Stellar winds

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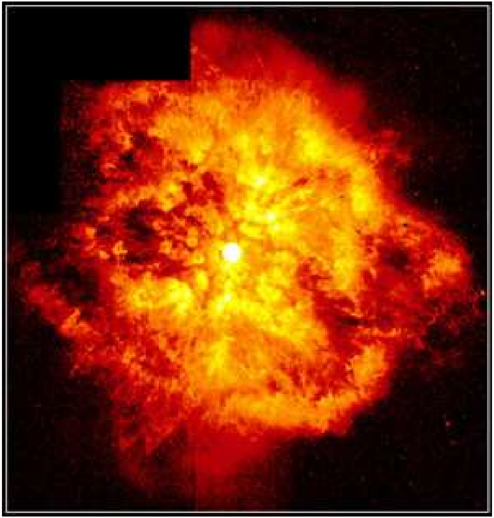
Introduction: why mass-loss?

Evidence for mass-loss: shells around stars

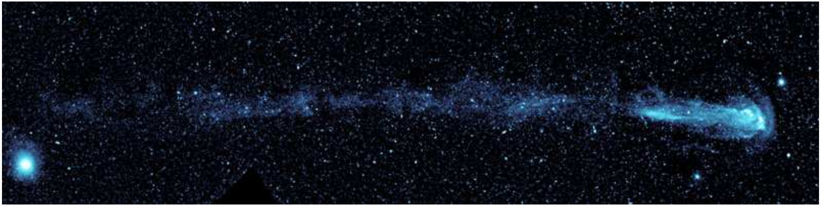
Abell 39 nebula

Evidence for mass-loss: shells around stars



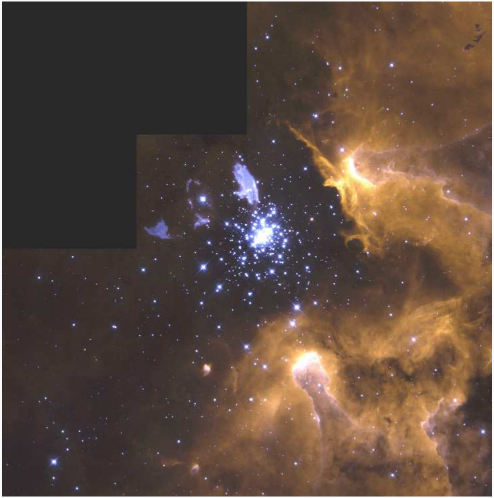
nebula around WR 124

Evidence for mass-loss: shells around stars



nebula around Mira (o Cet)

Evidence for mass-loss: interstellar medium



NGC 3603 cluster

Evidence for mass-loss: heavy elements

After the period of primordial nucleosynthesys, the Universe was composed mostly form H and He (with a tiny amount of heavier elements like Li). Heavy elements (C, N, O, Fe, . . .) were completely missing.

However, there are heavy elements around us. Where do they come from? Heavier elements are synthethised during thermonuclear reactions in the stellar interiors. How do they get into the interstellar medium?

How the mass can espape gravitational wells of stars?

Let us start with the momentum equation with gravity

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = \rho g_{\mathrm{w}} - \frac{\partial p}{\partial r} - \frac{\rho G M}{r^2},$$

which can be simplified assuming stationary isothermal outflow ($p = a^2 \rho$)

$$v \frac{\mathrm{d} v}{\mathrm{d} r} = g_{\mathrm{w}} - \frac{a^2}{\rho} \frac{\mathrm{d} \rho}{\mathrm{d} r} - \frac{G M}{r^2},$$

where g_{w} gives the force that drives the wind (force per unit of mass, i.e., the acceleration) and a is the isothermal sound speed.

We can integrate the equation from the stellar surface R_* to infinity

$$\int_{R_*}^{\infty} v \frac{\mathrm{d} v}{\mathrm{d} r} \,\mathrm{d} r = \int_{R_*}^{\infty} g_{\mathrm{w}} \,\mathrm{d} r - \int_{R_*}^{\infty} \frac{a^2}{\rho} \frac{\mathrm{d} \rho}{\mathrm{d} r} \,\mathrm{d} r - \int_{R_*}^{\infty} \frac{G M}{r^2} \,\mathrm{d} r$$

yielding

$$\frac{1}{2} v_{\infty}^2 - \frac{1}{2} v_0^2 = \int_{R_*}^{\infty} g_{\mathrm{w}} \,\mathrm{d} r - a^2 \ln \frac{\rho_{\infty}}{\rho_0} - \frac{G M}{R_*}.$$

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_{\mathrm{w}}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

There are three ways to initiate the outflow. Either the initial velocity is larger than the escape speed v_{esc},

$$\frac{1}{2}v_0^2 \geq \frac{GM}{R_*}, \quad v_0 \geq v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R_*}}, \quad v_{\mathrm{esc}} = 620\,\mathrm{km\,s^{-1}}\sqrt{\frac{M}{M_\odot}\frac{R_\odot}{R}}.$$

The initial kinetic energy of the flow should be larger than the absolute value of the potential energy. This is fullfilled for **explosive outflows** like supernovae or supernova impostors (e.g., η Car).

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

The other possibility is that the driving force is large enough

$$\int_{R_*}^{\infty} g_\mathrm{w}\,\mathrm{d}r \geq \frac{GM}{R_*}.$$

This is true for **winds driven radiatively**, either due to the absorption in lines (in hot stars) or on dust particles (in luminous cool stars).

Three ways

The individual terms in

$$\frac{1}{2}v_\infty^2 - \frac{1}{2}v_0^2 = \int_{R_*}^{\infty} g_{\mathrm{w}}\,\mathrm{d}r - a^2 \ln\frac{\rho_\infty}{\rho_0} - \frac{GM}{R_*}$$

describe (from left to right) change of the kinetic energy (per unit of mass), work of driving forces, work of pressure force, and the potential energy (per unit of mass).

The last possibility is that the work done by pressure forces is large,

$$a^2 \ln\frac{\rho_0}{\rho_\infty} \geq \frac{GM}{R_*}.$$

Because $\ln(\rho_0/\rho_\infty)$ is of the order of ten at most, this implies that $a \approx v_{\mathrm{esc}}$. This happens in **coronal winds** of cool main-sequence stars.

Coronal winds

Is there any evidence for the wind of our Sun?

- two types of the comet tails (Biermann 1951)

Is there any evidence for the wind of our Sun?

- aurorae

Is there any evidence for the wind of our Sun?

- satellite observations
  - flux of particles streaming from our Sun (protons, electrons, He, ...)
  - speed about $\sim 500\,\mathrm{km\,s^{-1}}$
  - number density ($r = 1\,\mathrm{a.u.}$) $\sim 10^7$ particles $\mathrm{m^{-3}}$
  - mass-loss rate

$$\dot{M} = 4\pi r^2 \rho v \approx 2 \times 10^{-14}\,\mathrm{M_\odot\,yr^{-1}}$$

Thermally driven wind

We have seen that the spherically symmetric atmosphere cannot be in hydrostatic equilibrium. However, the root mean square of the total velocity of particles $v_{\mathrm{th}} = \sqrt{\frac{3kT}{m_{\mathrm{H}}}}$ in the atmosphere with $T = 6000\,\mathrm{K}$ is about $v_{\mathrm{th}} = 12\,\mathrm{km\,s^{-1}}$, which is significantly lower than the escape speed $v_{\mathrm{esc}} = 620\,\mathrm{km\,s^{-1}}$.

Thermally driven wind

The Sun has a large outer atmosphere called *corona*. The corona can be in optical light observed only during the solar eclipses or using satellites. The detection of lines of highly ionized atoms (Ca XII, Fe XIII, Ni XVI,. . . , "coronium", Grotrian 1939, Edlén 1942) shows that the temperature of the solar corona is about $10^5 - 10^6\,\mathrm{K}$. Corresponding root mean square of the total velocity of the order of $100\,\mathrm{km\,s^{-1}}$ is comparable with the escape speed. Consequently, the thermal expansion of the solar corona is thought to be the source of the solar wind (Parker 1958).

This coins the term coronal wind.

Parker model of the coronal wind

Let us assume that the coronal wind wind can be described as a spherically symmetric, stationary, and isothermal outlow. Then the corresponding hydrodynamical equations are

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0,$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{\rho GM}{r^2}.$$

The integration of the continuity equation gives the **mass-loss rate** $\dot{M} \equiv 4\pi r^2\rho v = \mathrm{const.}$

Inserting $\mathrm{d}\rho/\mathrm{d}r$ from the continuity equation into the equation of motion gives ordinary differential equation for velocity

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2},$$

which can be solved analytically.

Parker model of the coronal wind

We shall study the momentum equation

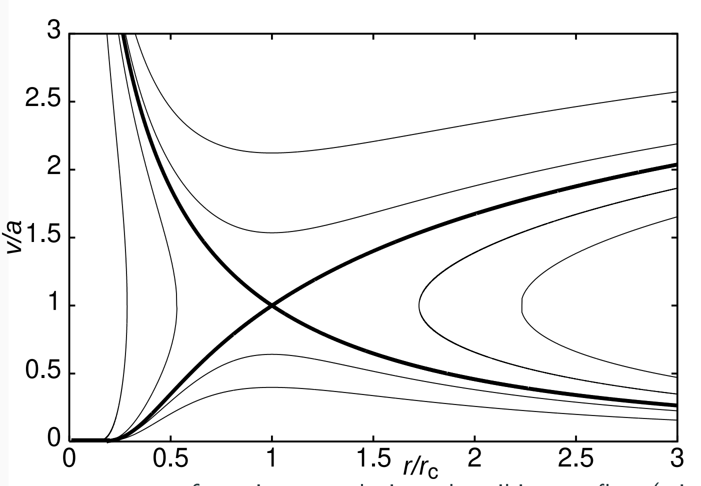
$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2}$$

in more detail.

At radius $r = r_\mathrm{c}$ given by $2a^2/r_\mathrm{c} = GM/r_\mathrm{c}^2$ the right-hand side of momentum equation is equal to zero. This implies either $v = a$ or $\mathrm{d}v/\mathrm{d}r = 0$.

At the **sonic point** $(v = a)$ either $r = r_\mathrm{c}$ or $\mathrm{d}v/\mathrm{d}r \to \infty$. At the sonic point the sound speed is equal to half of the escape speed.

Solution of the Parker equation



There are two types of continuous solutions describing outflow (wind) and inflow (accretion). There is one outflow solution that is supersonic at large distances from the Sun (wind). Other outflow solutions are subsonic ("breeze"). Observations show supersonic flow at the location of Earth.

