995, Graz)
- B) Günter Porod (1919 near Villach, 1984 Graz)
- C) Dmitri I. Svergun
- I) Scattering
- II) Scattering curve
- III) Guinier plot
- IV) PDF (Pair-distribution function)
- a) Bead model
- b) Bead model / SAXS envelope

Experimental setup



# Crystalized Sample Sample in Solution



(very tiny) bit of theory





Often *q* is denoted as *s*

1<sup>st</sup> step: scattering to scattering curve



What can we learn from the scattering curve:



#### Is it really that easy?



2<sup>nd</sup> step: Guinier plot & Kratky plot (from the initial region of the scattering curve)



2<sup>nd</sup> step: Guinier plot & Kratky plot (from the initial region of the scattering curve)





rd step: PDF Pair-distribution function





#### All information from the scattering curve together



### **Bead model**

Spherical harmonics problem to be solved



# Solutions with similar "goodness" of fit may be obtained





# Summary

Parameter	Formula	Range of data used and variable definitions	Comments
Radius of gyration $(R_G)$ : Guinier approximation	$\ln [I(q)] = \ln [I(0)] - \frac{q^2 R_G^2}{3}$	$qR_G$ <1.3 globular, $qR_G$ <0.8 elongated. $I(0)$ : Intensity of the scattering	Most common method of estimating $R_G$ Measured via the slope of the plot $\ln[I(q)]$ vs. $q^2$
Radius of gyration $(R_G)$ : Debye approximation	$I(q) = \frac{2I(0)}{q^4 R_G^4} (q^2 R_G^2 - 1 + e^{-q^2 R_G^2})$	profile extrapolated to $q=0$ $qR_G$ <1.4 for elongated macromolecules	Particularly useful for elongated proteins where the Guinier approximation is valid over narrower range
Radius of gyration $(R_G)$ : defined by $P(r)$	$R_{\rm G}^2 = \int_0^{D_{\rm max}} r^2 P(r) \mathrm{d}r / \int_0^{D_{\rm max}} P(r) \mathrm{d}r$	Entire q-range. $D_{\text{max}}$ : Maximum dimension of particle	Good consistency check for $R_G$ , $D_{\text{max}}$ , and $P(r)$
Pair distribution function $(P(t))$	$P(r) = \frac{r}{2\pi^2} \int_0^\infty I(q) q \sin(qr) dq$	Entire q-range	Indirect Fourier transform methods have been developed for calculating $P(r)$
Maximum dimension $(D_{\text{max}})$	$D_{\text{max}}$ is the value of r at $P(r) = 0$ for large $r$	Requires data $q \leq \pi/D_{\text{max}}$	Assignment of $D_{\text{max}}$ may be complicated by flexibility or multimerization
Particle volume $(V)$ : defined by Porod Invariant	$V = 2\pi^2 I_{\rm exp}^2(0) / \left( \int_0^\infty I(q) q^2 dq \right)$	Entire <i>q</i> -range. $I_{exp}(0)$ is the experimental intensity at $q=0$ and does not require an absolute scale	The integral portion of this equation is known as the Porod invariant. Accuracy varies for shape and size; however absolute scale and concentration
$I(0)$ : Intensity at $q=0$ which is also proportional to mass and volume	$I(0) = 4\pi \left( \int_{0}^{D_{\text{max}}} P(r) dr \right)$	Entire q-range	information are unnecessary Calculation of M and V using this version of $I(0)$ is less susceptible to aggregation and inter-particle correlations than extrapolation of low $q$ data
Mass $(M)$	$M = \frac{I(0)\mu^2}{N_A(1-(\rho_s/\rho_s))^2}$	$\mu$ : Average mass per number of electrons. $\rho_s$ : Solvent electron density $\rho_b$ : Particle electron density $N_A$ : Avagadro's number	I(0) must be on an absolute scale and normalized by mass/volume and not molar concentration
Formulas for elongated or flexibly linked linear macromolecules			
Radius of gyration of cross-section $(R_{\text{XC}})$	$\ln [qI(q)] = \ln [qI(0)] - \frac{q^2 R_{\text{XC}}^2}{2}$	Intermediate $q$ values	The slope of the linear portion of a plot of $\ln[qI(q)]$ vs. $q^2$ is $R_{\text{XC}}^2$ ; however, $R_{\text{XC}}^2$ goes to 0 as q goes to 0 in regimes where scattering is dominated by $R_G$
Length $(L)$	$L = (12(R_{\rm G}^2 - R_{\rm XC}^2))^{\frac{1}{2}}$	See $R_G$ and $R_{XC}$	The co-axial length rather than the hypotenuse $(D_{\text{max}})$

Table 1. Common parameters defined by SAXS for monodisperese and homogeneous scatterers

