Unit 11 Functions and trigonometry

1. *What is a function in mathematics? Complete the missing words in the definition, the first letters are given.*

A rule that assigns to every e______________ *x* of a set X a unique element y of a set Y, written *y = f(x)* where *f* d_________ the function. X is called the d_________ and Y the r__________ (or codomain).

The Penguin Dictionary of Mathematics, ed. J. Daintith and R,D, Nelson. Penguin Books 1989.

2. *The text below expands the definition of a function. Several articles have been taken out of the text. Read it and fill in the blanks with "a(n)", or "the" if necessary.*

In mathematics, a **function** is (1)___ relation between a set of inputs and a set of permissible outputs with (2)___ property that each input is related to exactly one output. (3)___ example is the function that relates each real number *x* to its square x^2. (4)___ output of a function *f* corresponding to an input *x* is denoted by *f*(*x*) (read "*f* of *x*"). In this example, if the input is −3, then (5)___ output is 9, and we may write *f*(−3) = 9. The input variable(s) are sometimes referred to as the argument(s) of the function and (6)___ output is called the value. There are many ways to describe or represent a function. Some functions may be defined by (7)___ formula or algorithm that tells how to compute the output for (8)___ given input. Others are given by (9)___ picture, called the graph of the function. In (10)___ science, functions are sometimes defined by a table that gives the outputs for selected inputs.

Adapted from: http://fileserver.net-texts.com/asset.aspx?dl=no&id=158138

3. *Watch the video and answer questions.*

https://www.youtube.com/watch?v=snHKEpCv0Hk

2:30

1) How are the yellow and blue points related?

………………………………………………………………………………………

2) What are possible answers to the question "What is sine?"

………………………………………………………………………………………

3) What is the difference between sine and cosine wave?

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4) What is the property of things going in cycles?

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6:30

5) Which other trigonometric functions are mentioned in the video? What are their properties?

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6) What does the final simulation show?

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4. ***Look, read and underline key verbs and nouns:*** (texts and pictures adapted from Hall, David. *Nucleus. English for Science and Technology.* Longman 1985)

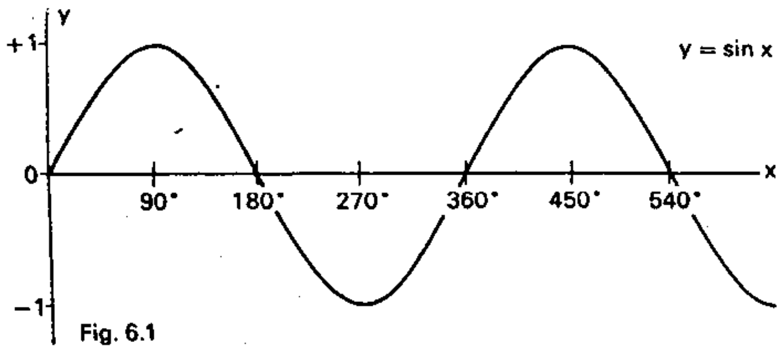

Fig. 6.1

Figure 6.1 is a graph of the function y=sin x. As x goes from 0° to 90°, sin x increases from 0 to 1. As x goes from 90°to 270°, sin X decreases from 1 to =1, crossing the x-axis at 180°. As x goes from 270°to 360°, sin x increases from -1 to 0. When x reaches 360° the graph repeats itself. The sine function is a periodic function, with a period of 360°, i.e. the graph repeats itself every 360°.