Nature of coronal heating: unresoloved problem

The nature of coronal heating is one of the most important open problems in astrophysics. It is most likley related to a deep hydrogen convective zone in cool stars. The convection magnifies seed magnetic fields. Sound waves generated by the convection interact with coronal magnetic field and create MHD waves. Hybrid MHD waves or Alfvén waves possibly heat the corona.

Coronal winds: importance

Coronal winds of main-sequence stars are weak and do not influence stellar evolution significantly. In earlier phases, they might be responsible for evolution of interplanetary medium. Coronal winds are important for the interaction of stars with exoplanets and for rotational braking of main-sequence stars.

In cool gaints, the coronal winds are important for the mass-loss (Cranmer & Saar 2011, Suzuki 2013). It is expected that our Sun will lose a fraction of its mass by this mechanism (about 0.2).

Dust-driven winds

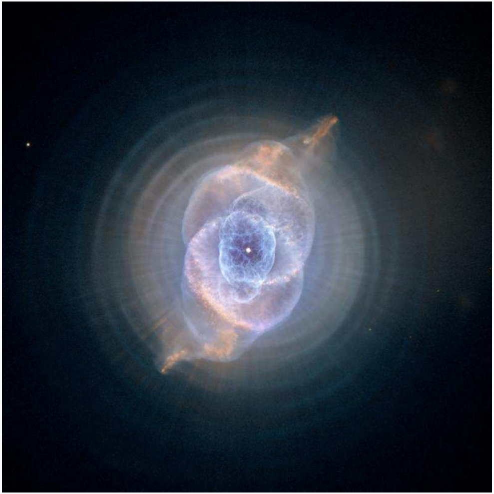
Evidence for wind in cool luminous stars

- envelopes around stellar remnants

planetary nebula Abell 39

Evidence for wind in cool luminous stars

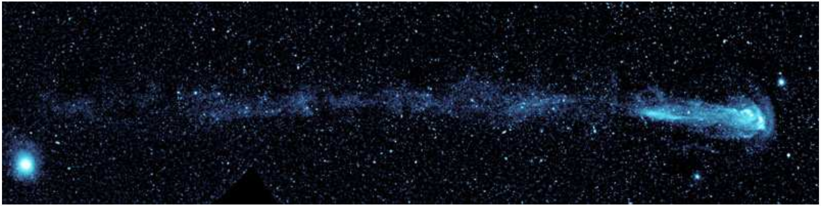
- envelopes around stellar remnants



Cat's Eye Nebula – NGC 6543 (HST)

Evidence for wind in cool luminous stars

- envelopes around stellar remnants



nebula around Mira (o Cet)

Driving the wind of cool luminous stars

We have seen that there are observational indications of wind in cool luminous stars (AGB stars, red supergiants). These winds are likely not connected with coronae, due to missing strong X-ray emission and chromospheric activity. On the other hand, these stars are luminous, consequently radiative force is capable to drive a wind in these stars. As a result of their low temperature, dust grains form in the envelopes of cool luminous stars. Dust grain absorb the stelar radiation and accelerate the wind giving rise to dust driven winds.

Dust driven wind equations

Again, let us assume that the dust driven winds can be described as a spherically symmetric, stationary, and isothermal outlow. Then the corresponding hydrodynamical equations are

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0,$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{\rho G M}{r^2} + \rho g_{\mathrm{rad}},$$

where g_{rad} is the radiative force.

The integration of the continuity equation gives the **mass-loss rate**

$$\dot{M} \equiv 4\pi r^2 \rho v = \text{const.}$$

Inserting $\mathrm{d}\rho/\mathrm{d}r$ from the continuity equation into the equation of motion gives ordinary differential equation for velocity

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a^2}{r} - \frac{GM}{r^2} + g_{\mathrm{rad}}.$$

In cool dust driven winds, one can neglect $\frac{2a^2}{r} \ll \frac{GM}{r^2}$.

Sonic point condition

From the momentum equation

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = g_{\mathrm{rad}} - \frac{GM}{r^2}$$

follows that at the sonic point $v = a$ the radiative force equals the gravity (in magnitude)

$$g_{\mathrm{rad}} = \frac{GM}{r^2}.$$

The gravity is stronger than the radiative force in subsonic region $v < a$

$$g_{\mathrm{rad}} < \frac{GM}{r^2},$$

while the radiation dominates in the supersonic region $v > a$

$$g_{\mathrm{rad}} > \frac{GM}{r^2}.$$

Radiative force due to the dust

The radiative force due to absorption of radiation on dust particles is

$$f_{\mathrm{rad}} = \rho g_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu,$$

where $\chi(r,\nu)$ is the absorption coefficient and $F(r,\nu)$ is the radiative flux. Because $\kappa(r,\nu) = \chi(r,\nu)/\rho(r)$ varies only due to a change of the dust fraction and $F(r,\nu)/F(r)$ varies mostly with frequency, the radiative acceleration is

$$g_{\mathrm{rad}} = \frac{1}{c}\,\overline{\kappa}(r)F(r),$$

where the mean opacity is

$$\overline{\kappa}(r) = \int_0^\infty \kappa(r,\nu)\frac{F(r,\nu)}{F(r)}\,\mathrm{d}\nu$$

and $F(r)$ is the integrated flux, $F(r) = L/(4\pi r^2)$.

Eddington parameter due to dust

Introducing the ratio of the radiative and gravity acceleration,

$$\Gamma_{\mathrm{d}}(r) = \frac{\overline{\kappa}(r)L}{4\pi c G M},$$

the momentum equation can be rewritten as

$$\frac{1}{v}\left(v^2 - a^2\right)\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{GM}{r^2}\left(1 - \Gamma_{\mathrm{d}}(r)\right).$$

There are three regions of the wind:

- subsonic region, $v < a$, $\Gamma_{\mathrm{d}}(r) < 1$,
- sonic point, $v = a$, $\Gamma_{\mathrm{d}}(r) = 1$ (formation of the dust),
- supersonic region $v > a$, $\Gamma_{\mathrm{d}}(r) > 1$.

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho G M}{r^2} + \Gamma_{\mathrm{d}} \frac{\rho G M}{r^2}$$

and let us assume that dust starts to form close to the sonic point r_{c}, where Γ_{d} changes from $\Gamma_{\mathrm{d}} \ll 1$ to $\Gamma_{\mathrm{d}} \gg 1$. We shall multiply the momentum equation by $4\pi r^2$ and integrate the equation from R_* to ∞

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho G M}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_{\mathrm{d}} \frac{\rho G M}{r^2} \mathrm{d}r.$$

The term in the first integral can be taken out assuming constant mass-loss rate $\dot{M} = 4\pi r^2 \rho v$. The integral can be evaluated assuming hydrostatic equilibrium in the atmosphere $v(R_*) \approx 0$ and using wind terminal velocity $v_\infty = v(r = \infty)$. Therefore,

$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r = \dot{M} \left[v\right]_{R_*}^{\infty} \approx \dot{M} v_\infty .$$

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho GM}{r^2} + \Gamma_\mathrm{d} \frac{\rho GM}{r^2}$$

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$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r.$$

The second integral gives the hydrostatic equilibrium density distribution for $r < r_\mathrm{c}$, while the pressure gradient becomes neglibible for $r < r_\mathrm{c}$. Therefore,

$$\int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{r_\mathrm{c}} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r + \\ + \int_{r_\mathrm{c}}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r \approx 4\pi GM \int_{r_\mathrm{c}}^{\infty} \rho \, \mathrm{d}r.$$

Estimating the mass-loss rate

Let us start with the momentum equation

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{\rho GM}{r^2} + \Gamma_\mathrm{d} \frac{\rho GM}{r^2}$$

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$$\int_{R_*}^{\infty} 4\pi r^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}r} \mathrm{d}r + \int_{R_*}^{\infty} 4\pi r^2 \left[\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\rho GM}{r^2}\right] \mathrm{d}r = \int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r.$$

The right-hand side integral can be evaluated using wind optical depth $\tau_\mathrm{W} = \int_{r_\mathrm{c}}^{\infty} \overline{\kappa}(r) \rho \, \mathrm{d}r$,

$$\int_{R_*}^{\infty} 4\pi r^2 \Gamma_\mathrm{d} \frac{\rho GM}{r^2} \mathrm{d}r = 4\pi GM \int_{r_\mathrm{c}}^{\infty} \Gamma_\mathrm{d} \rho \, \mathrm{d}r = \frac{L}{c} \int_{r_\mathrm{c}}^{\infty} \overline{\kappa}(r) \rho \, \mathrm{d}r = \tau_\mathrm{W} \frac{L}{c}.$$

Estimating the mass-loss rate

Putting all the terms together we derive

$$\dot{M} v_\infty = 4\pi G M \int_{r_\mathrm{c}}^{\infty} \rho \, \mathrm{d}r + \tau_\mathrm{W} \frac{L}{c}.$$

For $\Gamma_\mathrm{d} \gg 1$ the gravity term can be neglected with respect to the radiative acceleration and we derive the mass-loss rate estimate

$$\dot{M} v_\infty = \tau_\mathrm{W} \frac{L}{c}.$$

We see that by assuming multiple scattering ($\tau_\mathrm{W} > 1$) the mass-loss rate can be significantly higher than the single-scattering limit $\dot{M} v_\infty = L/c$ (Gail a Sedlmayr 1986, Netzer a Elitzur 1993).

