

# F7030 Rentgenový rozptyl na tenkých vrstvách

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Prezentace k přednášce  
Numerické simulace  
Příklady experimentů  
Vybrané vztahy

# Syllabus

1. Experimentální technika: zdroje, vznik rtg záření, goniometry, optické prvky (monochromátory, kolimátory, zrcadla, fokusační optika), detektory. Základní experimenty: polykrystalové a monokrystalové metody, mapování reciprokého prostoru
2. Kinematická teorie rozptylu: úvod do teorie rozptylu, rozptyl na elektronu, izolovaném atomu, krystal, strukturní a geometrický faktor, omezená velikost krystalu
3. Difrakce na polykrystalech I: strukturní faktor, velikost krystalitu (Scherrerova formule), vliv deformace na polohy a šířky difrakčních maxim, zbytková napětí, kvantifikace fázového složení (vnitřní normál)
4. Polykrystaly II: Full profile fitting; Texture, ODF (orientation distribution function); Debyeův vztah, PDF (pair distribution function).
5. SAXS: teoretický popis, řídké roztoky – Guinierův a Porodův vztah, uspořádané částice – long range a short-range order
6. Dokonalé, téměř dokonalé krystaly, epitaxní vrstvy: Kinematická teorie na monokrystalu a epitaxní vrstvě – polohy difrakcí, truncation rod, deformace v epitaxní vrstvě, relaxace. Mozaikový krystal
7. Dynamická teorie rtg reflexe: Jednovlnná aproximace – hloubka vniku, reflexe na hladkém rozhraní, multivrstvy (formalismus přenosové matice), TRXRF
8. Dynamická teorie rtg difrakce: Dvojevlnná aproximace: případ Bragg a Laue, Borrmannův jev, stojatá vlna, GID, epitaxní vrstvy
9. Semikinematická teorie I: DWBA, Rozptyl na drsných rozhraních – popis drsného rozhraní, příklady: fraktálové rozhraní, dvouúrovňové, vicinální, spekulární odraz a nespekulární rozptyl, drsné multivrstvy
10. Semikinematická teorie II: GISAXS na částicích na povrchu a uvnitř vzorku, Difuzní rozptyl na defektech v krystalu v okolí difrakce
11. Experimentální rozlišení Experimentální rozlišení v recipročném prostoru: analyzer streak, detector streak, monochromator streak, DuMondovy grafy, disperzní a nedisperzní uspořádání, koherenční šířka a délka
12. Další rentgenové metody: Fluorescenční spektroskopie, absorpční spektroskopie – XAFS, XMCD.

# Semikinematická teorie

$$\hat{V}(\mathbf{r}) = \hat{V}_A(\mathbf{r}) + \hat{V}_B(\mathbf{r})$$

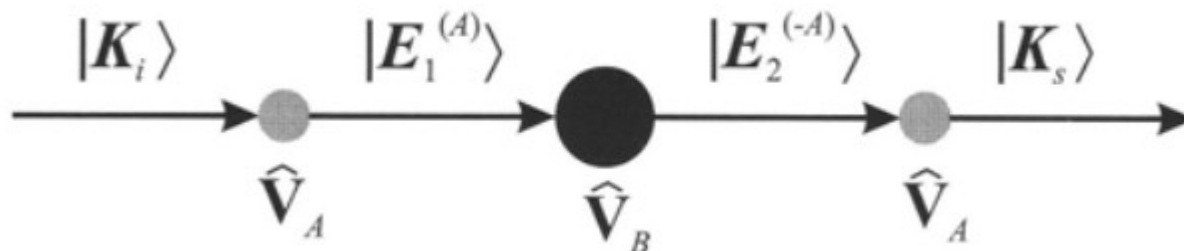
Dokážeme najít přesné řešení pro

$$(\Delta + K^2)\mathbf{E}^{(A)}(\mathbf{r}) = \hat{V}_A(\mathbf{r})\mathbf{E}^{(A)}(\mathbf{r})$$

$$|K_i\rangle \quad |E_1^{(A)}\rangle \quad \text{-----} \quad |E_2^{(-A)}\rangle \quad |K_s\rangle$$

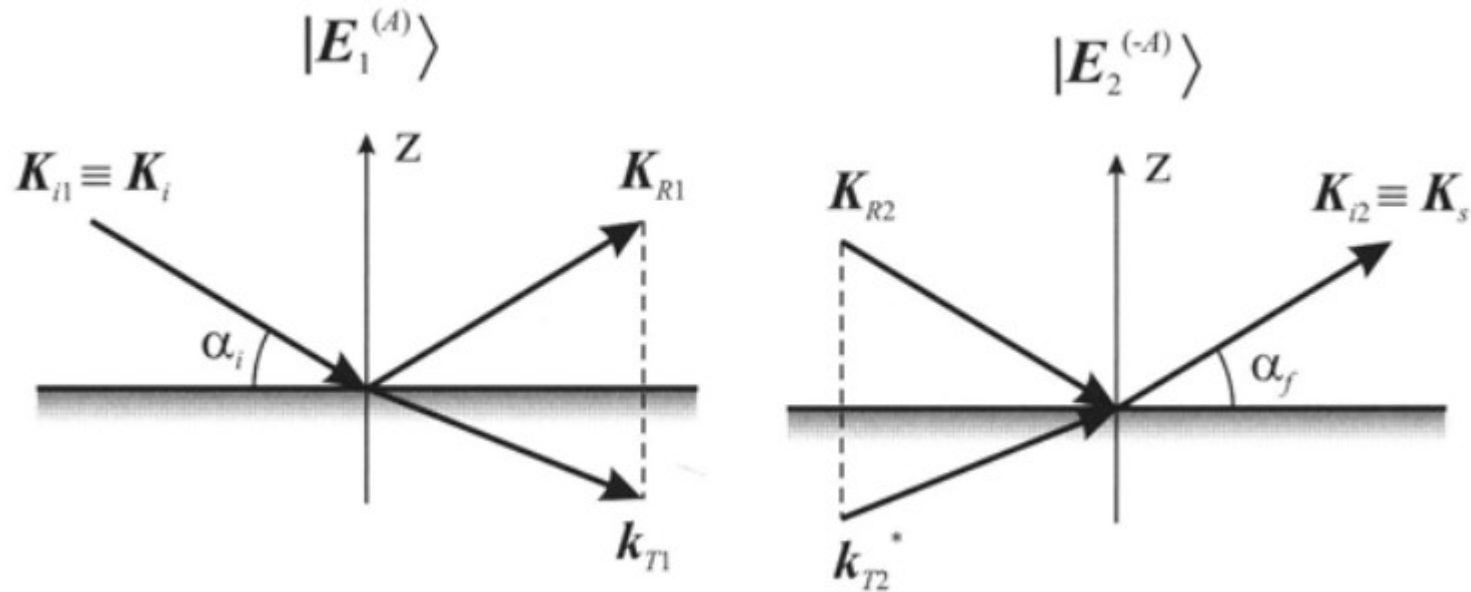
$$\left(\frac{d\sigma}{d\Omega}\right)_A = \frac{1}{16\pi^2} \left| \langle K_s | \hat{T}_A | K_i \rangle \right|^2 \equiv \frac{1}{16\pi^2} \left| \langle E_2^{(-A)} | \hat{V}_A | K_i \rangle \right|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{DWBA} = \frac{1}{16\pi^2} \left| \langle E_2^{(-A)} | \hat{V}_A | K_i \rangle + \langle E_2^{(-A)} | \hat{V}_B | E_1^{(A)} \rangle \right|^2$$



# Semikinematická teorie

Příklad: polonekonečný substrát



$$E_1^{(A)}(\mathbf{r}) \equiv \langle \mathbf{r} | E_1^{(A)} \rangle = \begin{cases} e^{i\mathbf{K}_{i1} \cdot \mathbf{r}} + r_1 e^{i\mathbf{K}_{R1} \cdot \mathbf{r}} & \text{for } z > 0 \\ t_1 e^{i\mathbf{k}_{T1} \cdot \mathbf{r}} & \text{for } z < 0 \end{cases}$$

$$E_2^{(-A)}(\mathbf{r}) \equiv \langle \mathbf{r} | E_2^{(-A)} \rangle = \begin{cases} e^{i\mathbf{K}_{i2} \cdot \mathbf{r}} + r_2^* e^{i\mathbf{K}_{R2} \cdot \mathbf{r}} & \text{for } z > 0 \\ t_2^* e^{i\mathbf{k}_{T2}^* \cdot \mathbf{r}} & \text{for } z < 0 \end{cases}$$

# Semikinematická teorie

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{coh}} = \frac{1}{16\pi^2} \left| \langle \mathbf{E}_2^{(-A)} | \hat{\mathbf{V}}_A | \mathbf{K}_i \rangle + \langle \mathbf{E}_2^{(-A)} | \langle \hat{\mathbf{V}}_B \rangle | \mathbf{E}_1^{(A)} \rangle \right|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{incoh}} = \frac{1}{16\pi^2} \text{Cov} \left( \langle \mathbf{E}_2^{(-A)} | \hat{\mathbf{V}}_B | \mathbf{E}_1^{(A)} \rangle, \langle \mathbf{E}_2^{(-A)} | \hat{\mathbf{V}}_B | \mathbf{E}_1^{(A)} \rangle \right)$$

# Semikinematická teorie

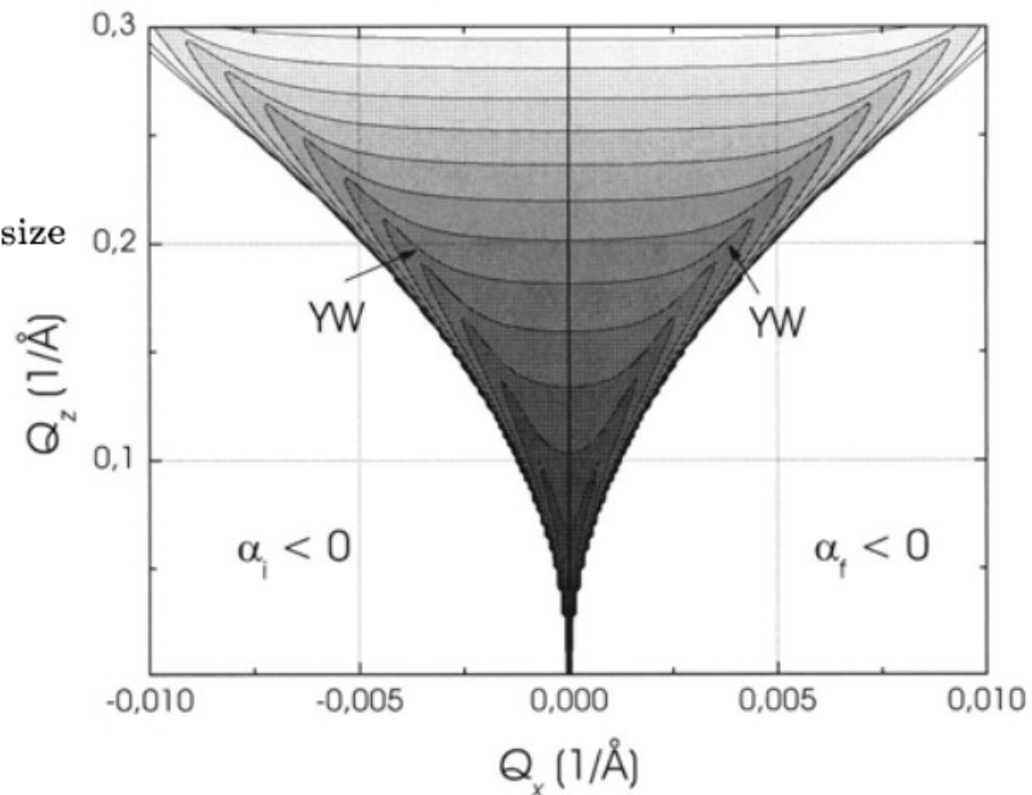
Otvory v krystalu

$$\hat{\mathbf{V}}_B(\mathbf{r}) = K^2 \chi_0 \sum_n \Omega_n(\mathbf{r} - \mathbf{R}_n)$$

