Second set of hand-in assignments in QED, fall semester 2003.

These are hand in assignments for the course in Quantum electrodynamics at the Masaryk University in the fall of year 2003. They are part of the requirement of the course and it is necessary to have passed these assignments to be able to take the final exam but no further grading will be used. **Do not leave out any part of the calculations and motivate your assumptions and approximations carefully**. You may answer in Czech *or* English.

1. Assume that the electromagnetic field interacts with electrons through the interaction Hamiltonian $ie\int d^4x\bar{\psi}\gamma^5\gamma^\mu\psi A_\mu$.(Here $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Calculate the differential scattering cross section for scattering of electrons in an external Coulomb potential if we assume that the incoming electrons are unpolarized and that we do not observe the spin of the outgoing electrons.

2. It is possible to show that in relativistic theories the spin projection along a particular direction (the m_s quantum number) is in general *not* a constant of motion. That means that for a particle moving in some general direction and with spin pointing up (say) along the z-axis at time $t = 0$, the particle will be in a superposition of spin up and spin down (along the z-axis) at later times. Only if we use the direction along which the particle travels as the axis on which we project is this quantum number conserved. These states are called *helicity* states and are denoted by $u^{(+)}$ for the spin in the same direction as the direction of motion and $u^{(-)}$ for the spin in the opposite direction. The states we have already found are helicity eigenstates only if the direction of motion is in the z-direction ($\mathbf{k} = (0, 0, k)$). They therefore look like

$$u^{(+)} = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{k}{E+m} \\ 0 \end{pmatrix} u^{(-)} = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{k}{E+m} \end{pmatrix}$$

If we instead have a particle moving in the xz-plane with an angle θ to the z-axis ($\mathbf{k} = (k\sin(\theta), 0, k\cos(\theta))$ we have the helicity eigenstates

$$u^{(+)} = \sqrt{E+m}\begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) \\ \frac{|\mathbf{k}|}{E+m}\cos\left(\frac{\theta}{2}\right) \\ \frac{|\mathbf{k}|}{E+m}\sin\left(\frac{\theta}{2}\right) \end{pmatrix} u^{(-)} = \sqrt{E+m}\begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{\theta}{2}\right) \\ \frac{|\mathbf{k}|}{E+m}\sin\left(\frac{\theta}{2}\right) \\ -\frac{|\mathbf{k}|}{E+m}\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Calculate the cross section for scattering of electrons in the electric field of a nucleus in the four different processes when the incoming and outgoing electrons are in helicity eigenstates. Show how the usual Mott cross section can be recovered from your result.