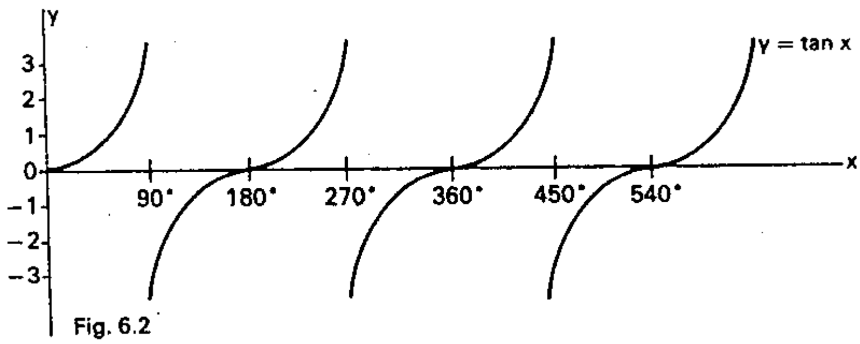

Fig. 6.2

Figure 6.2 is a graph of the function y=tan x. As x approaches 90°, tan x tends to infinity. After 90°, tan x reappears on the negative side. As x goes from 90°to 180°, tan x increases to 0. As x approaches 270°, tan x again tends to infinity, reappearing again after 270°on the negative side. The tangent function is a periodic function, with a period of 180°, i.e. the graph repeats itself every 180°.

5. ***Now use these verbs and nouns to write a short description of the following trigonometric functions:***

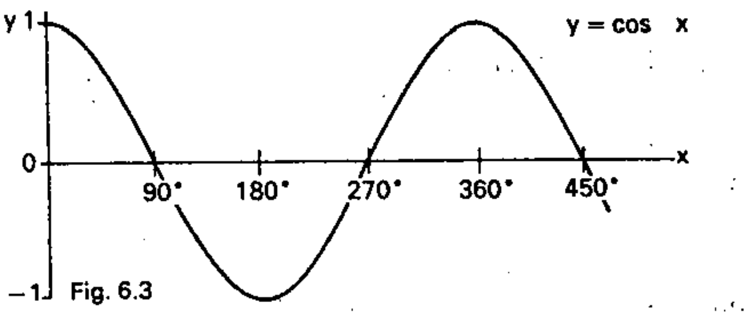

Fig. 6.3

Figure 6.3 is a graph of the function ____
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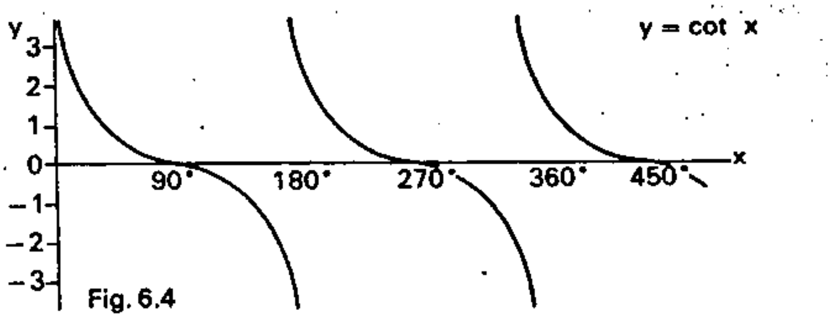

Figure 6.4 is a graph of the function ____
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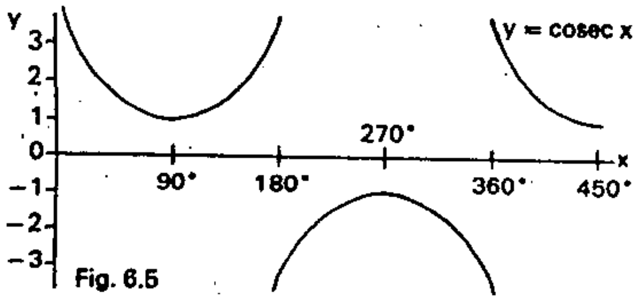

Figure 6.5 is a graph of the function ____
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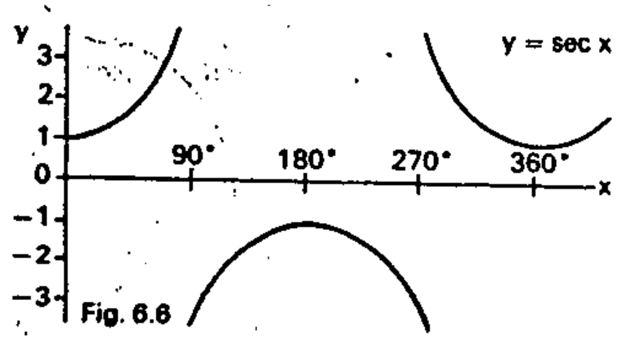

Figure 6.5 is a graph of the function ____
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6. What type of a function is this?