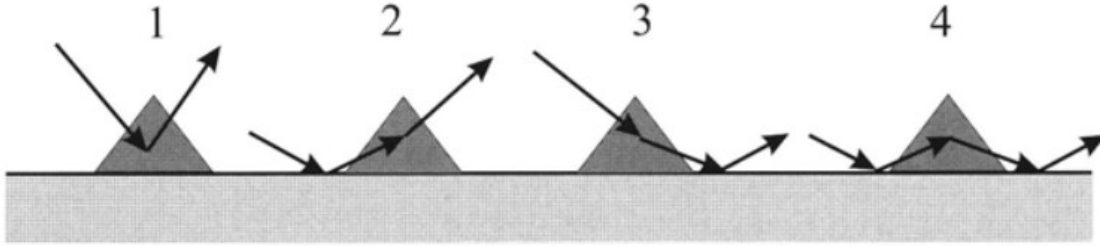
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{incoh}} = \frac{N}{16\pi^2} |K^2 \chi_0 t_1 t_2|^2 \left\langle |\Omega^{\text{FT}}(\mathbf{Q}_T)|^2 \right\rangle_{\text{size}}$$

$$\mathbf{Q}_T = \mathbf{k}_{T2} - \mathbf{k}_{T1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{incoh,kin}} = \frac{N}{16\pi^2} |K^2 \chi_0|^2 \left\langle |\Omega^{\text{FT}}(\mathbf{Q})|^2 \right\rangle_{\text{size}}$$



# Semikinematická teorie

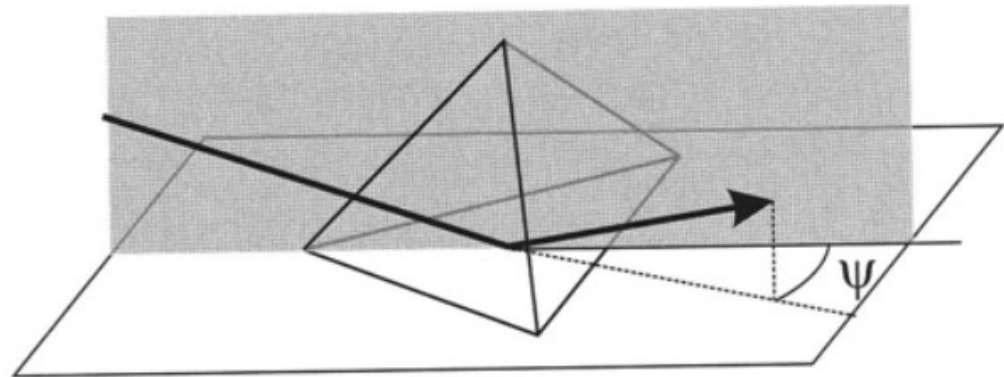
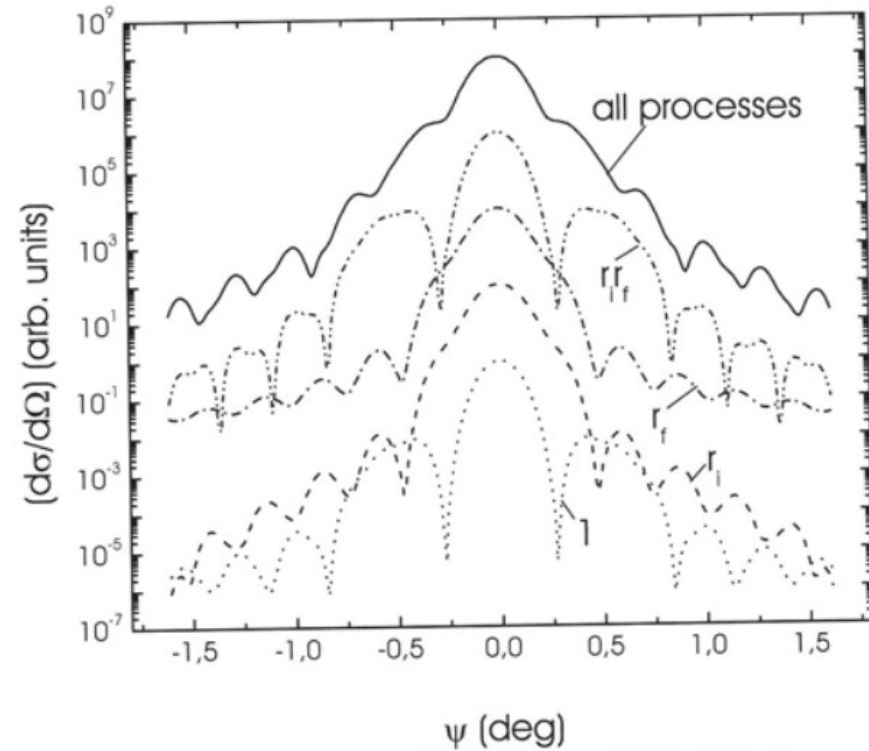


$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{incoh}} = \frac{N}{16\pi^2} |K^2 \chi_0|^2 \left| \sum_{p=1}^4 A_p \Omega^{\text{FT}}(\mathbf{Q}_p) \right|^2$$

$$A_1 = 1, A_2 = r_1, A_3 = r_2, A_4 = r_1 r_2$$

$$Q_1 = K_{i2} - K_{i1} \equiv Q, \quad Q_2 = K_{i2} - K_{R1}$$

$$Q_3 = K_{R2} - K_{i1}, \quad Q_4 = K_{R2} - K_{R1}$$



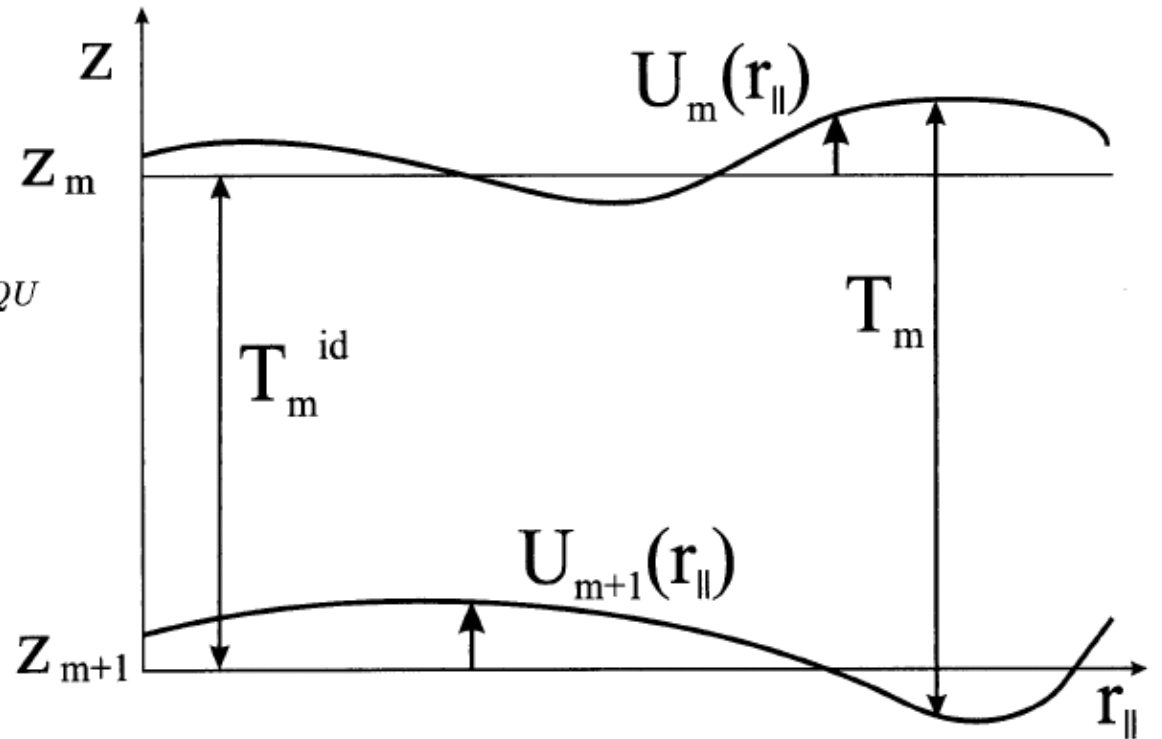
# Drsné rozhraní

$$w_m(U) = \frac{1}{S} \int_S d^2 \mathbf{r}_{\parallel} \delta(U_m(\mathbf{r}_{\parallel}) - U)$$

$$\chi_{U_m}(Q) = \langle e^{-iQU_m} \rangle = \int_{-\infty}^{\infty} dU w_m(U) e^{-iQU}$$

$$\langle U_m \rangle = 0$$

$$\sigma_m^2 \equiv \langle U_m^2(\mathbf{r}_{\parallel}) \rangle = \int_{-\infty}^{\infty} dU w_m(U) U^2$$



$$\Phi_m(z) = \int_{-\infty}^{z-z_m} dU w_m(U)$$



# Drsné rozhraní

$$w(U, U') = w(U(\mathbf{r}_{\parallel}), U(\mathbf{r}'_{\parallel}))$$

$$\chi_{UU'}(Q, Q') = \left\langle e^{-i(QU - Q'U')} \right\rangle$$

$$C(\mathbf{x}, \mathbf{x}') = \langle U(\mathbf{x})U(\mathbf{x}') \rangle \quad C(\mathbf{x}, \mathbf{x}') = C(\mathbf{x} - \mathbf{x}')$$

$$C(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) = \left\langle U(\mathbf{r}_{\parallel})U(\mathbf{r}'_{\parallel}) \right\rangle$$

$$g(\mathbf{x}, \mathbf{x}') = \langle [U(\mathbf{x}) - U(\mathbf{x}')]^2 \rangle$$

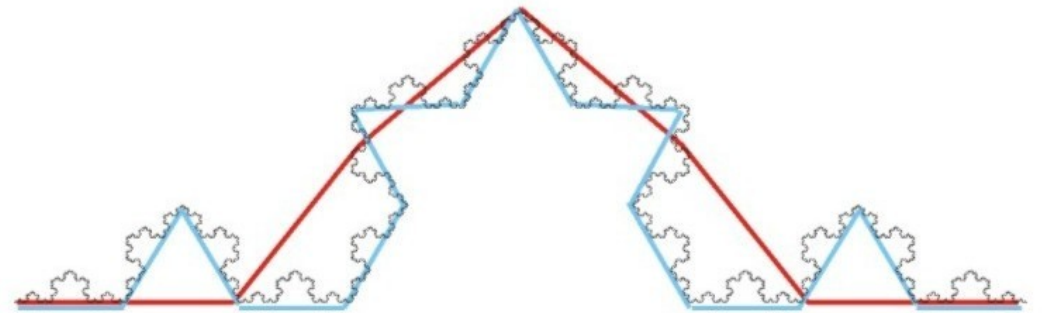
$$g(\mathbf{x} - \mathbf{x}') = 2(\sigma^2 - C(\mathbf{x} - \mathbf{x}'))$$

