First set of hand-in assignments in QED, fall semester 2003.

These are hand in assigments for the course in Quantum electrodynamics at the Masaryk University in the fall of year 2003. They are part of the requirement of the course and it is necessary to have passed these assignments to be able to take the final exam but no further grading will be used. **Do not leave out any part of the calculations and motivate your assumptions and approximations carefully**. You may answer in Czech *or* English.

1. Give in natural units: Mass of the earth, 1 minute, the charge of an electron, the distance between Prague and Brno, the magnetic moment of the muon $4.49 \cdot 10^{-23}$ erg/G (G = gauss, this one is a little bit tricky).

2. Give in normal units: Somebody is $1.581 \cdot 10^{24}$ eV^{-1} old (answer in years), a car moving with the speed $9.259 \cdot 10^{-8}$ eV^0 (answer in km/h), the mass of the heaviest quark, the top quark is 174.3 GeV (it is around 200 times heavier than the proton!), the magnetic moment of the muon: $4.04 \cdot 10^{-10}$ eV^{-1}.

3. Show, using *only* the anti-commutation relations of the gamma matrices, that:

$$
\begin{aligned}
\mathrm{Tr}\left(\gamma_\mu \gamma_\nu\right) &= 4 g_{\mu\nu} \\
\mathrm{Tr}\left(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma\right) &= 4\left(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}\right) \\
\gamma_\mu \gamma^\mu &= 4\,\mathbf{1} \\
\gamma_\mu \gamma_\nu \gamma^\mu &= -2\gamma_\nu \\
\gamma_\mu \gamma_\nu \gamma_\sigma \gamma^\mu &= 4 g_{\nu\sigma} \\
\gamma_\mu \gamma_\nu \gamma_\sigma \gamma_\rho \gamma^\mu &= -2 \gamma_\rho \gamma_\sigma \gamma_\nu
\end{aligned}
$$

4. Defining the matrix $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ find the anti-commutation relations between γ^5 and the original gammas. Calculate $\gamma^5\gamma^5$. Use this to show that the trace of an odd number of gamma matrices is always zero.

5. Use the explicit representation of the gamma matrices to write the hermitian conjugate of a gamma matrix, $\gamma_\mu^\dagger$, as a product of three

gamma matrices. What is $(\bar{u}\gamma^{\mu_1}\cdots\gamma^{\mu_n}u)^\dagger$, for an arbitrary number of gamma matrices, n?