Problem with too high condensation radii

The condition of radiative equilibrium on dust particles

$$\int_0^\infty \kappa_\lambda \, B_\lambda(T) \, \mathrm{d}\lambda = \int_0^\infty \kappa_\lambda \, J_\lambda \, \mathrm{d}\lambda$$

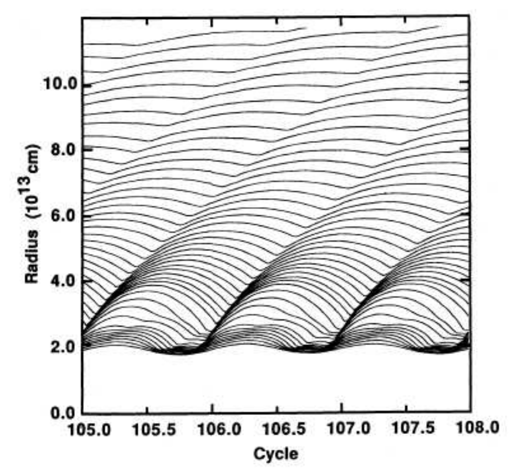
predicts too high dust temperature in the atmosphere. The temperature is higher than the condensation temperature T_c. The condensation may appar only at larger distances from the star, which are higher than the condensation radius, $r > r_\mathrm{c}$. This is a problem for radiative driving.

	silicate	graphite	amorphous carbon
	$T_\mathrm{c} = 1500\,\mathrm{K}$	$T_\mathrm{c} = 1500\,\mathrm{K}$	$T_\mathrm{c} = 1500\,\mathrm{K}$
T_eff	r_c/R_*	r_c/R_*	r_c/R_*
3000	2.99	4.03	3.42
2500	1.85	2.34	2.12
2000	1.15	1.29	1.24

(Lamers a Cassinelli 1999)

Solution of the problem with too high condensation radii

Luminous cool stars pulsate, consequently, pulsations may transfer the stellar matter to a large distances from the star.



trajectories of pulsating particles without radiative acceleration on dust particles (Bowen 1988)

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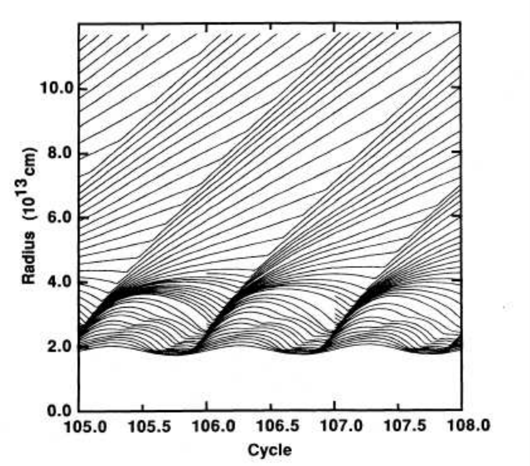


trajectories of pulsating particles with radiative acceleration on dust particles (Bowen 1988)

Wind radial velocity

Neglecting the gas pressure term, the momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = (\Gamma_\mathrm{d}(r) - 1)\frac{GM}{r^2}$$

can be integrated. Substituting $v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{1}{2}\frac{\mathrm{d}(v^2)}{\mathrm{d}r}$, we have for the radial wind velocity

$$v^2(r) = v^2(r_\mathrm{c}) + 2GM\int_{r_\mathrm{c}}^{r}\frac{\Gamma_\mathrm{d} - 1}{r'^2}\,\mathrm{d}r'.$$

The wind speed at the critical point can me typically neglected. Moreover, one can assume that the Eddingtin parameter is constant $\Gamma_\mathrm{d} = \mathrm{const}$. This yelds for the wind velocity "beta" velocity law in the form of

$$v(r) = v_\infty\sqrt{1 - \frac{r_\mathrm{c}}{r}}$$

with **wind terminal velocity**

$$v_\infty = \sqrt{2GM\,(\Gamma_\mathrm{d} - 1)/r_\mathrm{c}}$$

propotional to he escape speed.

Minimum stellar luminosity to drive a wind

As a necessary condition for the wind existence, the radiative force should overcome the gravity,

$$\Gamma_\mathrm{d} > 1.$$

Inserting the formula for the Eddington parameter,

$$\frac{\overline{\kappa} L}{4\pi c G M} > 1,$$

this gives the condition for the stellar luminosity,

$$L > \frac{4\pi c G M}{\overline{\kappa}}.$$

Assuming a typical mean opacity $\overline{\kappa} \approx 30\,\mathrm{cm}^2\,\mathrm{g}^{-1}$, the minimum stellar luminosity to drive a wind is (in scaled quantities)

$$L > 400\,\mathrm{L}_\odot \left(\frac{M}{1\,\mathrm{M}_\odot}\right).$$

This means, that evolved solar-mass stars with $L > 400\,\mathrm{L}_\odot$ may drive a dust driven wind.

Dust driven winds: importance

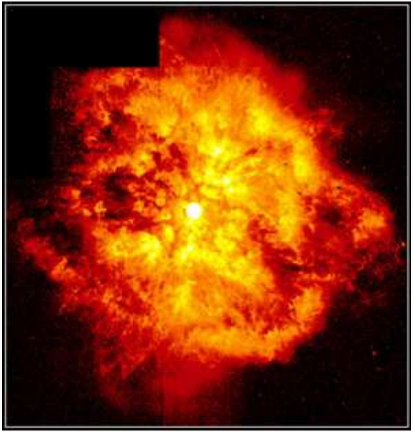
Dust driven winds appear during the AGB phase of low-mass stars with initial mass $0.4\,\mathrm{M}_\odot \lesssim M_0 \lesssim 8\,\mathrm{M}_\odot$. A typical wind mass-loss rate due to dust driven wind is of the order of $10^{-7}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1} - 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$. Given a typical duration of AGB phase (which is of the order of $10 \times 10^6\,\mathrm{yr}$), low-mass stars lose a significant amount of their mass via dust driven winds.

Part of the material lost during the AGB phase is ionized in a subsequent evolutionary phase and form a planetary nebula. Dust driven winds carry freshly synthethised s-process elements, consequently AGB stars are an important source of elements heavier than iron.

Line-driven winds of hot stars

Evidence for wind in hot stars

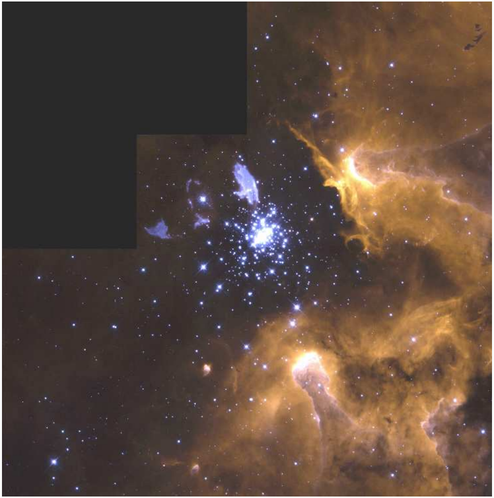
- shells in the surroundings of hot stars



nebula close to the star WR 124 (HST)

Evidence for wind in hot stars

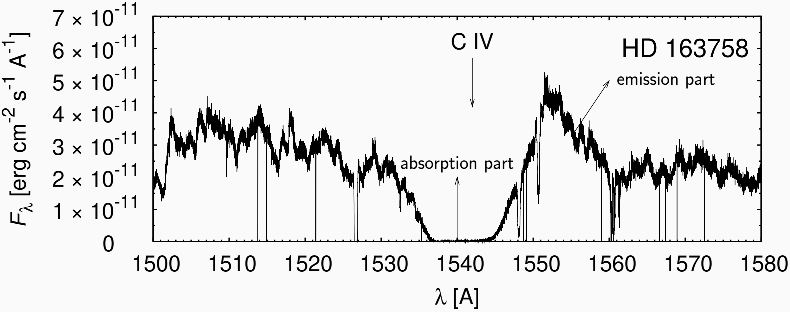
- the interstellar medium around hot stars



open cluster NGC 3603 (HST)

Evidence for wind in hot stars

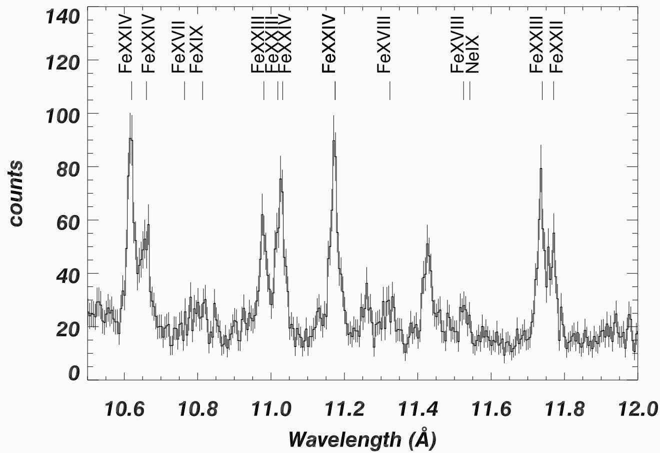
- P Cyg line profiles in UV



HD 163758 (HST)

Evidence for wind in hot stars

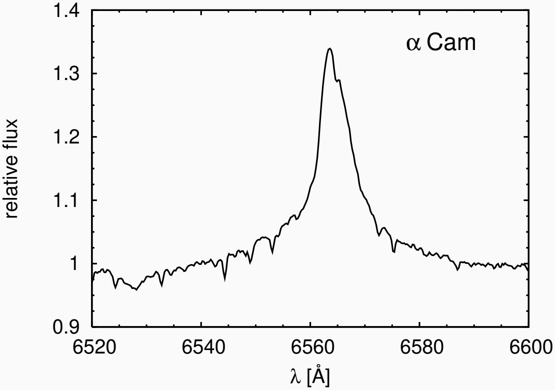
- X-ray emission



X-ray spectrum θ^1 Ori C
(CHANDRA, Schulz et al. 2003)

Evidence for wind in hot stars

- Hα emission line



α Cam, 2m telescope in Ondřejov (Kubát 2003)

Radiative driving

Hot stars are very luminous, consequently the radiative force can be suspected to drive the wind. In a spherically symmetric case, the radiative force is

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu,$$

where $\chi(r,\nu)$ is the absorption coefficient and $F(r,\nu)$ the radiative flux.

Radiative driving: free electrons

The free electrons are the most numerous particles in the wind. In a nonrelativistic limit, the opacity due to the light scattering on free electrons is $\chi(r,\nu) = \sigma_{\mathrm{Th}} n_{\mathrm{e}}(r)$, where σ_{Th} is the Thomson scattering cross-section and $n_{\mathrm{e}}(r)$ is the electron density. Because the σ_{Th} is frequency independent, the integral can be easily evaluated as

$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c},$$

where $L = 4\pi r^2 \int_0^\infty F(r,\nu)\,\mathrm{d}\nu$ is the stellar luminosity. This can be easily compared with the force due to gravity $f_{\mathrm{grav}} = \rho(r) GM/r^2$ to give the **Eddington parameter**

$$\Gamma \equiv \frac{f_{\mathrm{rad}}}{f_{\mathrm{grav}}} = \frac{\sigma_{\mathrm{T}} \frac{n_{\mathrm{e}}(r)}{\rho(r)} L}{4\pi c G M}, \quad \text{in scaled quantities,} \quad \Gamma \approx 10^{-5} \left(\frac{L}{1\,\mathrm{L}_\odot}\right) \left(\frac{M}{1\,\mathrm{M}_\odot}\right)^{-1}.$$

For a typical stars Γ is roughly constant and $\Gamma < 1$, therefore free electrons are not suitable to drive a wind.