### Summary (graphically)



# SAXS Hamburg



















#### References:

- 1) Koch, M.H.J., Vachette, P., Svergun, D.I. *Quart Rev Biophys* **2003**
- 2) Jacques, D.A., Trewhella, J. *Prot Sci* **2010**
- 3) Svergun, D.I., Petoukhov, M.V., Koch, M.H.J. *Biophys J* **2001**
- 4) Wriggers, W. *Biophys Rev* **2010**
- 5) Putnam, C.D., Hammel, M., Hura, G.L., Tainer, J.A. *Quart Rev Biophys* **2007**
- 6) Madl, T. Gabel, F., Sattler, M. *J Struct Biol* **2010**



https://deepmind.com/blog/article/alphafold-a-solution-to-a-50-year-old-grand-challenge-in-biology



# Confidence in structural features of proteins determined by X-ray crystallography

(estimates are very rough and strongly depend on the quality of the data)



XAS = X-ray Absorption Spectroscopy

XANES = X-ray Absorption Near Edge Structure EXAFS = Extended X-Ray Absorption Fine Structure





 $I = I_0 \exp(-\mu x)$ 

EXAFS (Extended X-ray Absorption Fine Spectroscopy) poskytuje informace o nejbližších slupkách atomů sousedících s absorbujícím atomem




## **A BLAST search over all non-redundant GenBank genomes**





# **Rentgenstrukturní analýza**

Krystalová mřížka působí na rentgenové záření jako optická mřížka na viditelné světlo. Nastávají ohybové jevy a na stínítku se objevuje difrakční obrazec. Tyto obrazce mohou být matematicky analyzovány, aby se získala informace o rozložení elektronů v molekulách tvořících krystal.

# **Bragg's law**

# $2d \sin\Theta = n\lambda$



 $10$ 



## Krystalografická soustava



## Raumgitter



 $a_0 = b_0 \neq c_0$ :

 $\alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$ ,<br>  $a_0 = b_0 = c_0$ :

















Phase diagram of solubility of a protein in solutions as a function of the concentration of the precipitant.



<https://doi.org/10.3390/cryst8110434>

[http://skuld.bmsc.washington.edu/~merritt/bc530/local\\_copies/phase\\_methods\\_files/vd\\_xtals.jpg](http://skuld.bmsc.washington.edu/~merritt/bc530/local_copies/phase_methods_files/vd_xtals.jpg)





### **X-Ray Goniometer**

a device that **permits the simultaneous recording of the direction of X rays diffracted by a specimen** under study **and** of **the position of the specimen at the time of diffraction**.

An X-ray goniometer can be an independent device recording the diffraction pattern on photographic film; in this case it is an X-ray camera. The term "X-ray goniometer" is also applied to goniometric devices that are components of X-ray diffractometers and are used for mounting the specimen and detector in positions corresponding to the conditions necessary for the occurrence of X-ray diffraction.





https://www.intechopen.com/books/x-ray-scattering/x-ray-diffraction-in-biology-how-can-we-see-dna-and-proteins-in-three-dimensions-

Krystalogram **B-DNA** získaný v **r. 1952** Rosalindou E. **Franklin**ovou, na jehož základě **Watson** a **Crick** navrhli dvoušroubovicový model struktury DNA. **C. & W.** dostali v **r.1962** společně s Mauricem Hugh Frederick Wilkinsem NC za fyziologii a medicínu "za jejich objevy týkající se molekulární struktury nukleových kyselin a jejich významu při přenosu informací v živých organizmech"



## **Mass spectrometry**

• Electron impact Mass spectrometry



- $\cdot$  70 eV = 1614 kcal/mol
	- contrast with energy from IR (1-10 kcal/mol) or NMR (0.2 cal/mol)
	- typical C-C bond = 100 kcal/mol
- Point: lots of energy in play here
	- you can eject electrons, break bonds, etc.

• don't call it spectroscopy (absorption of electromagnetic radiation)

## • Electron impact Mass spectrometry



uniform circular motion in the magnetic field

> $F = zvB$ where z= point charge v= velocity  $B = Field strength$



**Figure 15.04** Copyright © W.W. Norton & Company, Inc. 2005

• Upon ionization, radical cations (M+) are accelerated toward a negatively charged plate with a slit. Some of the ions pass through the slit to form a beam.

. ions follow a curved path between poles of a magnet.

particles follow circular pathway as a function of time



## Molecular Weight vs Exact Mass

**Molecular Mass** refers the average mass of molecules made from their natural isotopic abundance:

**Exact Mass:** The mass of the most abundant isotopic form of a molecule.

Example: HOCH<sub>2</sub>CH<sub>2</sub>Cl

Molecular Weight:

 $2 \times C$ :  $2 \times 12.011$ : 24.022

 $1 \times Q$ :  $1 \times 15.999$ : 15.999

 $1 \times$  Cl:  $1 \times 35.453$ :  $35.453$ 

 $H x 5: 5 x 1.008: 5.040$ 

MW: 80.514



Exact Mass:  $80.003$ 



•Lone pair electrons are more easily displaced than bonding electrons.

•Electrons in pi-bonds are more easily displaced than those in single bonds



The McLafferty Rearrangment:



must have gamma proton

R.