# Drsné rozhraní

Fraktálový model

$$L_x = N(\varepsilon) \cdot \varepsilon^D,$$

	$\varepsilon$	$N(\varepsilon)$	$L(\varepsilon)=\varepsilon \cdot N(\varepsilon)$
	10	6	60
	5	16	80



$D=1.2619$

$$g(\mathbf{x} - \mathbf{x}') \rightarrow |\mathbf{x} - \mathbf{x}'|^{2h} \text{ for } |\mathbf{x} - \mathbf{x}'| \rightarrow 0.$$

$$D_f = 3 - h$$

$$g(\mathbf{x} - \mathbf{x}') \rightarrow \sigma^2 \text{ for } |\mathbf{x} - \mathbf{x}'| \rightarrow \infty$$

$$C(\mathbf{x} - \mathbf{x}') = \sigma^2 \exp \left[ - \left( \frac{|\mathbf{x} - \mathbf{x}'|}{\Lambda} \right)^{2h} \right]$$

Náhodné kráčení (Brownův pohyb)

$h=0.5$

# Nekoherentní rozptyl na drsném rozhraní

$$\mathcal{V}_A = \langle E_s^{(-A)} | \hat{\mathbf{V}}_A | E_i^{(0)} \rangle$$

$$\mathcal{V}_B = \langle E_s^{(-A)} | \hat{\mathbf{V}}_B | E_i^{(B)} \rangle \quad \left( \frac{d\sigma}{d\Omega} \right)_{\text{coh}} = \frac{1}{16\pi^2} |\mathcal{V}_A + \langle \mathcal{V}_B \rangle|^2, \quad \left( \frac{d\sigma}{d\Omega} \right)_{\text{diff}} = \frac{1}{16\pi^2} \text{Cov}(\mathcal{V}_B, \mathcal{V}_B)$$

$$\mathcal{V}_B = \frac{K^2}{iq_z} t_i t_s \int d^2 \mathbf{x} e^{-i\mathbf{q}_{\parallel} \cdot \mathbf{x}} \sum_{m=1}^N \Delta \chi_{m} e^{-q_z (z_m + U_m(\mathbf{x}))}$$

$$J(\mathbf{Q}) = \frac{K^2}{16\pi^2} I_i \frac{|\chi_0|^2}{Q_z^2} \int d^2(\mathbf{x} - \mathbf{x}') \chi_{UU}(Q_z; \mathbf{x} - \mathbf{x}') e^{-i\mathbf{Q}_{\parallel} \cdot (\mathbf{x} - \mathbf{x}')}, \quad \mathbf{Q} = \mathbf{K} - \mathbf{K}_i$$

$$\chi_{UU}(Q_z; \mathbf{x} - \mathbf{x}') = \langle e^{-iQ_z(U(\mathbf{x}) - U(\mathbf{x}'))} \rangle$$

Nekoherentní část

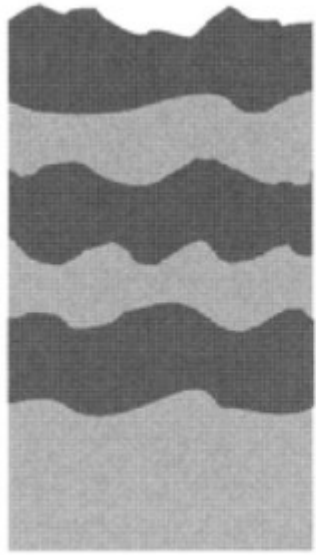
$$Q(Q_z; \mathbf{x} - \mathbf{x}') = \langle e^{-iQ_z(U(\mathbf{x}) - U(\mathbf{x}'))} \rangle - \langle e^{-iQ_z U(\mathbf{x})} \rangle \langle e^{iQ_z U(\mathbf{x}')} \rangle^*$$

$$\chi_{UU}(Q_z; \mathbf{x} - \mathbf{x}') = \exp \left[ -\frac{1}{2} Q_z^2 g(\mathbf{x} - \mathbf{x}') \right] = e^{-\sigma^2 Q_z^2} e^{Q_z^2 C(\mathbf{x} - \mathbf{x}')}$$

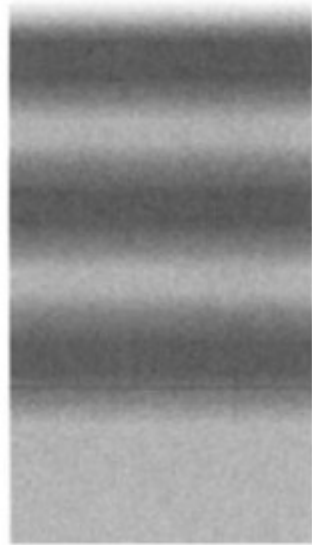
$$Q(Q_z; \mathbf{x} - \mathbf{x}') = e^{-\sigma^2 Q_z^2} \left[ e^{Q_z^2 C(\mathbf{x} - \mathbf{x}')} - 1 \right]$$

$$C(\mathbf{x} - \mathbf{x}') = \sigma^2 \exp \left[ -\left( \frac{|\mathbf{x} - \mathbf{x}'|}{\Lambda} \right)^{2h} \right]$$

# Koherentní rozptyl na drsném rozhraní



(a)



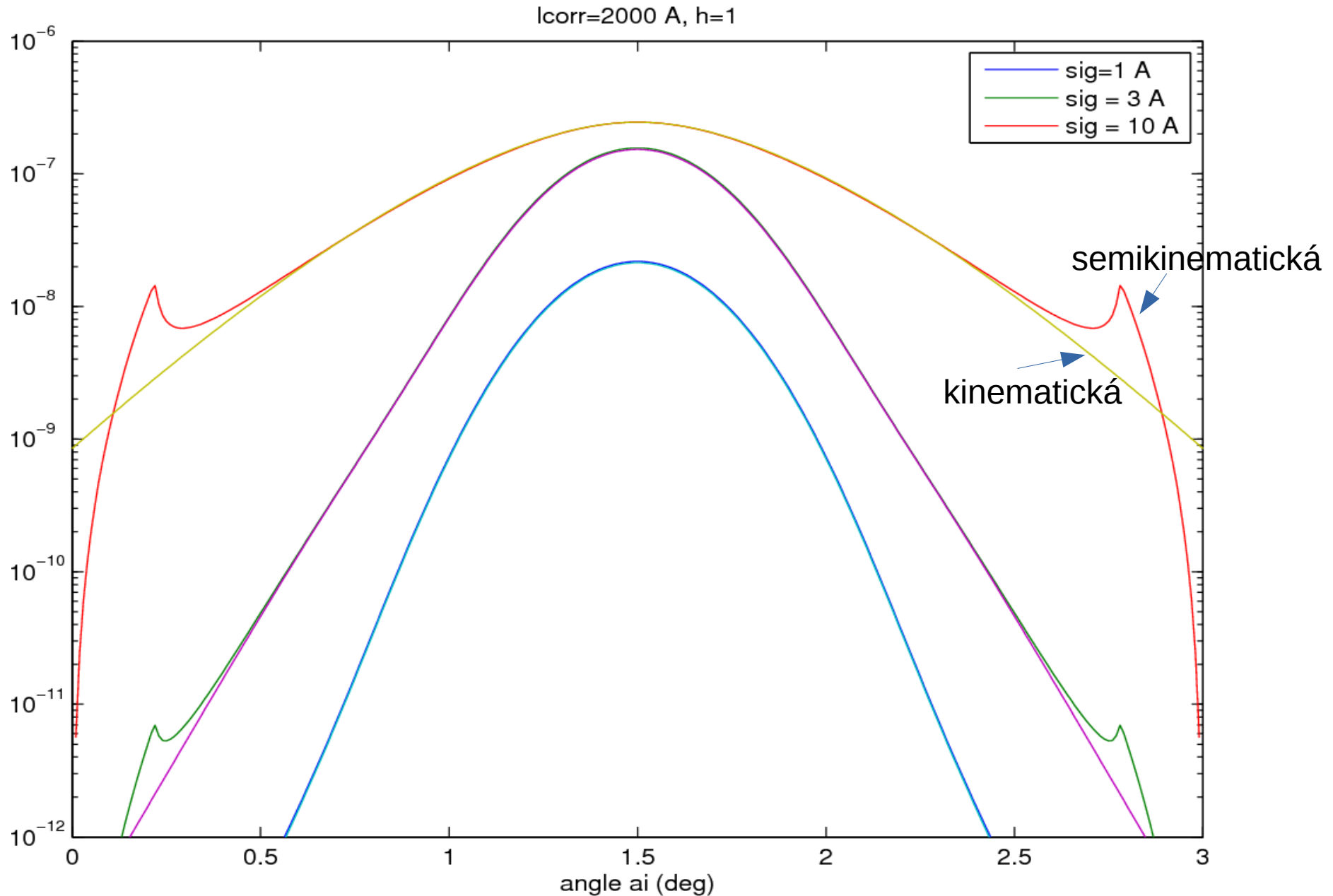
(b)

$$(\Delta + K^2)\langle E(\mathbf{r}) \rangle = \langle \hat{\mathbf{V}}(\mathbf{r})E(\mathbf{r}) \rangle \equiv \hat{\mathbf{V}}_{\text{eff}}(\mathbf{r})\langle E(\mathbf{r}) \rangle$$

$$t'_j = \frac{t_j}{\chi_{U_j}(k_z^{(j-1)} - k_z^{(j)})}, \quad r'_j = r_j \frac{\chi_{U_j}(k_z^{(j-1)} + k_z^{(j)})}{\chi_{U_j}(k_z^{(j-1)} - k_z^{(j)})}$$