Radiative driving: line transitions

Line (bound-bound) transitions provide another viable source of the radiative force. The opacity due to the line transitions is

$$\chi(r,\nu) = \frac{\pi e^2}{m_\mathrm{e} c} \sum_{\mathrm{lines}} \varphi_{ij}(\nu) g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right),$$

where $\varphi_{ij}(\nu)$ is the line profile normalized over frequencies, $\int_0^\infty \varphi_{ij}(\nu)\,\mathrm{d}\nu = 1$, f_{ij} is the oscillator strength, and $n_i(r)$, $n_j(r)$ are level occupation number with statistical weights g_i, g_j.

The radiative force due to the line transitions is then

$$f_{\mathrm{line}} = \frac{\pi e^2}{m_\mathrm{e} c^2} \int_0^\infty \sum_{\mathrm{line}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) \varphi_{ij}(\nu) F(r,\nu)\,\mathrm{d}\nu.$$

This cannot be easily evaluated due to the dependence of flux on frequency (lines may be optically thick). Therefore, $F(r,\nu)$ should be derived from the radiative transfer equation. This introduces coupling with hydrodynamical equations.

Radiative driving: optically thin lines

A crude estimate of the radiative force (maximum) can be obtained assuming that the lines are optically thin. In such case the flux $F(r, \nu)$ is constant over the frequencies corresponding to a given line yilding

$$f_{\text{lines}}^{\max} = \frac{\pi e^2}{m_{\text{e}} c^2} \sum_{\text{lines}} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j} \right) F(r, \nu_{ij}),$$

where ν_{ij} is the frequency of the line center. Again, we can compare the radiative force with gravity, which gives the ratio

$$\frac{f_{\text{lines}}^{\max}}{f_{\text{grav}}} = \Gamma \sum_{\text{lines}} \frac{\sigma_{ij}}{\sigma_{\text{Th}}} \frac{n_i}{n_{\text{e}}} \frac{\nu_{ij} L_\nu(\nu_{ij})}{L},$$

where we have neglected upper level population $n_j(r) \ll n_i(r)$ and where

$$\sigma_{ij} = \frac{\pi e^2 f_{ij}}{\nu_{ij} m_{\text{e}} c} \qquad \text{and} \qquad L_\nu(\nu_{ij}) = 4\pi r^2 F(r, \nu_{ij}).$$

For lines of heavier elements $\sigma_{ij}/\sigma_{\text{Th}} \approx 10^7$ and therefore $f_{\text{line}}^{\max}/f_{\text{grav}}$ is up to 10^3 (Abbott 1982, Gayley 1995).

This enables the appearance of line driven winds.

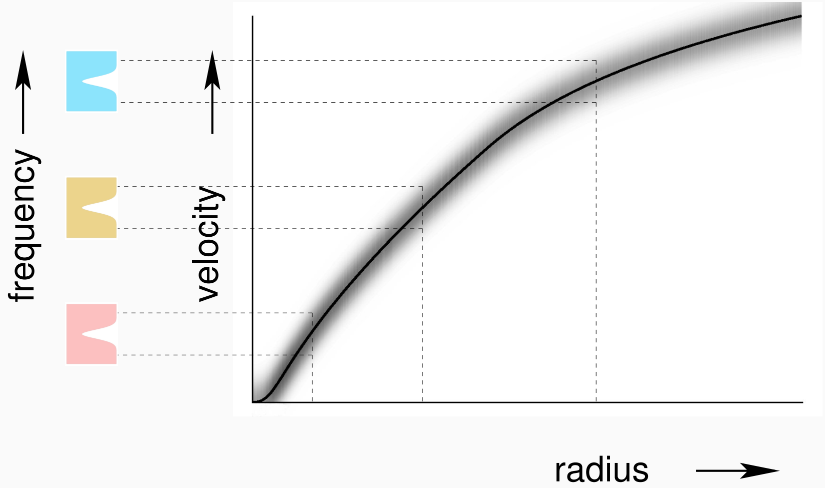
Radiative driving: general line depths

In general, lines may be optically thick. Therefore, the dependence of the line force on density and velocity (due to the Doppler efect) becomes very complex. Moreover, the level populations depend on the radiative field. This introduces intricate feedback that has to be resolved numerically in general.

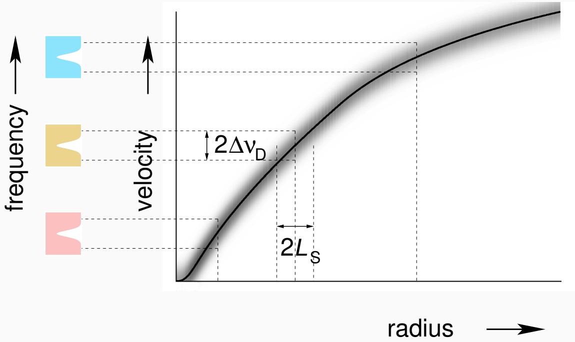
However, there exists an approximation named after Sobolev, which allows to derive an approximate expression for the radiative force in supersonic flows. This allows to obtain approximate solutions of wind equations.

Sobolev approximation: the essence

Stellar wind velocity increases with frequency. This causes a Doppler shift of the frequency at which the line absorbs the stellar radiation. In a supersonic wind, the width of the line (gray area) is smaller than the shift.



Sobolev approximation: going into details



If the Doppler width of the line $\Delta\nu_\mathrm{D}$ is smaller than the Doppler shift due to the radial expansion, then the radiative transfer equation has to be solved only accross the Sobolev length

$$L_\mathrm{S} \equiv \frac{v_\mathrm{th}}{\frac{\mathrm{d}v}{\mathrm{d}r}} = c\frac{\Delta\nu_\mathrm{D}}{\nu_{ij}}\frac{1}{\frac{\mathrm{d}v}{\mathrm{d}r}}.$$

This is possible if the Sobolev length is smaller than the density scale height H, $L_\mathrm{S} = v_\mathrm{th}/\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right) \ll H = \rho/\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right) \approx v/\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)$, i.e., for $v \gg v_\mathrm{th}$.

Sobolev approximation: analytical approximation

Solution of the comoving frame (CMF) radiative transfer equation provides a straightforward way to derive the Sobolev approximation. This equation for spherically symmetric stationary flow reads

$$\mu\frac{\partial}{\partial r}I(r,\mu,\nu)+\frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}I(r,\mu,\nu)-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)= =\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu).$$

When the spatial derivatives can be neglected (Sobolev approximation), $\frac{\partial}{\partial r}I(r,\mu,\nu)\sim\frac{I(r,\mu,\nu)}{r}\ll\frac{\nu v(r)}{cr}\frac{\partial}{\partial\nu}I(r,\mu,\nu)\sim\frac{\nu v(r)}{cr}\frac{I(r,\mu,\nu)}{\Delta\nu_{\mathrm{D}}}$, i.e, for $v(r)\gg v_{\mathrm{th}}$, the CMF radiative transfer equation is

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)= =\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu).$$

CMF radiative transfer equation equation: one line

We shall solve the CMF radiative transfer equation for *one* line,

$$
\begin{aligned}
-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu) = \\
= \chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\left(S_{\mathrm{L}}(r)-I(r,\mu,\nu)\right),
\end{aligned}
$$

where

$$\chi(r,\nu) = \chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

$$\eta(r,\nu) = \chi_{\mathrm{L}}(r)S_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

$$\text{where } \chi_{\mathrm{L}}(r) = \frac{\pi e^2}{m_{\mathrm{e}}c}g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right).$$

Solving the CMF equation

We shall introduce a new variable

$$y = \int_{\nu}^{\infty} \mathrm{d}\nu' \varphi_{ij}(\nu'),$$

which is $y = 0$ for the incoming side of the line and $y = 1$ for the outgoing side of the line.

This transforms the CMF radiative transfer equation

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu) = \\ = \chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\left(S_{\mathrm{L}}(r)-I(r,\mu,\nu)\right),$$

into

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y) = \\ = \chi_{\mathrm{L}}(r)\left(S_{\mathrm{L}}(r)-I(r,\mu,y)\right).$$

Integrating the CMF equation

The transformed CMF radiative transfer equation

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y) =$$

$$= \chi_{\mathrm{L}}(r)\left(S_{\mathrm{L}}(r)-I(r,\mu,y)\right).$$

can be integrated assuming that the variables do not significantly vary with r within the Sobolev "resonance zone". Replacing ν by center-of-line frequency ν_0, the integration gives

$$I(y) = I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)y\right] + S_{\mathrm{L}}\left\{1-\exp\left[-\tau(\mu)y\right]\right\},$$

where the Sobolev optical depth is given by

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

and where we assumed the boundary condition $I(y=0)=I_{\mathrm{c}}(\mu)$. Clearly, the Sobolev optical depth is inversely proportional to the velocity gradient, $\tau \sim \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$.

Calculating the radiative force

The radial component of the radiative force is

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu.$$

Inserting the expression for the specific intensity and opacity gives

$$f_{\mathrm{rad}} = \frac{2\pi}{c}\int_0^\infty \mathrm{d}\nu\,\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\int_{-1}^1 \mathrm{d}\mu\,\mu I(r,\mu,\nu).$$

Transforming the intregral using variable y yields.

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\mu\,\mu I(r,\mu,y).$$

Inserting the solution of CMF equation

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\mu\,\mu\left\{I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)y\right] + S_{\mathrm{L}}\left\{1-\exp\left[-\tau(\mu)y\right]\right\}\right\}$$

shows that there is no net contribution of the emission to the radiative force, because the Sobolev optical depth is an even function of μ (assuming that S_{L} is isotropic in the CMF).

The radiative force

The Sobolev radiative force

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)y\right]$$

after the integration over μ is

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_{-1}^1 \mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\frac{1-\exp\left[-\tau(\mu)\right]}{\tau(\mu)}$$

and after inserting the expression for the optical depth we arrive at the radiative force in the form of (with $\sigma(r) = \frac{r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r} - 1$)

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{rc^2}\int_{-1}^1 \mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\left[1+\mu^2\sigma(r)\right]\times$$
$$\times\left\{1-\exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]\right\},$$

which was derived by Sobolev (1957), Castor (1974), and Rybicki & Hummer (1978).