Imagine that you had the information shown in the table about some function *f.* What would you expect the output *f*(1) to be? What would a graph of this function look like?

x	0.9	0.99	0.999
f(x)	2.93	2.9954	2.9999997

Read this definition and underline linking expression. Which of these are typical for mathematical texts?

Definition *Continuous function.* Let *f* be a function whose domain is the *x* axis or is made up of open intervals. Then *f* is a continuous function if it is continuous at each number *a* in its domain.

Thus x^2 is a continuous function. So is $1/x$, whose domain consists of the intervals $(-\infty, 0)$ and $(0, \infty)$. Although this function explodes at 0, this does not prevent it from being a continuous function. *The key to being continuous is that the function is continuous at each number in its domain.* The number 0 is not in the domain of $1/x$.

Only a slight modification of the definition is necessary to cover functions whose domains involve closed intervals. We will say that a function whose domain is the closed interval $[a,b]$ is *continuous* if it is continuous at each point in the open interval (a,b), continuous from the right at *a,* and continuous from the left at *b*. Thus $\sqrt{1-x^2}$ is continuous on the interval $[-1, 1]$.

In a similar spirit, we say that a function with domain (a, ∞) is continuous if it is continuous at each point in (a, ∞) and continuous form the right at *a.* Thus *x* is a continuous function. A similar definition covers functions whose domains are of the form $(-\infty, b)$.

Many of the functions met in algebra and trigonometry are continuous. For instance, 2^x, sin *x,* tan *x* and any polynomial are continuous. So is any rational function (the quotient of two polynomials). Moreover, algebraic combinations of continuous functions are continuous. For example, since x^3 and sin *x* are continuous, so are $x^3 + \sin x$, $x^3 - \sin x$, and $x^3 \sin x$. The function $x^3/\sin x$, which is not defined when $\sin x = 0$, is continuous on its domain.

7. Study three definitions of continuous functions. Supply the missing parts of headings.

Definition *Continuity* ___________________. Assume that *f* (x) is defined at *a* and in some open interval *(a, b).* Then the function *f* is continuous at *a* from the right if $\lim_{x \to a+} = f(a)$. This means that
1 $\lim_{x \to a+} f(x)$ exists and
2 that limit is $f(a)$.

Definition *Continuity* ____________________. Assume that *f*(x) is defined at *a* and in some open interval *(c, a).* Then the function *f* is continuous at *a* from the left if $\lim_{x \to a-} f(x) = f(a)$. This means that
1 $\lim_{x \to a-} f(x)$ exists and
2 that limit is $f(a)$.

The next definition applies if the function is defined in some open interval that includes the number *a.* It essentially combines the first two definitions.

Definition *Continuity* ________________. Assume that *f*(x) is defined in some open interval *(b,c)* that contains the number *a.* Then the function *f* is continuous at *a* if $\lim_{x \to a} f(x) = f(a)$. This means that
1 $\lim_{x \to a} f(x)$ exists and
2 that limit is $f(a)$.

This third definition amounts to asking that the function be continuous both from the right and from the left at *a.*

8. *REVISION* & *LANGUAGE FOCUS.*

a) In the set of real numbers, how large is the highest number? However large a number is, there is always a higher number.
b) In the set of numbers < 1, what number is the highest member of the set? Whatever number we choose, there is always a higher number in the set.
c) How many points are there on a line? However many points we choose, there are always more points.

What is the difference in usage between *whatever* and *however?*

9. *Now make correct statements from the table:*

In the set of real numbers		large we make one angle,	there is always a smaller value.
On a line		distance we take between two points,	the sum does not reach one.
In the set x > 0	however	small a number is,	there is always a shorter distance.
In the series ½+1/4+1/8+…	whatever	many values we add,	it cannot be more than 180°.
In the series √2, $\sqrt[3]{2}$, $\sqrt[4]{2}$, $\sqrt[5]{2}$…		root of 2 is taken,	there is always a smaller number.
In a triangle		value of x we take,	its value is always greater than one.