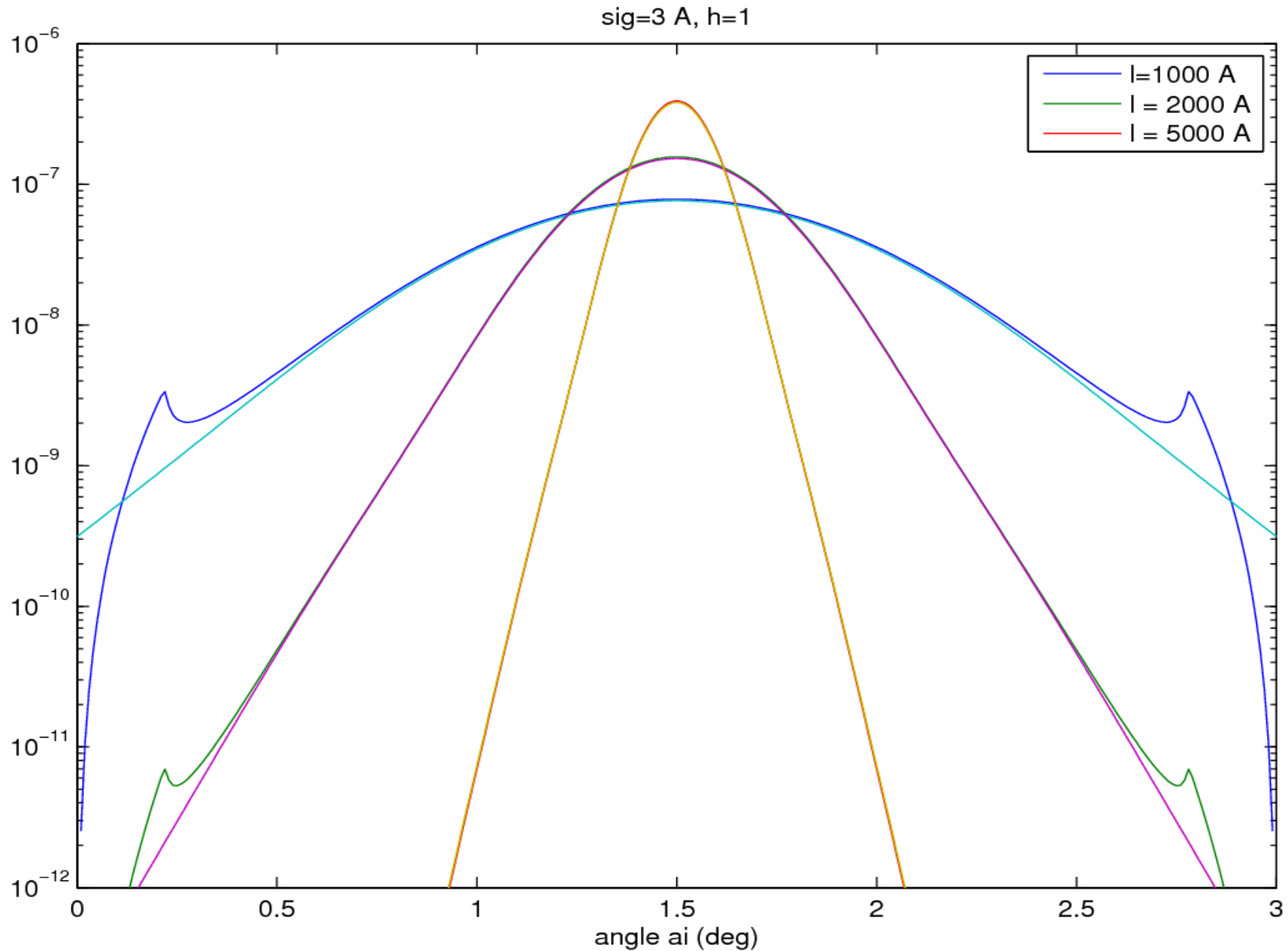
# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry fraktálové drsnosti



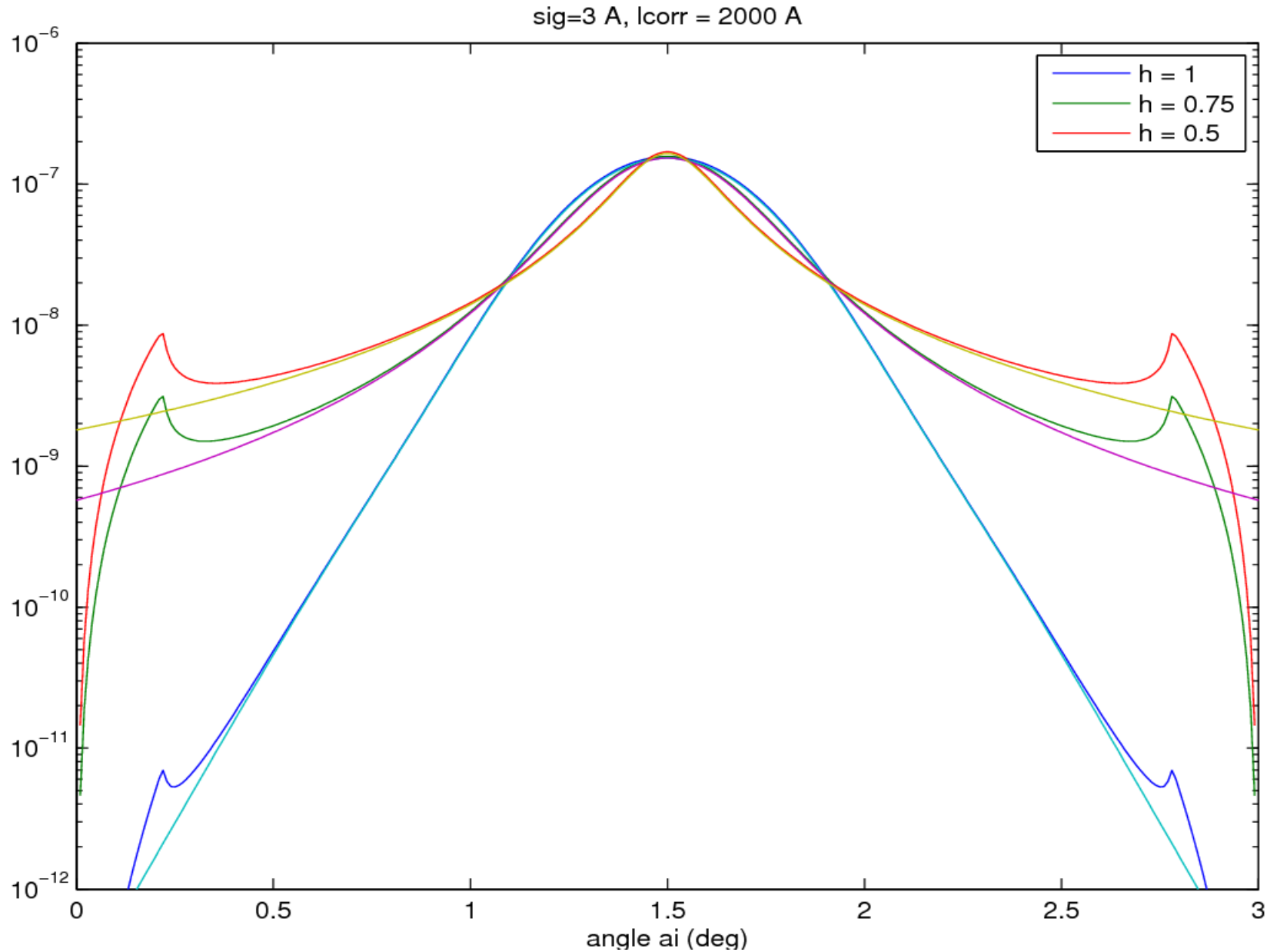
# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry fraktálové drsnosti



# Odraz na drsných rozhraních

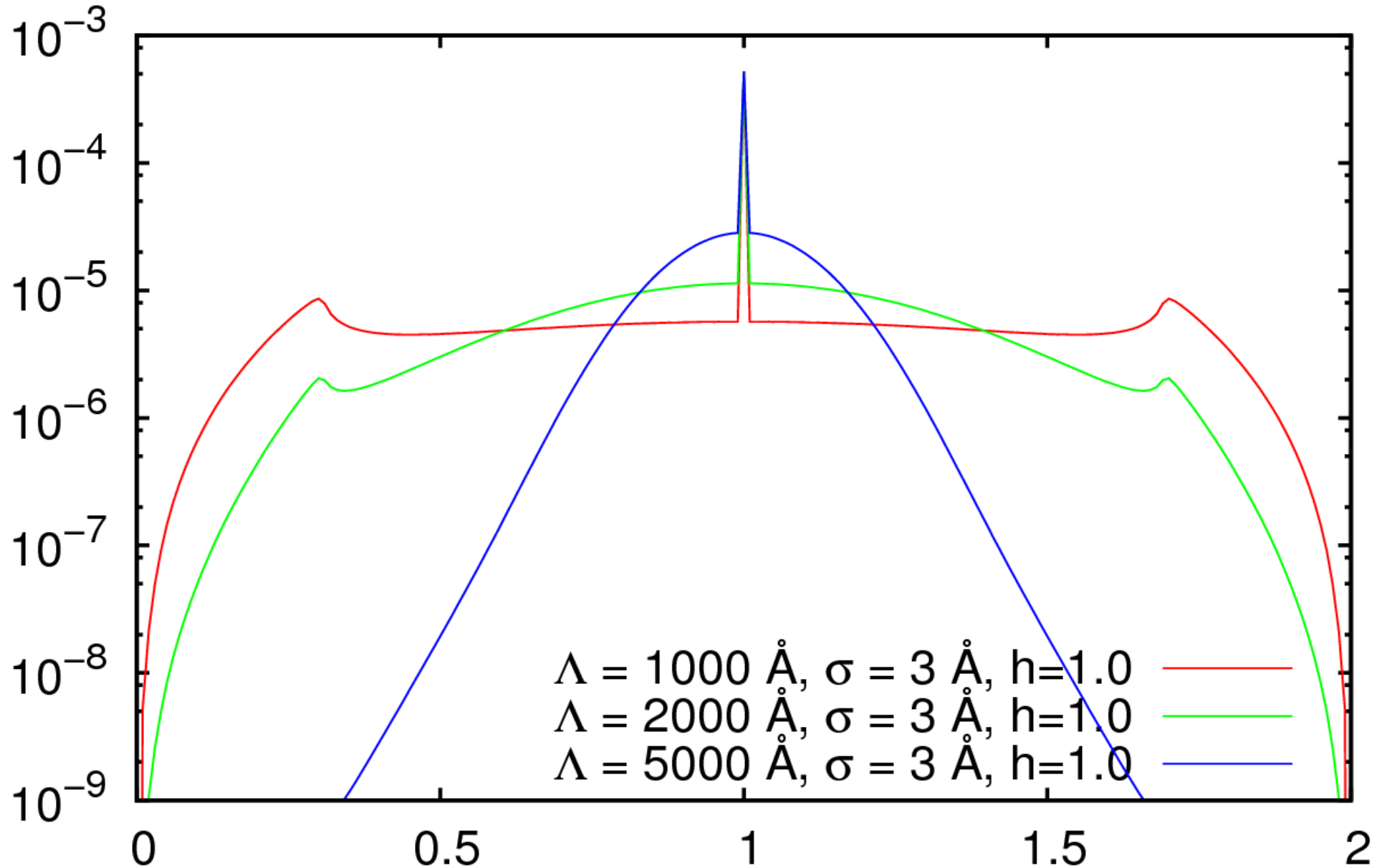
Nekoherentní rozptyl, různé parametry fraktálové drsnosti