Optically thin lines: checking the consistency

For optically thin lines $\tau(\mu) \ll 1$ the radiative force

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\frac{1-\exp\left[-\tau(\mu)\right]}{\tau(\mu)}$$

can be simplified using $\exp\left[-\tau(\mu)\right] \approx 1-\tau(\mu)$, which gives

$$\frac{1-\exp\left[-\tau(\mu)\right]}{\tau(\mu)} \approx 1$$

and for the radiative force

$$f_{\mathrm{rad}} = \frac{2\pi}{c}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\chi_{\mathrm{L}}(r) = \frac{1}{c}\,\chi_{\mathrm{L}}(r)F(r).$$

Therefore, the optically thin radiative force proportional to the radiative flux $F(r)$ and to opacity (density). The same result can be derived for the static medium.

Optically thick lines

For optically thick lines $\tau(\mu) \gg 1$ one can neglect the $\exp[-\tau(\mu)]$ term in

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\frac{1-\exp[-\tau(\mu)]}{\tau(\mu)},$$

which, after inserting of the optical depth cancels out the opacity,

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{rc^2}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\left\{1+\mu^2\left[\frac{r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}-1\right]\right\}.$$

Neglecting the limb darkening one can assume that

$$I_{\mathrm{c}}(\mu) = \begin{cases} I_{\mathrm{c}} = \mathrm{const.}, & \mu \geq \mu_*, \\ 0, & \mu < \mu_* \end{cases}, \quad \text{where} \quad \mu_* = \sqrt{1-\frac{R_*^2}{r^2}},$$

and the radiative force is

$$f_{\mathrm{rad}} = \frac{\nu_0 v(r) F(r)}{rc^2}\left\{1+\left[\frac{r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}-1\right]\left[1-\frac{1}{2}\frac{R_*^2}{r^2}\right]\right\},$$

where $F = 2\pi\int_{\mu_*}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}} = \pi\frac{R_*^2}{r^2}I_{\mathrm{c}}$.

Optically thick lines

At large distances from the star $r \gg R_*$ the optically thick radiative force

$$f_{\rm rad} = \frac{\nu_0 v(r) F(r)}{rc^2} \left\{ 1 + \left[\frac{r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r} - 1 \right] \left[1 - \frac{1}{2} \frac{R_*^2}{r^2} \right] \right\}$$

is

$$f_{\rm rad} \approx \frac{\nu_0 F(r)}{c^2} \frac{\mathrm{d}v(r)}{\mathrm{d}r}.$$

Therefore, the radiative force is proportional to the radiative flux and to the velocity gradient, but does not depend on the level populations or on the density.

Solving the hydrodynamical equations

Because the optically thick lines significantly contribute to the radiative force, one can solve stationary hydrodynamical equations

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} + f_{\mathrm{rad}} - \frac{\rho GM(1-\Gamma)}{r^2}$$

with optically thick line force only.

As always, the continuity equation gives the **mass-loss rate**

$$\dot{M} \equiv 4\pi r^2\rho v = \mathrm{const.}$$

Solving the equation of motion

Neglecting the gas pressure term, the equation of motion reads

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{f_{\mathrm{rad}}}{\rho} - \frac{GM(1-\Gamma)}{r^2}.$$

Inserting the optically thick line force, the equation of motion can be rewritten as

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right)\right]\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2},$$

where the monochromatic luminosity is $L_\nu = 4\pi r^2 F(r)$. This equation has a **critical point**, which at large distances from the star is given by

$$v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2} = 0.$$

This provides the mass-loss rate estimate

$$\dot{M} \equiv 4\pi r^2 \rho v(r) = \frac{\nu_0 L_\nu}{c^2} \approx \frac{L}{c^2}.$$

The wind mass-loss rate is proportional to the equivalent photon mass-loss rate.

Example: α Cam

The wind nass-loss rate estimate

$$\dot{M} \approx \frac{L}{c^2}$$

was derived for one optically thick line. Assuming now that the mass-loss is due to N_{thick} optically thick lines, the mass-loss rate prediction reads

$$\dot{M} \approx N_{\text{thick}} L/c^2.$$

The NLTE calculations for the stars α Cam ($T_{\text{eff}} = 30\,900\,\text{K}$, $R_* = 27.6\,\text{R}_\odot$, and $M = 43\,\text{M}_\odot$) give $N_{\text{thick}} \approx 1000$. This gives the mass-loss rate prediction $\dot{M} \approx 4 \times 10^{-5}\,\text{M}_\odot\,\text{yr}^{-1}$ not far from a more detailed calculation, which gives $1.5 \times 10^{-6}\,\text{M}_\odot\,\text{yr}^{-1}$ (Krtička & Kubát 2008).

CAK theory

We have seen that the optically thin line force is

$$f_{\mathrm{rad}} = \frac{1}{c}\,\chi_{\mathrm{L}}(r)F(r),$$

while the optically thick line force is

$$f_{\mathrm{rad}} = \frac{\nu_0 F(r)}{c^2}\frac{\mathrm{d}v}{\mathrm{d}r}.$$

This gives a possibility to introduce the Sobolev optical depth $\tau_{\mathrm{S}} = \frac{\chi_{\mathrm{L}}(r)c}{\nu_0 \frac{\mathrm{d}v}{\mathrm{d}r}}$, which enables us to rewrite the radiative force in a unified form

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)\left(\tau_{\mathrm{S}}^{-1}\right)^{\alpha},$$

where $\alpha = 0$ for optically thin line and $\alpha = 1$ for optically thick line.

CAK theory

We derived the radiative force accounting for optically thick lines only. However, in reality the wind is driven by a mixture of optically thick and thin lines. Therefore, Castor, Abbott & Klein (1975) introduced the radiative force in the form of

$$f_{\mathrm{rad}} = k\frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}} L}{4\pi r^2 c}\left(\frac{1}{\sigma_{\mathrm{Th}} n_{\mathrm{e}} v_{\mathrm{th}}}\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{\alpha},$$

where k, α are constants (force multipliers) derived from a given line list using NLTE calculations. Here σ_{Th} is the Thomson scattering cross-section, n_{e} is the electron number density, and v_{th} is hydrogen thermal speed (for $T = T_{\mathrm{eff}}$).

Gayley (1995) introduced a more physically motivated paremeter $\bar{Q}$, which scales the line force in terms of optically thin line force,

$$f_{\mathrm{rad}} = \frac{1}{1-\alpha}\frac{\kappa_{\mathrm{e}}\rho F\bar{Q}}{c}\left(\frac{\mathrm{d}v/\mathrm{d}r}{\rho c\bar{Q}\kappa_{\mathrm{e}}}\right)^{\alpha}.$$

Solving the momentum with CAK radiative force

Neglecting the gas pressure term, the momentum equation with CAK line force reads after some manipulation

$$r^2 v \frac{\mathrm{d}v}{\mathrm{d}r} = k \frac{\sigma_{\mathrm{Th}} L}{4\pi c} \frac{n_{\mathrm{e}}}{\rho} \left(\frac{\rho}{n_{\mathrm{e}}} \frac{4\pi r^2 v}{\sigma_{\mathrm{Th}} \dot{M} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha} - GM(1-\Gamma).$$

We will express the velocity in terms of the escape speed

$$w \equiv \frac{v^2}{v_{\mathrm{esc}}^2}, \text{ where } v_{\mathrm{esc}}^2 = \frac{2GM(1-\Gamma)}{R_*}.$$

Introducing a new variable

$$x \equiv 1 - \frac{R_*}{r}$$

the momentum equation can be rewritten as algebraic equation

$$1 + w' = C \left(w' \right)^{\alpha},$$

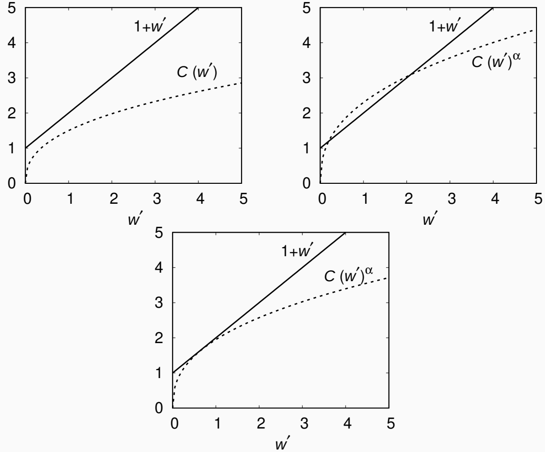
where $w' \equiv \frac{\mathrm{d}w}{\mathrm{d}x}$ and C is some uggly constant that depends on the mass-loss rate.

Solving for the mass-loss rate

The algebraic equation

$$1 + w' = C\,(w')^{\alpha}$$

has zero, one or two solutions depending on the value of C (or $\dot{M}$).



Solving for the mass-loss rate and terminal velocity

Because C is inversely proportional to $\dot{M}$, the maximum mass-loss rate appears for the minimum C, where both curves are tangent. This gives

$$w_c' = \frac{\alpha}{1-\alpha},$$

$$C_c = \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha}.$$

The second equation gives the wind mass-loss rate and the first one (after the integration) the velocity profile

$$w = \frac{\alpha}{1-\alpha} x \Rightarrow v = v_\infty \left(1 - \frac{R_*}{r}\right)^{1/2},$$

in the form of the β-velocity law with $\beta = 1/2$ and **wind terminal velocity**

$$v_\infty = v_{esc} \sqrt{\frac{\alpha}{1-\alpha}}$$

proportional to the escape speed. Because v_{esc} is of order of $100\,\mathrm{km\,s^{-1}}$, hot star winds are supersonic. For α Cam $v_{esc} = 620\,\mathrm{km\,s^{-1}}$ ($\alpha = 0.61$) prediction $v_\infty = 780\,\mathrm{km\,s^{-1}}$ is close to the empirical $1500 \pm 200\,\mathrm{km\,s^{-1}}$.

