

8. Sets and Operations on Sets

1. Relations between sets. How is a set defined? How is it denoted? How is an element denoted?

a) In naive set theory, a set is a collection of objects (called members or elements) that is regarded as being a single object. To indicate that an object x is a member of a set A one writes $x \in A$, while $x \notin A$ indicates that x is not a member of A . A set may be defined by a membership rule (formula) or by listing its members within braces. For example, the set given by the rule “prime numbers less than 10” can also be given by $\{2, 3, 5, 7\}$. In principle, any finite set can be defined by an explicit list of its members, but specifying infinite sets requires a rule or pattern to indicate membership; for example, the ellipsis in $\{0, 1, 2, 3, 4, 5, 6, 7, \dots\}$ indicates that the list of natural numbers \mathbb{N} goes on forever. The empty (or void, or null) set, symbolized by $\{\}$ or \emptyset , contains no elements at all. Nonetheless, it has the status of being a set.

Let set A be {all animals}

Let set G be {positive integers <10}

Let set B be {lions}

Let set H be {1,2,3,4,5,6,7,8,9}

Let set C be {all animals except lions}

Let set I be {1,2,3}

Let set D be {all human beings}

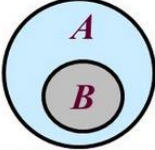






Let set J be $\{\}$ (This set is called the *empty set*)

Let set E be {all solid figures}

Let set K be {3,4,5}

Let set F be {all geometrical figures}

b) We can use *Venn diagrams* or *set notation* to show the relations between sets. Fill in the table.

Relation	Venn diagram	Set notation
Set B is a _____ of set A. Set B is contained in set A.		$B \subset A$
Set G is a _____ of set H and set H is a subset of set G.		$G \subseteq H$ $G \supseteq H$
Set C is the _____ of set B.		$C = B'$
Set I added to set K is the _____ of I and K.		$I \cup K = \{1,2,3,4,5\}$
Set I subtracted from set K is the _____ between K and I.		$K - I = \{4,5\}$
The members common to set I and set K form the _____ of I and K.		$K \cap I = \{3\}$
Set B and D are _____.		$B \cap D = \{\}$

c) Now use a similar table to express the following relations:

- a) A and D
- b) E and F
- c) $H - I$
- d) $H \cap I$
- e) $H \cup I$
- f) All triangles are plane figures.
- g) No triangles are solid figures.
- h) The set {figures with straight sides} and the set {figures with curved sides} have some members in common.

2. Match the terms to their definitions:

1. The union of two sets	is	a) a set which contains all the elements which do not belong to A.
2. The intersection of two sets		b) a third set which contains all the members of both sets.
3. The difference between two sets		c) a set in which every member of A is also in B, but there is at least one member in B but not in A.
4. The complement of a set A		d) a third set which contains all the members of one set which are not common to both sets.
5. The subset A of a set B		e) a third set which contains all the members common to both sets.
6. The proper subset A of a set B		f) a set in which every member of A is also in B.

3. Explain the relations between the sets of numbers:

- a) All integers and all natural numbers
- b) All real and all imaginary numbers
- c) All whole numbers and all positive integers
- d) All real and all irrational numbers.

4. Solve these problems and present your solution to your fellow students:

(a) Draw a Venn diagram to show these data.

65 persons were polled

47 liked calculators

15 liked computers and calculators

Use your diagram: How many liked computers? How many liked only calculators?

(b) The total membership in the chess and badminton club is 57. Eleven students belong to the chess club and also the badminton club. Thirty-five students belong to the badminton club only. How many students belong to the chess club?

(c) In a recent poll of 91 students, it was found that 34 students liked chocolate K and chocolate C. Sixty-two students liked chocolate C. How many students liked only chocolate K?

(d) Use the following statistics to answer the question:

185 persons liked only T.V.

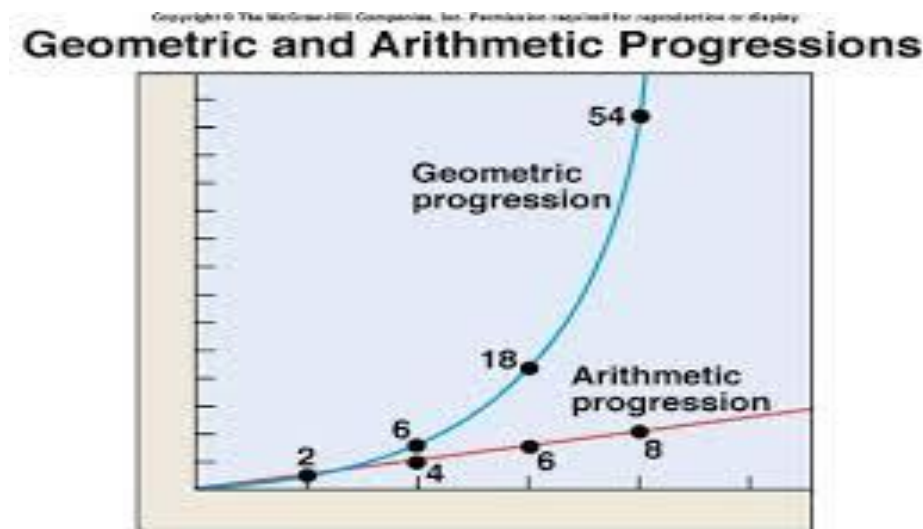
463 persons liked T.V. and movies

832 persons liked movies.