# Odraz na drsných rozhraních

Koherentní a nekoherentní rozptyl, různé parametry fraktálové drsnosti

$\omega$  scan, GaAs substrate

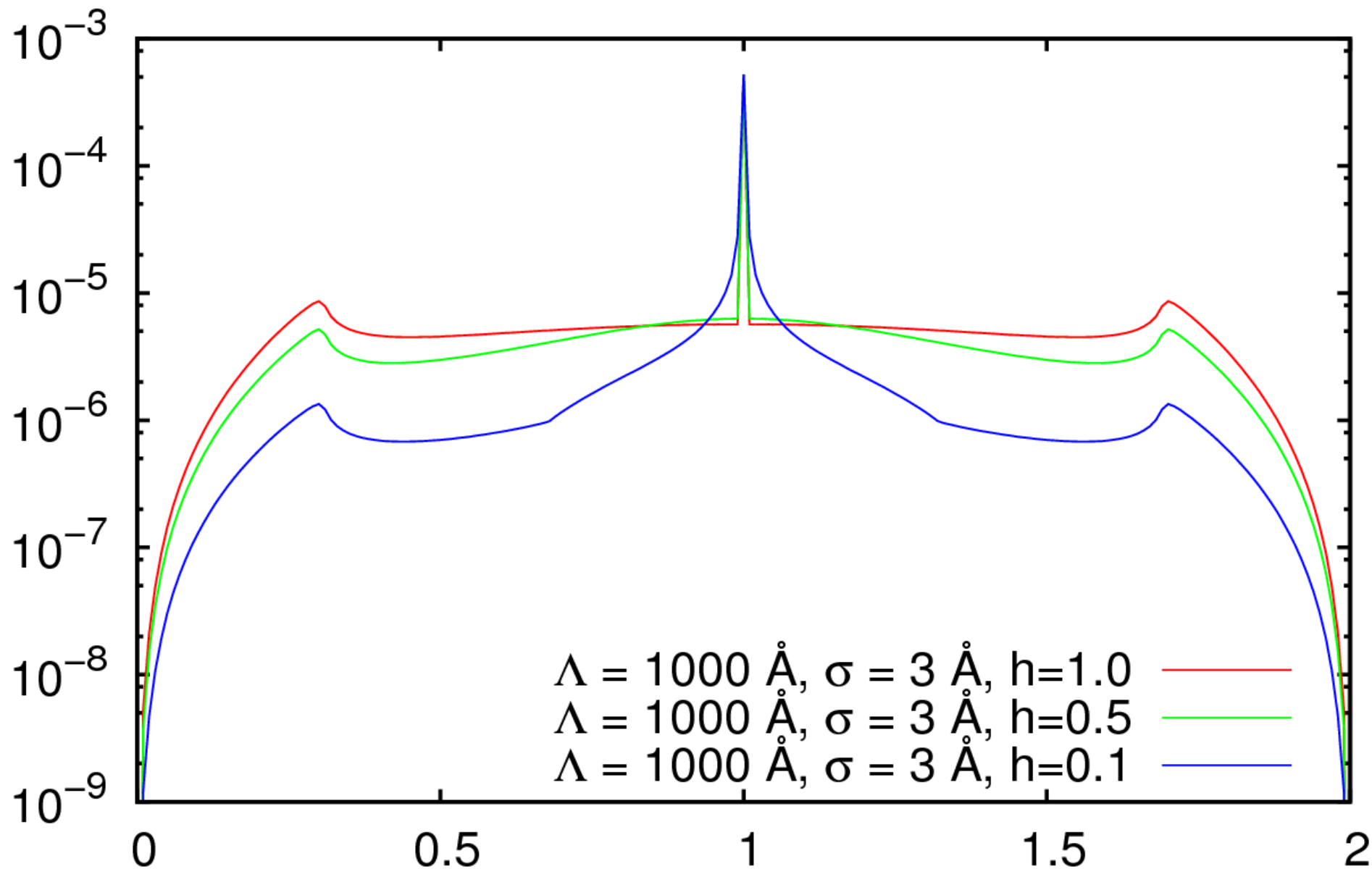




# Odraz na drsných rozhraních

Koherentní a nekoherentní rozptyl, různé parametry fraktálové drsnosti

$\omega$  scan, GaAs substrate



# Drsné rozhraní

Multivrstvy

$$w(U_m, U'_n) = w(U_m(\mathbf{r}_{\parallel}), U_n(\mathbf{r}'_{\parallel}))$$

$$C_{mn}(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) = \langle U_m(\mathbf{r}_{\parallel}) U_n(\mathbf{r}'_{\parallel}) \rangle$$

$$U_m(\mathbf{r}_{\parallel}) = \int_S d^2 \mathbf{r}'_{\parallel} U_{m+1}(\mathbf{r}'_{\parallel}) a_m(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) + \Delta_m(\mathbf{r}_{\parallel})$$

$$C_{mn}^{\text{FT}}(\mathbf{Q}_{\parallel}) = C_{m+1, n+1}^{\text{FT}}(\mathbf{Q}_{\parallel}) a_m^{\text{FT}}(\mathbf{Q}_{\parallel}) a_n^{\text{FT}}(\mathbf{Q}_{\parallel}) + \delta_{mn} K_m^{\text{FT}}(\mathbf{Q}_{\parallel})$$

$$K_m(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) = \langle \Delta_m(\mathbf{r}_{\parallel}) \Delta_m(\mathbf{r}'_{\parallel}) \rangle$$

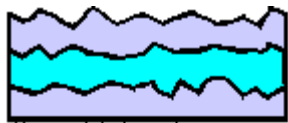
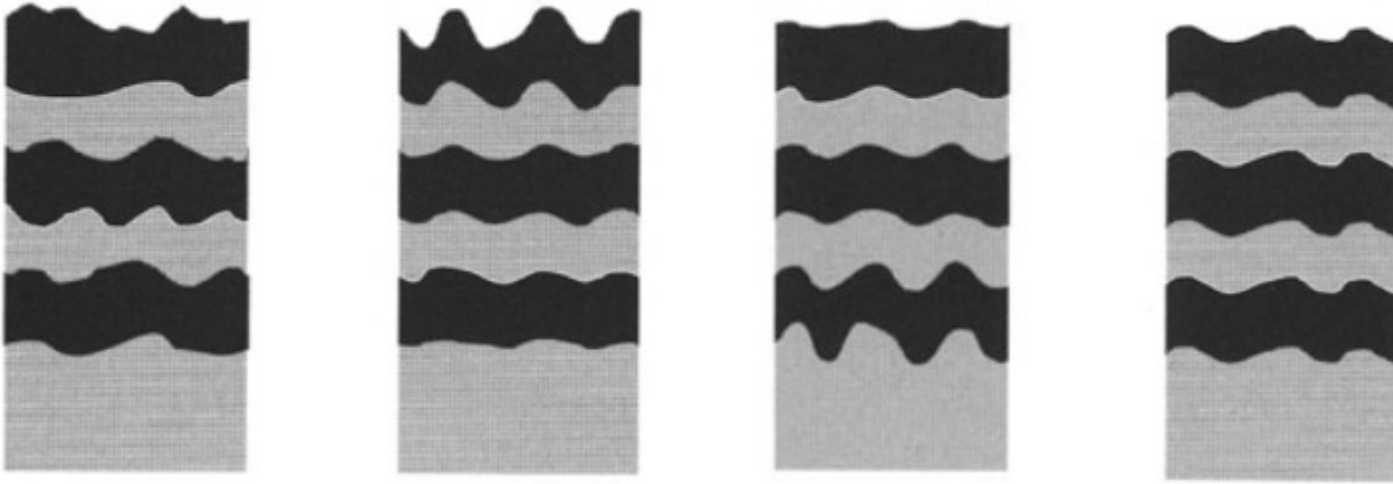
Aproximace

$$C_{mn}(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) = C_{jj}(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}) e^{-|z_m - z_n|/\Lambda_{\perp}}$$

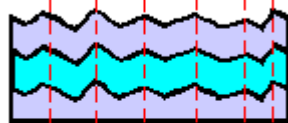
Realističtější aproximace

$$C_{mn}(\mathbf{Q}_{\parallel})^{\text{FT}} = C_{jj}(\mathbf{Q}_{\parallel})^{\text{FT}} \exp[-\zeta |\mathbf{Q}_{\parallel}|^n |z_m - z_n|]$$

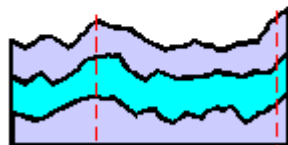
# Drsné rozhraní



Uncorrelated roughness



Correlated (repeated) roughness



Wavelength-dependent inheritance of roughness.

Fig.2. Different models of roughness correlations in multilayers

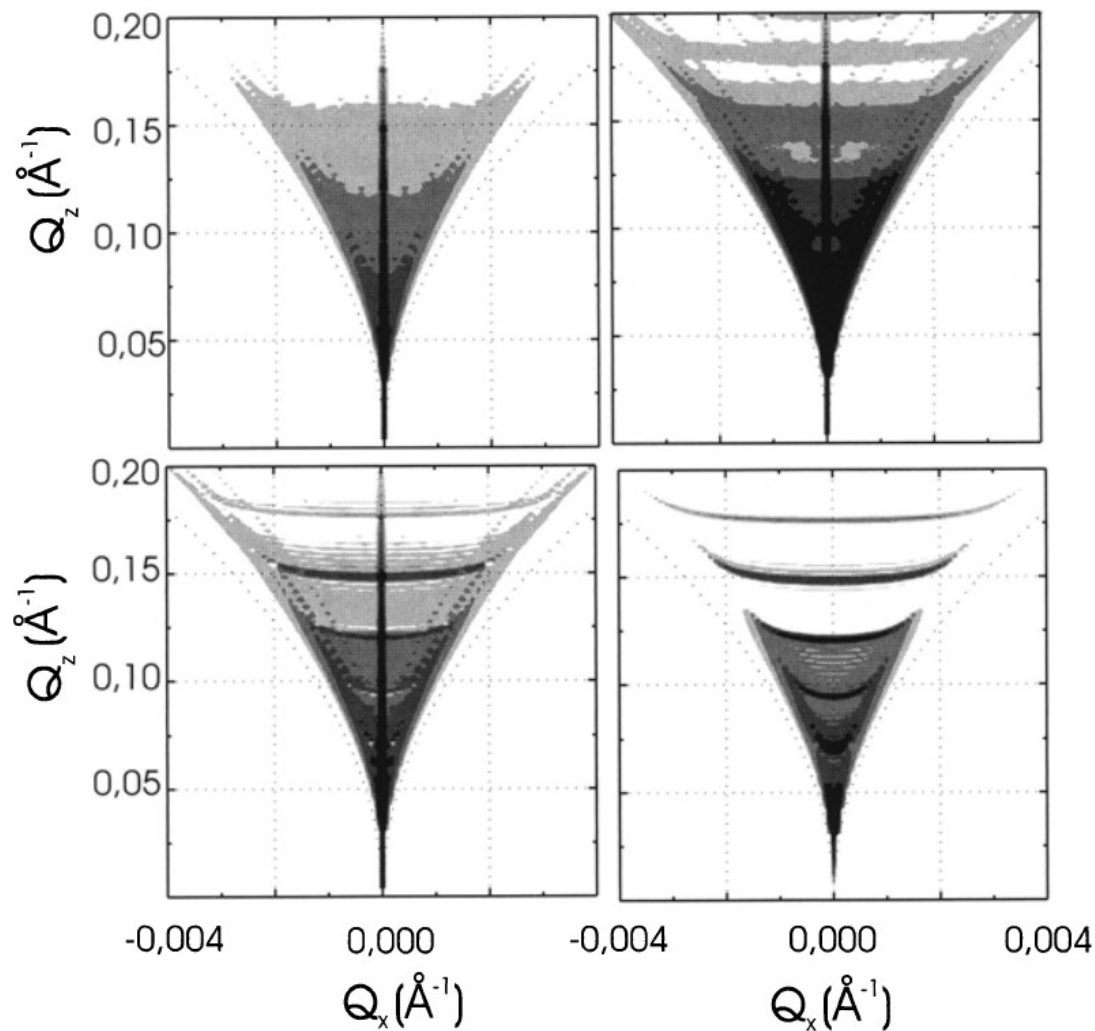
# Semikinematická teorie

$$E_1^{(A)}(\mathbf{r}) = \sum_{m=0}^{N+1} e^{i\mathbf{k}_{1\parallel} \cdot \mathbf{r}_{\parallel}} \\ \times \left[ T_1^{(m)} e^{ik_{1z}^{(m)}(z-z_m)} + R_1^{(m)} e^{-ik_{1z}^{(m)}(z-z_m)} \right] \Omega_{\text{id}}^{(m)}(z)$$