Wind instabilities I.: The ansatz

The Sobolev approximation gives reliable prediction of wind structure. Therefore, it is reasonable to assume that is also provides a sound basis for the study of instabilities. We shall start with time-dependent hydrodynamical equations

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = -a^2 \frac{\partial \rho}{\partial r} + f_{\mathrm{rad}} - \frac{\rho G M (1-\Gamma)}{r^2}$$

We will study the instabilities in the comoving fluid-frame and assume small perturbations $\delta\rho$ of stationary solution desctibed by ρ_0 and v_0:

$$\rho = \rho_0 + \delta\rho,$$

$$v = v_0 + \delta v, \qquad v_0 = 0.$$

Wind instabilities I.: Wave equation

Inserting the perturbation into hydrodynamical equations and neglecting second order term we derive equations for perturbations $\delta\rho$ and δv

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \frac{\partial \delta v}{\partial r} = 0,$$

$$\rho_0 \frac{\partial \delta v}{\partial t} = -a^2 \frac{\partial \delta\rho}{\partial r} + \delta f_{\mathrm{rad}},$$

where $\delta f_{\mathrm{rad}} = \rho_0 g'_{\mathrm{rad}} \, \partial \delta v / \partial r$ and $g'_{\mathrm{rad}} \equiv \partial g_{\mathrm{rad}} / \partial \left(\mathrm{d}v / \mathrm{d}r \right)$. Combining the radial derivative of the equation of continuity with the temporal derivative of the momentum equation we arrive into the wave equation in the form of

$$\frac{\partial^2 \delta v}{\partial t^2} = a^2 \frac{\partial^2 \delta v}{\partial r^2} + g'_{\mathrm{rad}} \frac{\partial^2 \delta v}{\partial t \partial r}.$$

Wind instabilities I.: Searching for the solution

We shall search for the solution of the wave equation

$$\frac{\partial^2 \delta v}{\partial t^2} = a^2 \frac{\partial^2 \delta v}{\partial r^2} + g'_{\rm rad} \frac{\partial^2 \delta v}{\partial t \partial r}$$

in the form of travelling waves, $\delta v \sim \exp\left[i\left(\omega t - kr\right)\right]$, which yields the dispersion relation

$$\omega^2 + g'_{\rm rad}\omega k - a^2 k^2 = 0,$$

which has a solution

$$\frac{\omega}{k} = -\frac{1}{2} g'_{\rm rad} \pm \left(\frac{1}{4} g'^2_{\rm rad} + a^2\right)^{1/2}.$$

With zero radiative force we obtain ordinary sound waves $\omega/k = \pm a$ once again as it should be. A general case gives a new type of waves – radiative-acoustic (Abbott) waves (Abbott 1980, Feldmeier et al. 2008) with downstream $(+)$ and upstream $(-)$ mode.

Wind instabilities I.: Understanding the critical point

A very special point is the critical point, at which the radial wind velocity equals to the speed of (upstream) Abbott waves; they move in opposite direction, therefore from $v_\mathrm{c} = -\omega/k$ the critical point conditions is

$$v_\mathrm{c} - \frac{1}{2} g'_\mathrm{rad} - \left(\frac{1}{4} g'^2_\mathrm{rad} + a^2 \right)^{1/2} = 0.$$

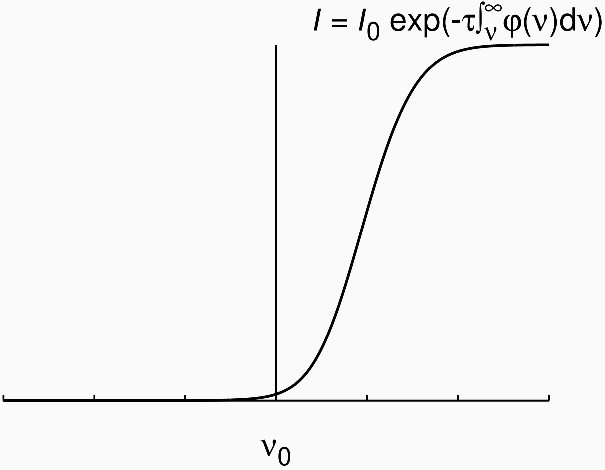
The siginificance of the critical point stems from the fact that no information can travel from the regions with $v > v_\mathrm{c}$ towards the stellar surface. In that sense the critical surface resembles the even horizon of a black hole (Feldmeier & Shloshman 2000). This also means that the mass-loss rate is determined there.

As a bottom line, we have not found any instability of hot-star winds. The hot star winds should be perfectly stable, as follows, for example, from hydrodynamical simulations of Votruba et al. (2007).

Wind instabilities II.

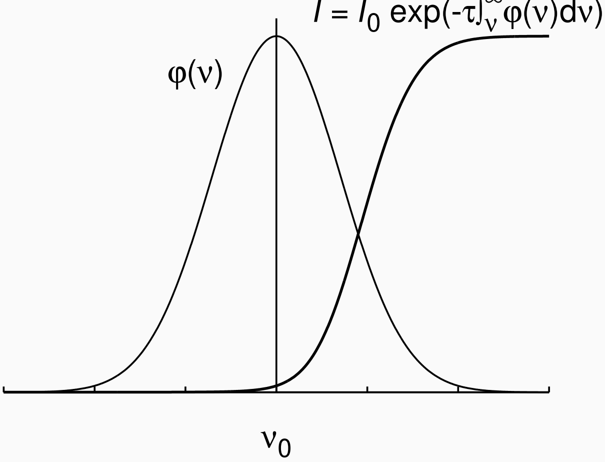
Our stability analysis showed that the wind should be stable. However, the question is what causes the occurrence of X-rays? Is there anything wrong with our stability analysis? The problem is that the Sobolev approximation is not valid for small-scale perturbations!

Wind instabilities II. – The origin of instability



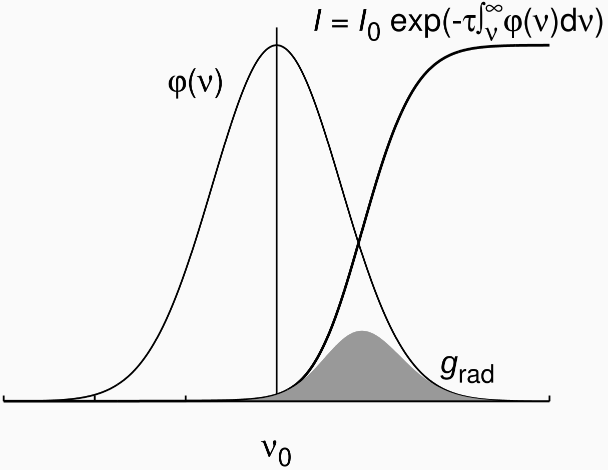
The plot shows the radiative transfer in the comoving frame. As the unabsorbed photon comes from blue, it hits the rezonance zone where it can be absorbed. This results in the decrease of the light intensity.

Wind instabilities II. – The origin of instability



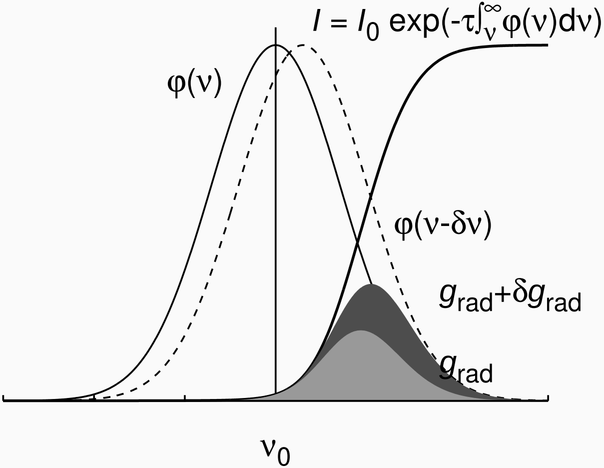
While the intensity in a given line decreases with decreasing frequency, the absorption line profile $\varphi(\nu)$ is symmetric accross the laboratory frequency of a given line ν_0 in the comoving frame.

Wind instabilities II. – The origin of instability



Therefore, the line force, which is given by a product of opacity and flux comes mostly from the blue part of a given line.

Wind instabilities II. – The origin of instability



The line force can significantly change after introducing a small change of the velocity. This is the essence of the line-driven wind instability (Lucy & Solomon 1970, Owocki et al. 1984, Feldmeier et al. 1997).

Wind instabilities II. – Line force perturbation

To describe the instability, we shall evaluate the perturbation of the radiative acceleration

$$g_{\mathrm{rad}} = \frac{2\pi}{c\rho} \int_0^\infty \mathrm{d}\nu\, \chi_{\mathrm{L}}(r)\varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu\, \mu I(r,\mu,\nu)$$

assuming optically thin perturbation. This means, we shall perturb just the line profile and not the intensity,

$$\delta g_{\mathrm{rad}} = \frac{2\pi}{c\rho} \int_0^\infty \mathrm{d}\nu\, \chi_{\mathrm{L}}(r)\delta\varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu\, \mu I(r,\mu,\nu).$$

The perturbed line profile due to velocity perturbation is

$$\delta\varphi_{ij}(\nu) = \frac{\partial\varphi_{ij}(\nu)}{\partial\nu_0}\delta\nu_0 = \frac{\partial\varphi_{ij}(\nu)}{\partial\nu_0}\nu_0\frac{\delta v}{c}.$$

In short, the radiative force perturbation is

$$\delta g_{\mathrm{rad}} = \Omega\delta v,$$

where the positive quantity Ω need not to be written down explicitly.

Wind instabilities II. – Wave equation

The equations for perturbations $\delta\rho$ and δv are the same as before,

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \frac{\partial \delta v}{\partial r} = 0,$$

$$\rho_0 \frac{\partial \delta v}{\partial t} = -a^2 \frac{\partial \delta\rho}{\partial r} + \delta f_{\mathrm{rad}}.$$

Combining these two gives the wave equation

$$\frac{\partial^2 \delta v}{\partial t^2} = a^2 \frac{\partial^2 \delta v}{\partial r^2} + \Omega \frac{\partial \delta v}{\partial t}.$$

As usual, we shall seek the solution in the form of travelling waves $\delta v \sim \exp\left[i\left(\omega t - kr\right)\right]$, which gives the dispersion relation

$$\omega^2 + i\Omega\omega - a^2 k^2 = 0.$$

This has a solution, which is

$$\omega = -\frac{1}{2} i\Omega \pm \left(-\frac{1}{4}\Omega^2 + a^2 k^2\right)^{1/2}.$$

Wind instabilities II. – The dispersion relation

The dispersion relation

$$\omega = -\frac{1}{2} i\Omega \pm \left(-\frac{1}{4}\Omega^2 + a^2 k^2 \right)^{1/2}$$

takes in the case of negligible gas pressure $\Omega^2 \gg a^2 k^2$ the simple form of

$$\omega = -i\Omega.$$

Therefore, the wave amplitude varies as $(\Omega > 0)$

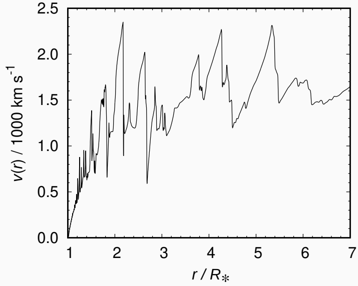
$$\delta v \sim \exp\left(i\omega t\right) = \exp\left(\Omega t\right).$$

This leads to a strong instability of the radiative driving (Lucy & Solomon 1970, MacGregor et al. 1979, Carlberg 1980, Owocki et al. 1984, Feldmeier et al. 1997).