∣a

b





PeptideMass

#### **PeptideMass**

The entered sequence is:

 $10$  $20$  $\frac{30}{40}$  $50$  $60$ MOODDDFONF VATLESFKDL KSGISGSRIK KLTTYALDHI DIESKIISLI IDYSRLCPDS

 $80$ 9<u>0</u> 10<u>0</u> 110  $120$  $70$ HKLGSLYIID SIGRAYLDET RSNSNSSSNK PGTCAHAINT LGEVIQELLS DAIAKSNQDH

 $130$  $140$  $150$  $160$ KEKIRMLLDI WDRSGLFQKS YLNAIRSKCF AMDLEHHHHH

The selected enzyme is: Trypsin

Maximum number of missed cleavages (MC): 0

All cysteines in reduced form.

Methionines have not been oxidized.

Displaying peptides with a mass bigger than 500 Dalton.

Using monoisotopic masses of the occurring amino acid residues and giving peptide masses as [M+H]<sup>+</sup>.

#### The peptide masses from your sequence are:

[Theoretical pl: 6.29 / Mw (average mass): 18149.49 / Mw (monoisotopic mass): 18138.14]



92.5% of sequence covered (you may modify the input parameters to display also peptides < 500 Da or > 100000000000 Da):



Display the list of masses in raw text format to be exported into an external application













Mass Spectrometry

ESI - electron spray ionization – high voltage applied to a liquid jet producing highly charged droplets

MALDI - matrix-assisted laser desorption/ionization

- three step process:

1) sample mix with matrix and deposition on a metal plate,

2) laser pulse desorbs the sample with matrix,

3) analyte molecules are ionized and analyzed (TOF – time of flight MS technique)

Quadrupole mass analyzer





### **UV MALDI Matrix List**


#### ARDD - average relative D-uptake difference



*Scientific Reports* **3**, Article number: 1247, 2013

Performing Hydrogen/Deuterium Exchange with Mass Spectrometry



Figure 1 (HDX/MS): A depiction of the relative deuterium uptake for interferon helps one visualize and interpret the higher order protein structure related to conformational change. The uptake measurements are made at the peptide level for multiple time points across the experiment. Each uptake measurement is superimposed on the 3D structure of the protein, typically obtained from an X-ray representation.

http://www.biopharminternational.com/performing-hydrogendeuterium-exchange-mass-spectrometry

#### ITC – isothermal titration calorimetry

- 1760s Black measured the heat capacity and latent heat of water
- 1780s Lavoisier designed an ice calorimeter and used this instrument to measure the metabolic heat produced by a guinea pig confined in the measurement chamber
- => Calorimeter was one of the earliest scientific instruments & first calorimetric experiment was a biologically relevant measurement



### **Why ITC?**

- 1) ITC is a quantitative technique
- 2) can determine:
	- I. binding affinity (**K<sup>a</sup>** ),
	- II. enthalpy changes (**ΔH**),
	- III. binding stoichiometry (**n**) of the interaction between two or more molecules in solution.

From these initial measurements:

- 3) Gibbs energy changes (**ΔG**)
- 4) entropy changes (**ΔS**)

$$
\Delta G = -RTInK_a = \Delta H - T\Delta S
$$

$$
Q = V_0 \Delta H_{\text{b}}[M]_{\text{t}} K_{\text{a}}[L]/(1 + K_{\text{a}}[L])
$$

$$
Q = V_0[M]_t \Sigma(n_i \Delta H_i K_{ai}[L])/(1 + K_{ai}[L]).
$$

## Thermodynamics Free energy change

 $K_B$  – binding constant

 $K_{D} = 1/K_{B} = \frac{[L] \times [M]}{[ML]}$ 

 $\cdot$   $\Delta G$  is change in free energy

 $\cdot \Delta G \leq 0$  for spontaneous process

 $\Delta G = RT \ln K_{D}$ 

 $\cdot$  More negative  $\Delta G$ , higher affinity  $\Delta G = \Delta H - T \Delta S$ 

### **Enthalpy change**

 $\cdot \Delta H$  – measure of the energy content of the bonds broken and created. The dominant contribution is from hydrogen bonds.

· Negative value indicates enthalpy change favoring the binding

· Solvents play a role

```
\Delta H_{observed} by ITC is total of :
 \Delta H_{binding}\Delta H_{\text{ionization}}\Delta H_{\text{conformation}}
```
**Entropy change** 

- $\cdot$   $\Delta$ S positive for entropically driven reactions
- · Hydrophobic interactions
- · Solvation entropy (favorable) due to release of water
- · Conformational degrees of freedom (unfavorable)

Three possibilities of calorimetric measurement:

- 1) temperature change (either adiabatic or isoperibol) [°C/time]
- 2) power compensation (often called isothermal) [ $\mu$ cal/time]
- 3) heat conduction (Lavoisier design)



## How Do ITCs Work?



# Performing an ITC experiment

**Ligand in syringe** Macromolecule in sample cell Heat of interaction is measured Parameters measured from a single ITC experiment: **Affinity - KD** Energy (Enthalpy) -  $\Delta H$ Number of binding sites - n



### **ITC - Before titration**



• Ligand – in syringe<br>
§ Macromolecule in ITC cell

### **Titration begins: First injection**









**Molar Ratio** 

### **The experimental binding isotherm can be characterized by the unitless value** *c*

 $c = K_{\rm a}[\mathbf{M}]n$ .