$$E_2^{(-A)}(\mathbf{r}) = \sum_{m=0}^{N+1} e^{i\mathbf{k}_{2\parallel} \cdot \mathbf{r}_{\parallel}} \\ \times \left[ T_2^{*(m)} e^{ik_{2z}^{*(m)}(z-z_m)} + R_2^{*(m)} e^{-ik_{2z}^{*(m)}(z-z_m)} \right] \Omega_{\text{id}}^{(m)}(z)$$

$$\hat{\mathbf{V}}_B = -K^2 \Delta \chi(\mathbf{r}) \quad \hat{\mathbf{V}}_B = \sum_{m=1}^{N+1} \hat{\mathbf{v}}^{(m)}$$

$$\hat{\mathbf{v}}^{(m)} = -K^2 \Delta \chi_0^{(m)} [H(z_m + U_m - z) - H(z_m - z)]$$



**Fig. 11.19.** Reciprocal-space maps of the diffusely scattered intensity calculated for a [GaAs(7 nm)/AlAs(15 nm)] $\times$ 10 multilayer using the DWBA method and the simpler replication model according to Eq. (11.28). All the interfaces have the same r.m.s. roughness, 1 nm; the same correlation length, 50 nm; but different vertical correlation lengths,  $\Lambda_{\perp}$ . Upper left panel: no replication,  $\Lambda_{\perp} = 0$  nm. Upper right panel: full replication  $\Lambda_{\perp} = \infty$ . Bottom left panel:  $\Lambda_{\perp} = 100$  nm. Bottom right panel: full replication  $\Lambda_{\perp} = \infty$ , calculated by the simpler DWBA. The full lines represent the arcs of the Ewald spheres for the limiting cases of  $\alpha_i = 0$  and  $\alpha_f = 0$ . Resonant diffuse scattering (RDS) disappears, if the roughness profiles are not replicated (upper left panel). Bragg-like resonance lines are visible in all maps calculated by the full DWBA. They are not reproduced by the simpler DWBA (bottom right panel).

# Nekoherentní rozptyl na drsném rozhraní

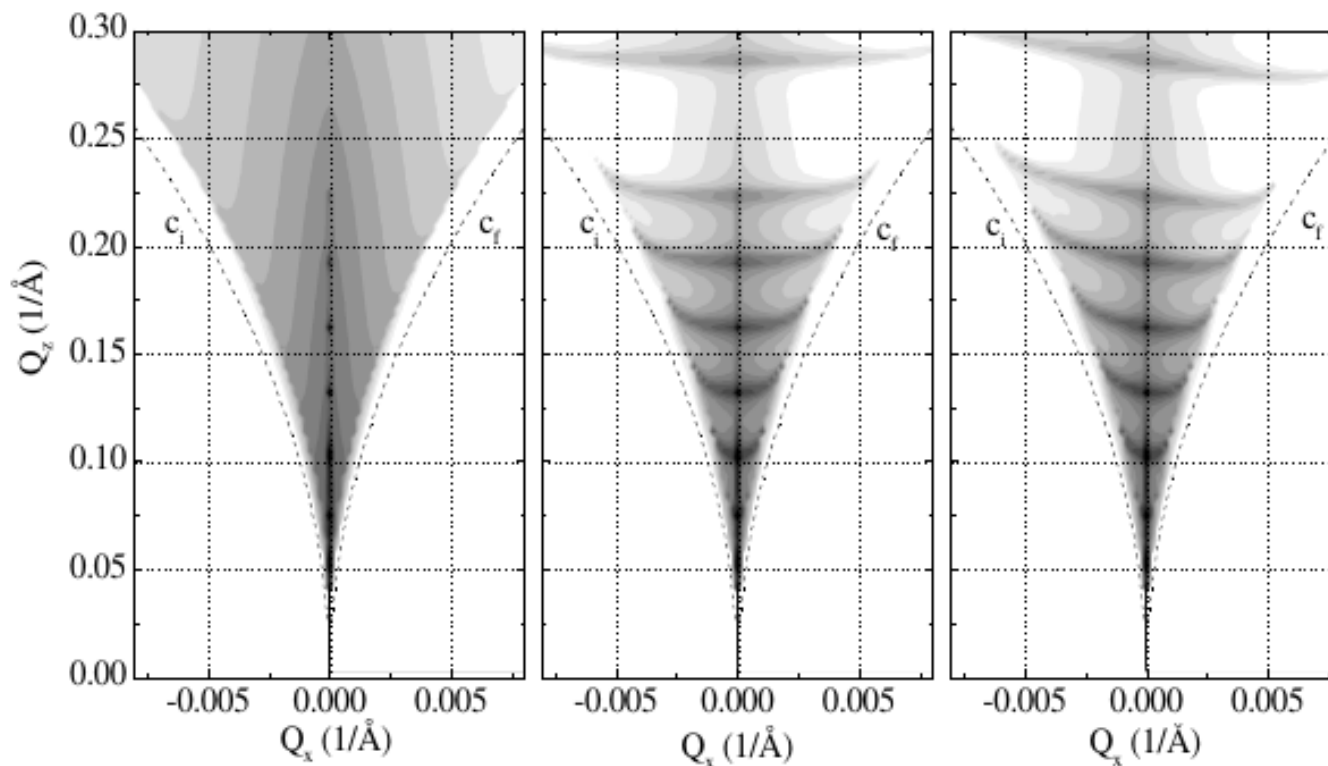


Figure 5.4: Simulation of the intensity distribution  $J(Q_x, Q_z)$  for a periodic Ni/C multilayer on glass,  $\sigma = 7 \text{ \AA}$ ,  $\Lambda = 1500 \text{ \AA}$ ,  $h = 0.5$ . Left panel – no roughness correlation between the interfaces, middle panel – vertical correlation  $\Lambda_{\perp} = 1000 \text{ \AA}$ , right panel – the correlation is inclined by angle  $\xi = 60^\circ$ .

# Nekoherentní rozptyl na drsném rozhraní

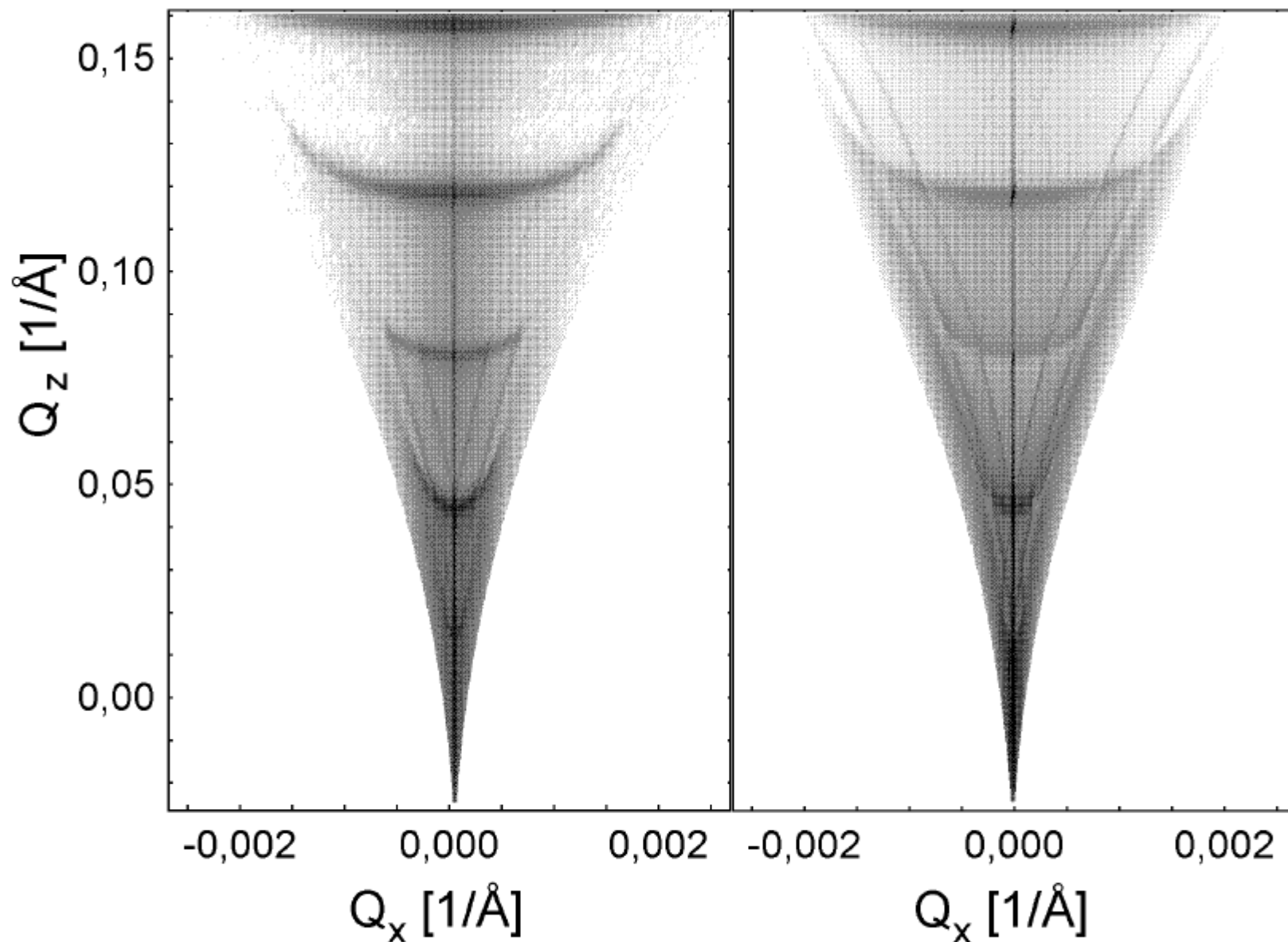


Figure 8.27: The measured (left) and calculated (right) reciprocal space map of the  $[\text{GaAs}(70\text{\AA})/\text{AlAs}(150\text{\AA})] \times 10$  multilayer.