Wind instabilities II. – Simulations

We have seen that line driven wind is stable for large scale perturbations, while it becomes unstable for small-scale perturbations. This enables us to derive basic wind properties from global models, while the small-scale structure may affect some observables. This is referred to as clumping.

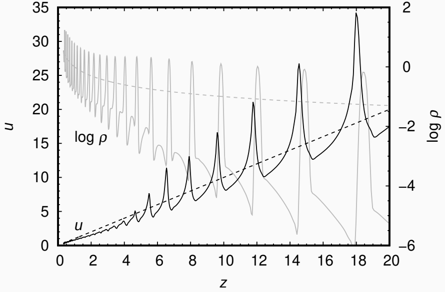
However, our instability analysis is linear only and hydrodynamical simulations are necessary to describe the instability in detail (Owocki et al. 1988, Feldmeier et al. 1997, Runacres & Owocki 2002).



Line-driven wind instability: the initiation

Here we see the simulations of Feldmeier & Thomas (2017) initiated by sinusoidal boundary perturbation. The initial perturbation steepens into shocks. However, most wind material appears at velocity that roughly corresponds to CAK solution (dashed lines). The low-density regions between individual clumps are formed due to strong radiative force acting on rarefied gas. Most of the wind material has negative velocity gradient.

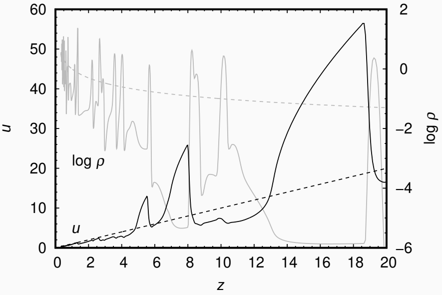
Note that while there are not strong velocity perturbations close to the photosphere, there are huge variations of density in this region. This is because even very small perturbation of velocity is able to evacuate the material between two forming clumps.



Line-driven wind instability: not a typical turbulence

Here we see the simulations of Feldmeier & Thomas (2017) close to the photosphere initiated by stochastic boundary perturbations. Again, typical structure of line-driven wind instability forms with thin overdensities close to the CAK solution but with negative velocity gradients and rarefied region between them.

Interestingly, although the boundary perturbation is stochastic, the structure of line-driven wind instability is rather regular. Therefore, the instability does not resemble the turbulence.

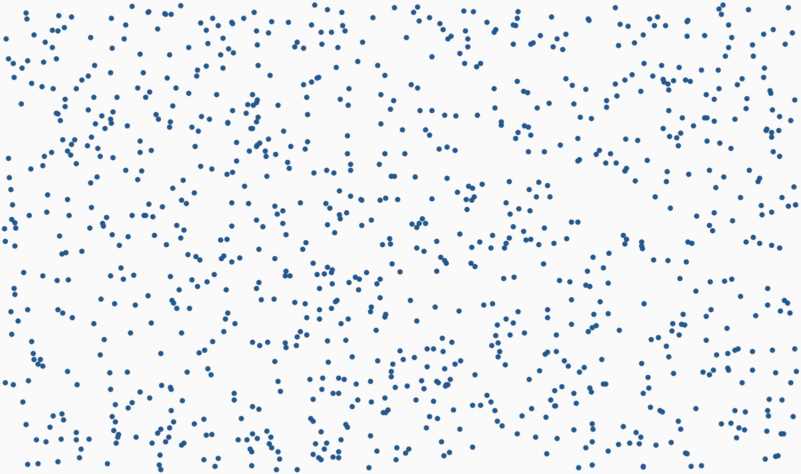


Hot star winds: micro-view

We have seen that the stellar wind of hot stars is accelerated due to the scattering of radiation in lines and on free electrons. However, mostly heavy elements such as carbon, nitrogen, silicon, and iron contribute to the radiative force. How does it work on a micro-level?

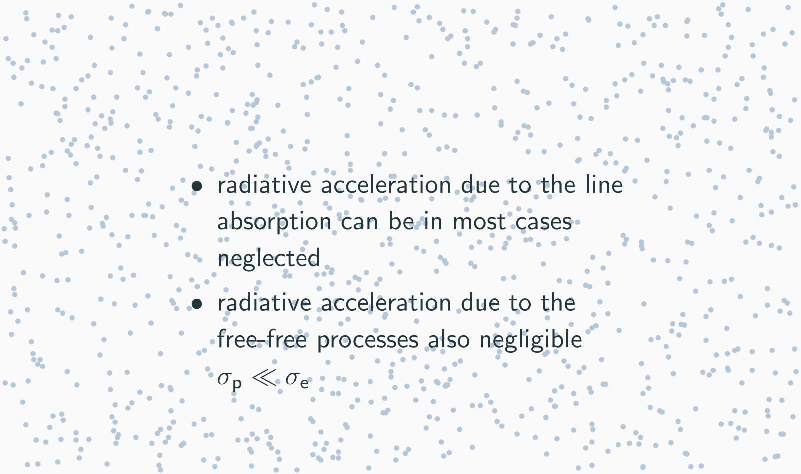
Hot star winds: micro-view

Typical volume with: 1000 H ions



Hot star winds: micro-view

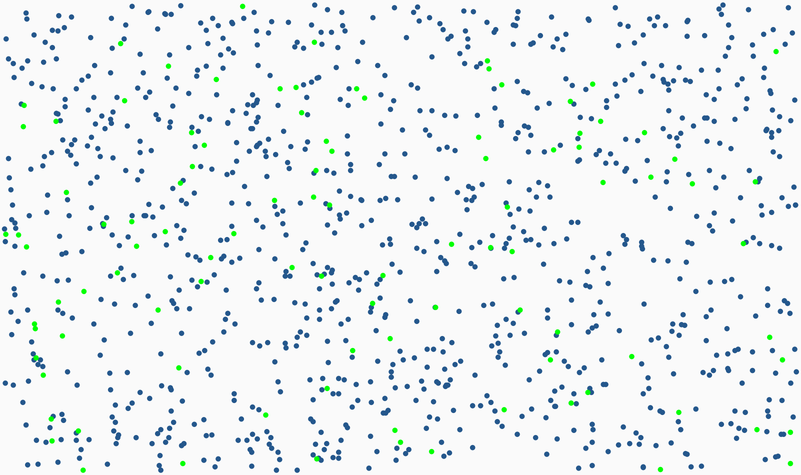
Typical volume with: 1000 H ions



- radiative acceleration due to the line absorption can be in most cases neglected
- radiative acceleration due to the free-free processes also negligible $\sigma_{\mathrm{p}} \ll \sigma_{\mathrm{e}}$

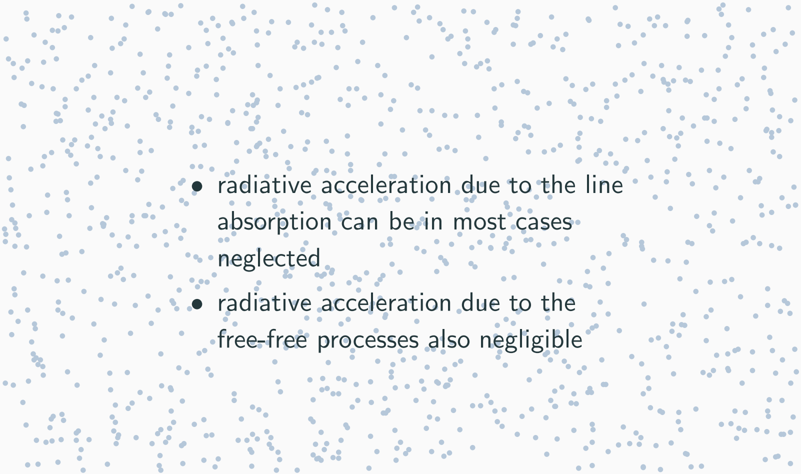
Hot star winds: micro-view

Typical volume with: 1000 H ions + 100 He ions



Hot star winds: micro-view

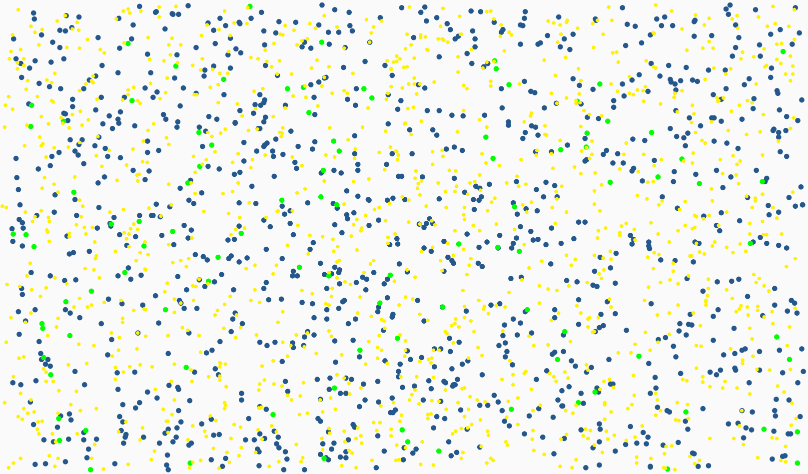
Typical volume with: 1000 H ions + 100 He ions



- radiative acceleration due to the line absorption can be in most cases neglected
- radiative acceleration due to the free-free processes also negligible