# Drsné rozhraní

dvouúrovňový povrch

$$w(ld; \Delta x) = \sum_{q=1,2} w_{q,q+l}(\Delta x),$$

$$w_{11} + w_{12} = \Phi_1$$

$$w_{22} + w_{21} = \Phi_2 = 1 - \Phi_1.$$

$$w_{11} = \Phi_1 - w_{12}, \quad w_{22} = 1 - \Phi_1 - w_{12}, \quad w_{21} = w_{12}.$$

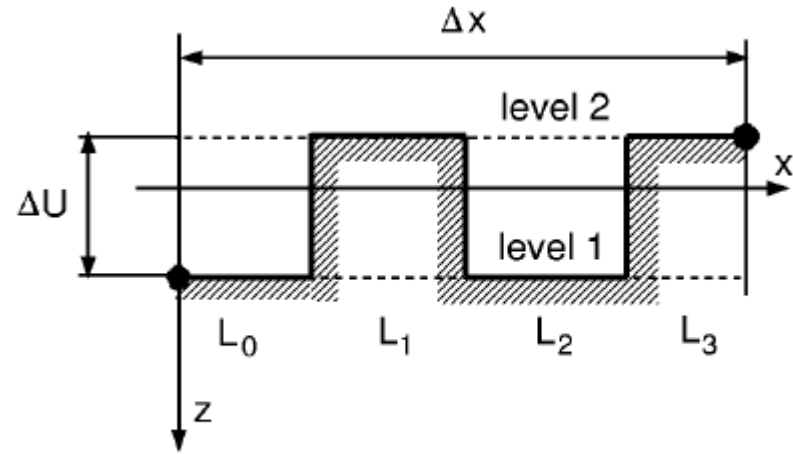


Figure 8.2: configuration of  $n = 3$  steps and  $\Delta U = d$

$$\int dL_0 \int dL_1 \cdots \int dL_n \delta \left( \Delta x - \sum_{j=0}^n L_j \right) P_{1,0}(L_0) P_2(L_1) P_1(L_3) \cdots P_1(L_{n-1}) P_{2,f}(L_n),$$

$$P(L) = \frac{\alpha^m}{\Gamma(m)} e^{-\alpha L} L^{m-1}$$

$$P_{1,0}(L) = \frac{\Phi_1}{\langle L_1 \rangle} \int_L^\infty d\tau P_1(\tau)$$

$$P_{2,f}(L) = \int_L^\infty d\tau P_2(\tau).$$

$$w_{12}(\Delta x) = \sum_{n=1, \text{odd}}^\infty \int dL_0 \int dL_1 \cdots \int dL_n \delta \left( \Delta x - \sum_{j=0}^n L_j \right)$$

$$\times P_{1,0}(L_0) P_2(L_1) P_1(L_3) \cdots P_1(L_{n-1}) P_{2,f}(L_n), \quad \Delta x > 0$$

$$P^{FT}(Q_x) = \int_{-\infty}^\infty dL P(L) e^{iLQ_x} = \left( 1 - \frac{i\langle L \rangle Q_x}{m} \right)^{-m}$$



# Drsné rozhraní

dvouúrovňový povrch

$$w_{12}^{FT}(Q_x) = \int_{-\infty}^{\infty} d\Delta x w_{12}(\Delta x) e^{i\Delta x Q_x} =$$
$$= 2\text{Re} \left[ \frac{P_{1,0}^{FT}(Q_x) P_{2,f}^{FT}(Q_x)}{1 - P_1^{FT}(Q_x) P_2^{FT}(Q_x)} \right] (1 - \delta_{Q_x,0}) + 2\pi \Phi_1 (1 - \Phi_1) \delta(Q_x).$$

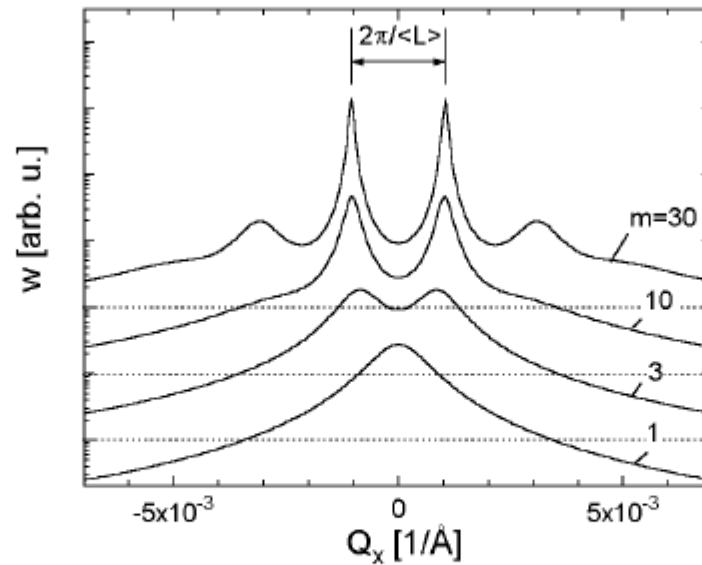


Figure 8.4: The probability  $w_{12}^{FT}(Q_x)$  calculated for a two-level system. The mean width of the terraces are  $\langle L_1 \rangle = \langle L_2 \rangle = 3000 \text{ \AA}$ , the step height is  $10 \text{ \AA}$ , the calculation has been performed for various values of the parameter  $m = m_1 = m_2$ .

# Drsné rozhraní

Vicinální povrch

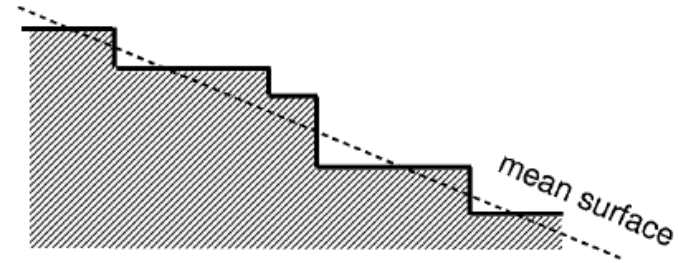


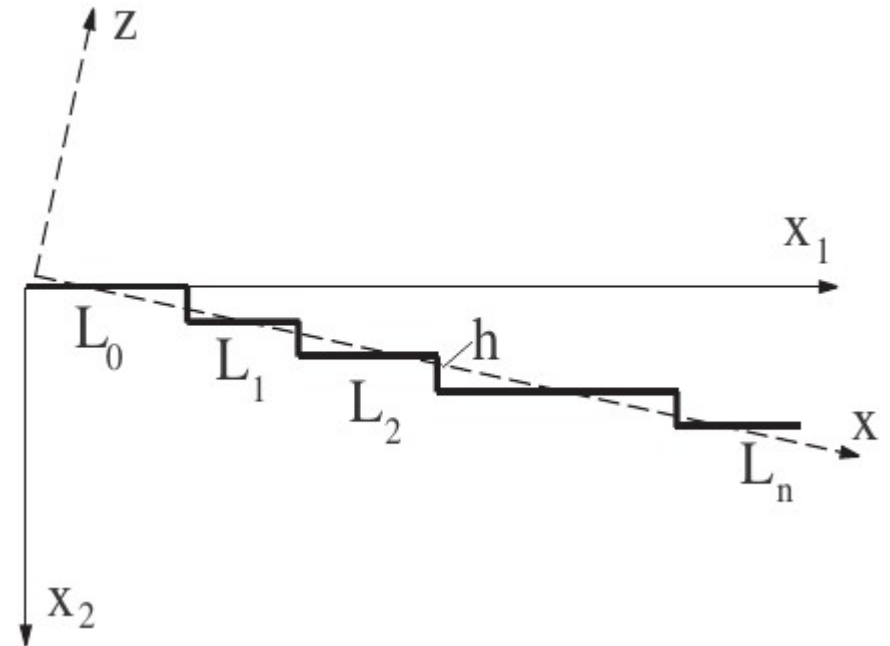
Figure 8.3: Sketch of a terraced surface with infinite number of levels

$$F(Q_1, Q_2) = \left\langle \int d(x_1 - x'_1) e^{-iQ_2(U(x_1) - U(x'_1))} e^{-iQ_1(x_1 - x'_1)} \right\rangle$$

$$F(Q_1, Q_2) = \frac{2}{Q_1^2 \langle L \rangle} \text{Re} \left( \frac{[1 - \xi(Q_1)][1 - \zeta(Q_2)]}{1 - \xi(Q_1)\zeta(Q_2)} \right)$$

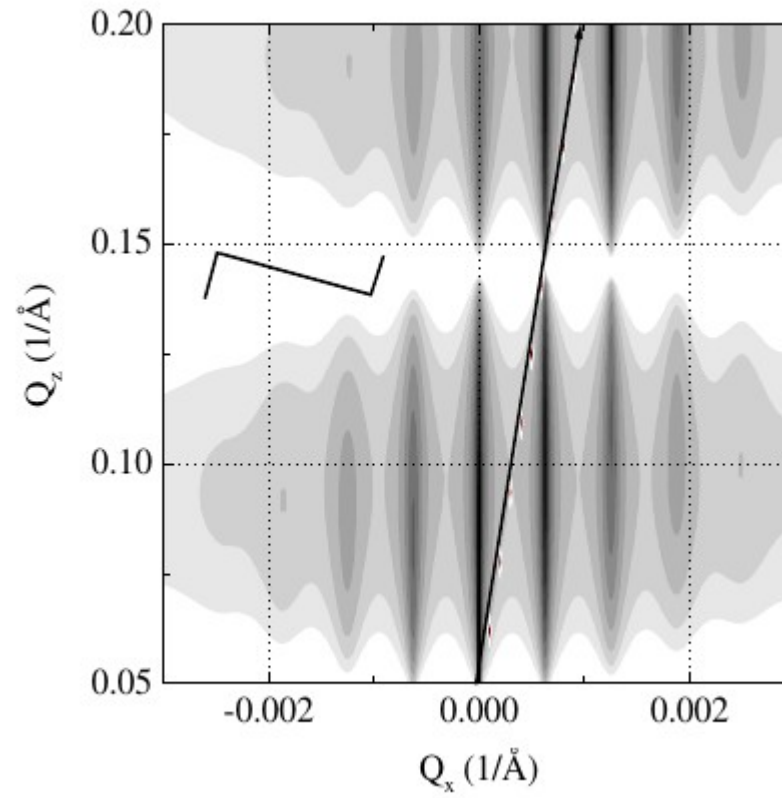
$$\xi(Q) = \int_0^\infty dL w(L) e^{-iQL}$$

$$\zeta(Q_2) = \exp(-iQ_2 h)$$



# Drsné rozhraní

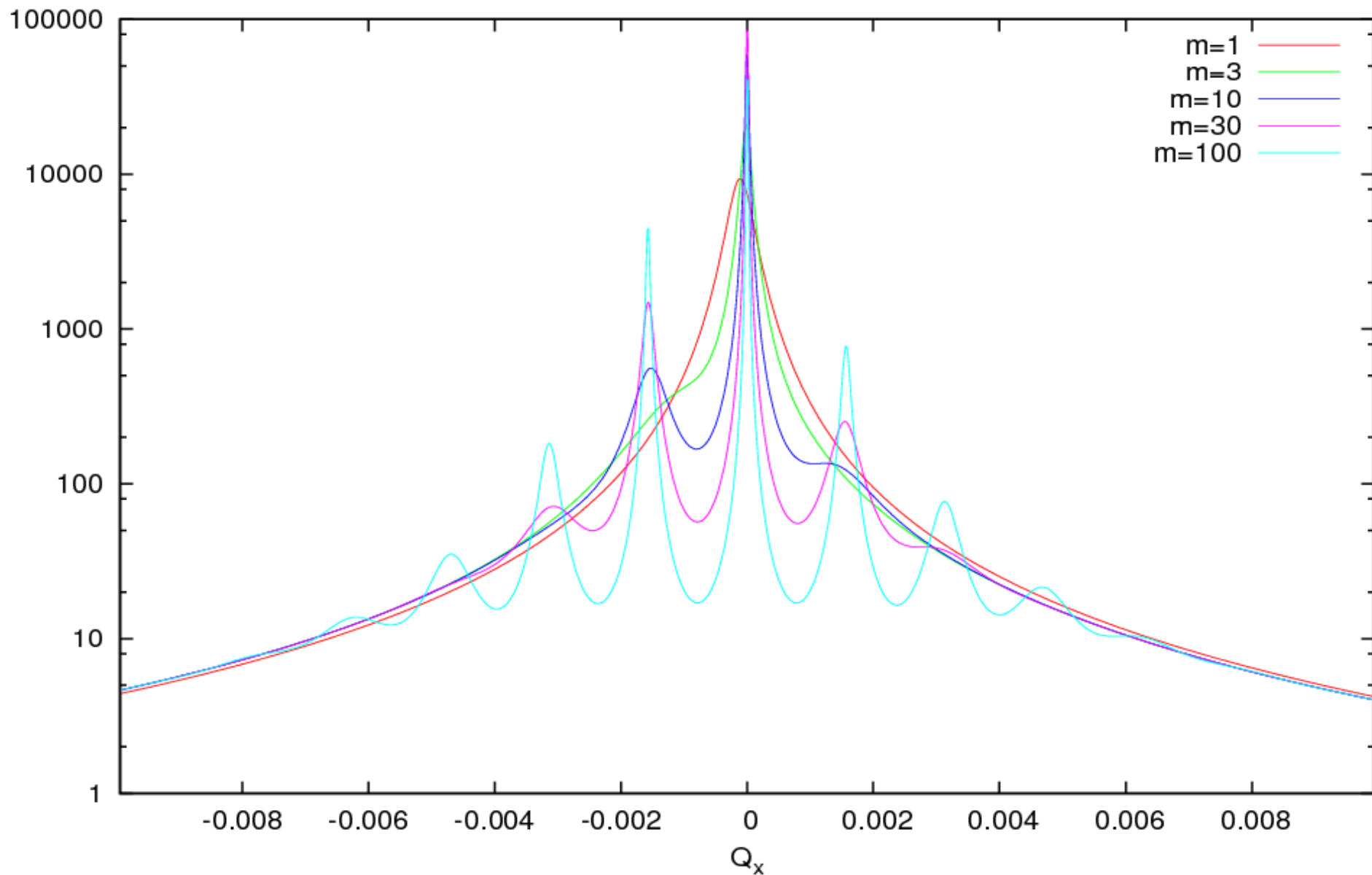
Vicinální povrch



# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry pro vicinální povrch

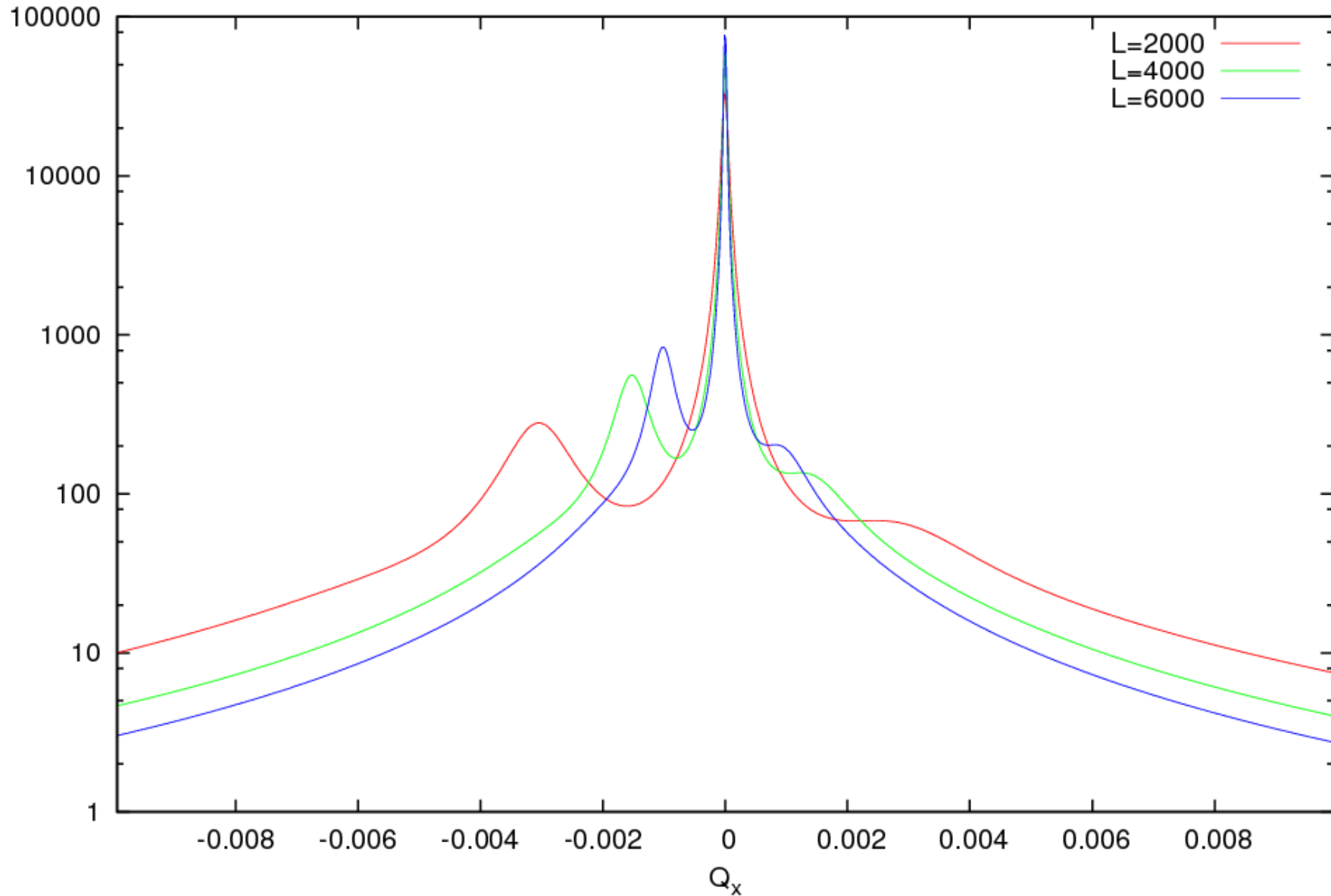
Kinematic approximation, gamma distribution,  $L=4000$ ,  $a_i+a_f=2.0\text{deg}$



# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry pro vicinální povrch

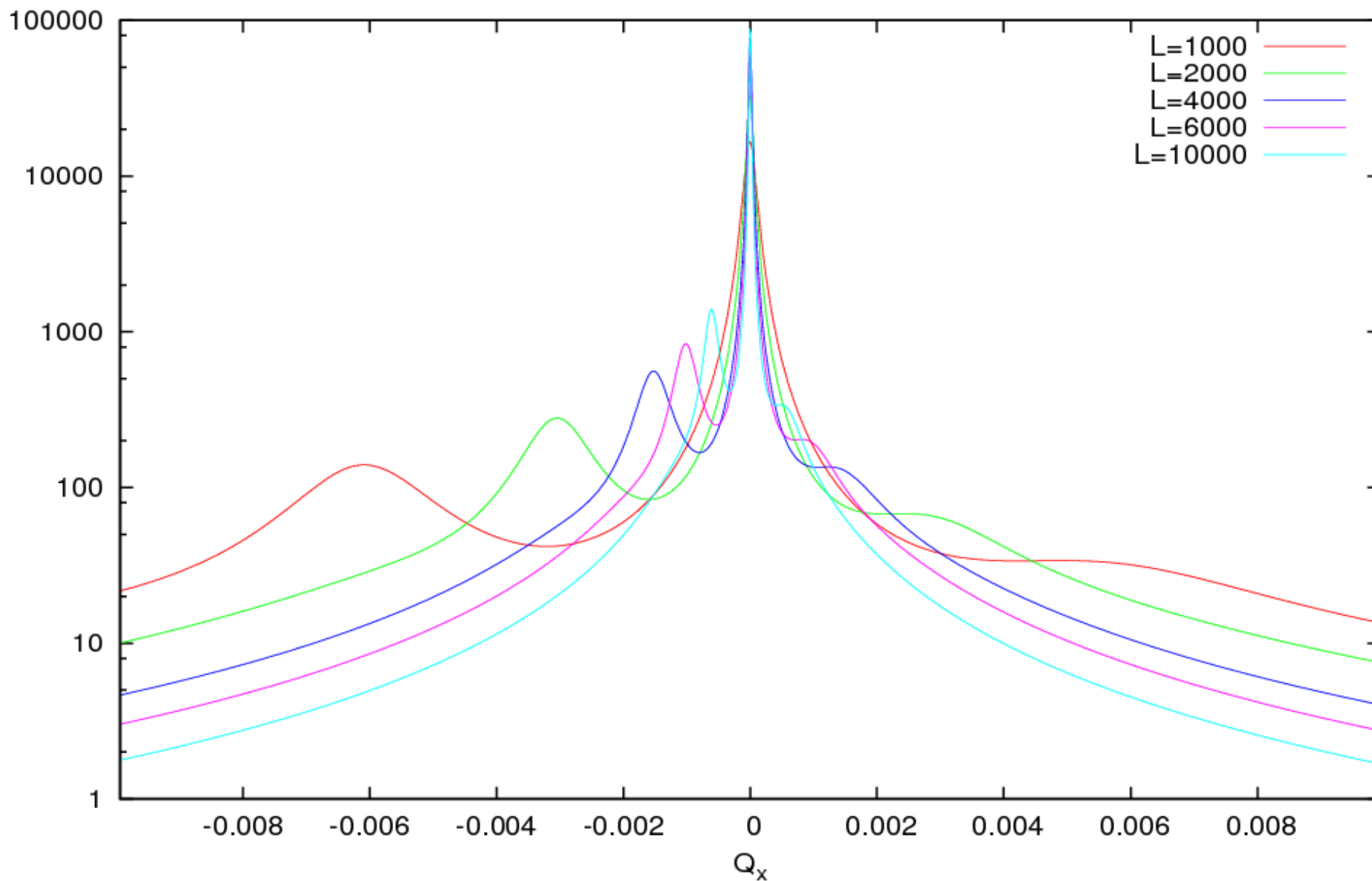
Kinematic approximation, gamma distribution,  $m=10$ ,  $a_i+a_f=2.0\text{deg}$



# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry pro vicinální povrch

Kinematic approximation, gamma distribution,  $m=10$ ,  $a_i+a_f=2.0\text{deg}$



# Odraz na drsných rozhraních

Nekoherentní rozptyl, různé parametry pro vicinální povrch

Semikinematic approximation, gamma distribution,  $L=4000\text{\AA}$ ,  $a_i+a_f=2.0\text{deg}$