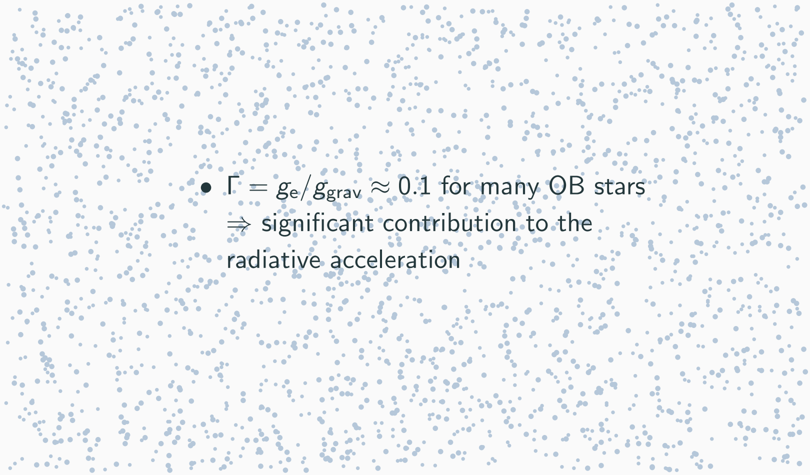
Hot star winds: micro-view

Typical volume with: 1000 H ions + 100 He ions + 1200 e^{-}



Hot star winds: micro-view

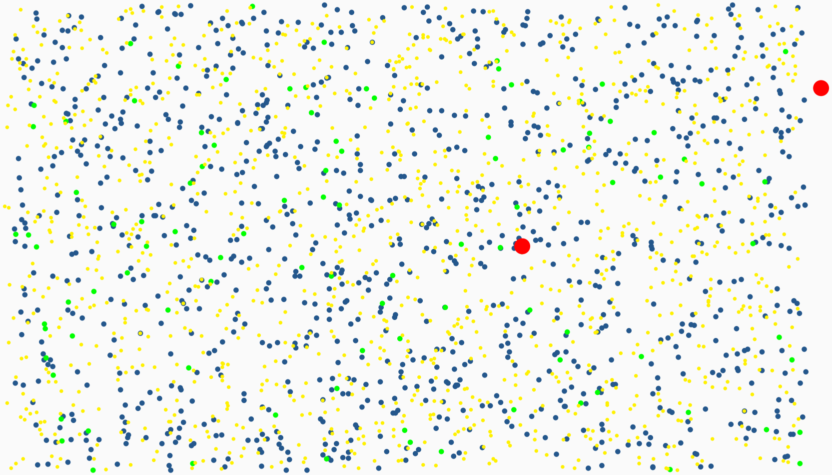
Typical volume with: 1000 H ions + 100 He ions + 1200 e$^-$



- $\Gamma = g_{\mathrm{e}}/g_{\mathrm{grav}} \approx 0.1$ for many OB stars
  $\Rightarrow$ significant contribution to the radiative acceleration

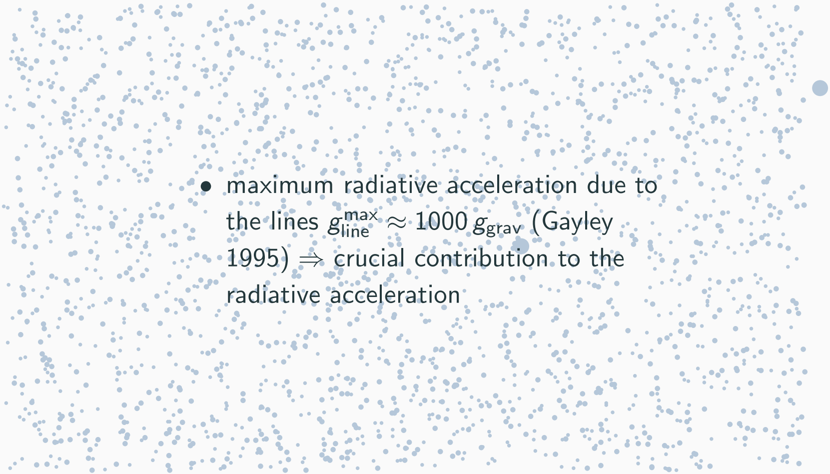
Hot star winds: micro-view

Typical volume with: 1000 H ions + 100 He ions + 1200 e$^-$ + 2 metals



Hot star winds: micro-view

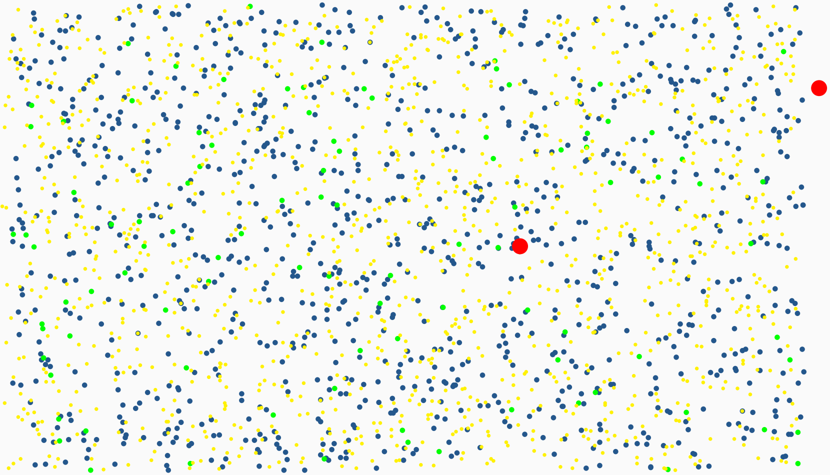
Typical volume with: 1000 H ions + 100 He ions + 1200 e$^-$ + 2 metals



- maximum radiative acceleration due to the lines $g_{\text{line}}^{\text{max}} \approx 1000\, g_{\text{grav}}$ (Gayley 1995) $\Rightarrow$ crucial contribution to the radiative acceleration

Hot star winds: micro-view

Typical volume with: 1000 H ions + 100 He ions + 1200 e^- + 2 metals



How can this work?

In order to accelerate the stellar wind, two efficient processes are necessary.

A process which transfers momentum from radiative field to heavier ions. This has to be efficitent in such a way that only a very small part of the radiative energy is used to heat the wind. Therefore, the frequency of absorbed and emitted photon should be precisely the same. Such absorption is called scattering in astrophysics. We have seen that both light scattering on free electrons and in spectral lines are exactly such processes.

However, since most of radiative momentum is received by heavy elements, a second process is necessary, which transfers momentum from heavier ions to the bulk flow (H and He).

How to transfer momentum?

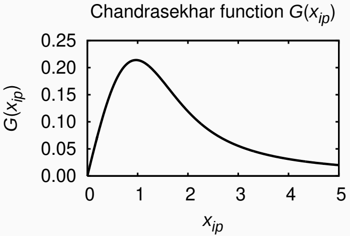
Stellar wind of hot stars is ionised. Therefore, Coulomb collisions are efficient to transfer momentum from heavier elements to the passive component. In such case the frictional force on passive component (p) due to heavy ions (i) is

$$f_{\mathrm{pi}} = \rho_{\mathrm{p}} g_{\mathrm{pi}} = n_{\mathrm{p}} n_{\mathrm{i}} \frac{4\pi q_{\mathrm{p}}^2 q_{\mathrm{i}}^2}{k T_{\mathrm{ip}}} \ln \Lambda \, G(x_{\mathrm{ip}}) \frac{v_{\mathrm{i}} - v_{\mathrm{p}}}{|v_{\mathrm{i}} - v_{\mathrm{p}}|},$$

where n_{p}, n_{i} are number densities of components, v_{i}, v_{p} are their radial velocities, and q_{p}, q_{i} their charges. The frictional force depends on the velocity difference between the wind componets via so-called Chandrasekhar function.

$$x_{\mathrm{ip}} = \frac{|v_{\mathrm{i}} - v_{\mathrm{p}}|}{\alpha_{\mathrm{ip}}}$$

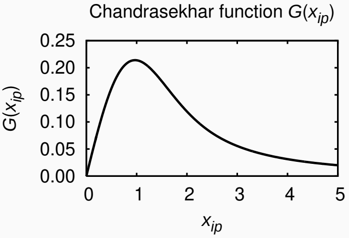
$$\alpha_{\mathrm{ip}}^2 = \frac{2k\,(m_{\mathrm{i}} T_{\mathrm{p}} + m_{\mathrm{p}} T_{\mathrm{i}})}{m_{\mathrm{i}} m_{\mathrm{p}}}$$



Momentum transfer efficiency: efficient transfer

The frictional force depends on the product of densities of individual components. Consequently, to transfer a given amount of momentum at high densities, just a small velocity difference is needed. In such a case $x_{\mathrm{ip}} \ll 1$, the transfer of momentum from heavier ions to hydrogen and helium is efficient and one-component models are sufficient in such case. Moreove, such winds are stable, a decrease of velocity difference leads to a stronger frictional force, which in turn leads to stronger coupling and decrease of the velocity difference.

Such winds are typically found at high densities or at high mass-loss rates. Therefore, wind of luminous OB stars and WR stars can treated as one-component flow neglecting all multicomponent effects.

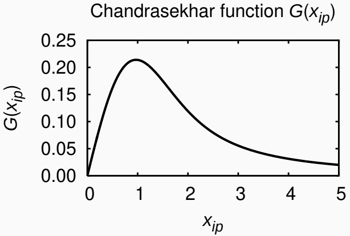


Momentum transfer efficiency: multicomponent effects

For lower wind densities (or lower wind mass-loss rates) the momentum transfer between heavy ions and hydrogen and helium becomes inefficient. For $x_{\mathrm{ip}} \lesssim 1$ part of transfered energy goes to heating, which leads to frictional heating of the wind.

For even lower densities $x_{\mathrm{ip}} > 1$ and a new type of instability appears. The Chandrasekhar function is a decreasing function of velocity difference. This means that small positive perturbation of velocity difference leads to decrease of the frictional force, which in turn allows for even larger velocity separation. This leads to runaway of heavy ions or to decoupling instability.

To model winds at low densities, multicomponent approach is needed. Such winds are typically found in main-sequence B stars or in stars at low metallicity.



Line-driven winds: importance

Despite the fact that massive stars are short lived (10^6 yr), with large mass-loss rate (of the order of $10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$) the massive stars may lose a significant fraction of their mass on their way from main sequence to the neutron star or a black hole. Amount of the mass-loss is one of the factors that determine the nature of the final evolutionary stage.

Hot star winds are important also for the dynamics of the interstellar medium. Moreover, cosmic-ray particles are generated at the boundary between hot star wind bubble and the interstellar medium.

Line-driven winds appear in O and early B main-sequence stars, in OBA supergiants, in hot subdwarfs and central stars of planetary nebulae and in Wolf-Rayet stars. Accretion disks around supermassive black holes may also be sources of line-driven wind.

Suggested reading

J. Castor: Radiation hydrodynamics

H. J. G. L. M. Lamers, & J. P. Cassinelli: Introduction to Stellar Winds

D. Mihalas & B. W. Mihalas: Foundations of Radiation Hydrodynamics

J. Puls, J. S. Vink, F. Najarro: Mass loss from hot massive stars

S. Owocki: Stellar wind mechanisms and instabilities

F. H. Shu: The physics of astrophysics: II. Hydrodynamics