How many persons were interviewed?

Video: Let's Talk About Sets

<https://www.youtube.com/watch?v=y12Tt3bOmKA>



5. Pre-listening. Answer questions.

What is the difference between arithmetic and geometric progression?

6. Listening. Answer the following questions.

- 1) What does the speaker do with the sets of positive integers?
.....
- 2) What does he do with the repeated members in the set $A + B$?
.....
- 3) What is the usual size of the resulting set when you add up two P sets?
.....
- 4) How are G and P sets different in relation to overlaps?
.....
- 5) What is the usual size of the resulting set when you add up two G sets?
.....
- 6) What are sum-free sets and how common are they?
.....
- 7) What did Paul Erdős show?

7. Solve these problems.

- 1) The sum of the first two terms of a geometric progression (G.P.) is 180 and the sum to infinity of the series is $182 \frac{6}{7}$. Find q (given that it is a **positive** common ratio) and the first term.
- 2) The sum of the first eight terms of an arithmetic progression (A. P.) is 124 and the sum of the first 15 terms is 390. Find the sum of the first 25 terms.

Paragraph writing– unity and coherence

<https://www.youtube.com/watch?v=vbMtBjoBalQ>

A. In pairs, answer these questions.

1. What is a paragraph?
2. What does unity and coherence mean in relation to paragraph writing?

B. Listening. Complete the missing parts of statements.

A paragraph always needs a 1. _____ that clearly states what the main idea is for that particular paragraph. This topic sentence will 2. _____ know what will be discussed in that paragraph. When the topic sentence is written clearly, it is easier to write 3. _____. Each sentence should add more information about the main idea by either adding 4. _____, or other details necessary to support the main idea. All of the sentences in the paragraph should 5. _____ the main idea. Do not add any 6. _____ information into the paragraph.

C. Example paragraph: Why do you think this paragraph is united? Can you identify individual parts of this paragraph?

Writing in English can be difficult for many foreign students. For some students, the English grammar can be a challenge as it is different from their own language. For example, the placement of the verb can be at the end of the sentence in many languages, whereas in English it usually comes at the beginning of a sentence. Also, the way that ideas are presented in an English paragraph may be different from their own language. This makes it difficult to write well-developed English paragraphs. All of these things can make writing in English a challenge for many students.

D. How can we achieve coherence in a paragraph?

1. Try to find linking expressions in the text.
2. Complete other linking words in the table.

In contrast to *In conclusion* *Therefore* *Furthermore* *In the same way*
Likewise *On the whole* *Accordingly* *On the contrary* *Additionally*
For the most part *In other words* *Unlike* *Equally* *If*

Adding information/examples	Cause and effect	Contrast	Same	Conclusion
In addition to	Consequently	On the other hand	Similarly	Finally
Also	Thus	However	Also	In summary
For example	In order to	Whereas	Moreover	As shown above
Another key point	Provided that	Conversely	Additionally	To summarize

E. Reading

a) Read a paragraph about set theory.

The theory had the revolutionary aspect of treating infinite sets as mathematical objects that are on an equal footing with those that can be constructed in a finite number of steps. Since antiquity, a majority of mathematicians had carefully avoided the introduction into their arguments of the actual infinite (i.e., of sets containing an infinity of objects conceived as existing simultaneously, at least in thought). As this attitude persisted until almost the end of the 19th century, Cantor's work was the subject of much criticism to the effect that it dealt with fictions—indeed, that it encroached on the domain of philosophers and violated the principles of religion. Once applications to analysis began to be found, however, attitudes began to change, and by the 1890s Cantor's ideas and results were gaining acceptance. By 1900, set theory was recognized as a distinct branch of mathematics.

b) In pairs, discuss answers to these questions.

1. What is the main idea of the paragraph? What is the topic sentence?
2. Which linking words does the writer use to make the paragraph coherent?

F. Connectors. Try to supply the missing connectors in sentences.

- 1) Official attendance figures for the protest stand at 5,000. It is clear, _____, that the true figure is much higher.
- 2) _____ eating smaller fish, the tiger shark has been known to consume weaker members of his own family.
- 3) Some peoples, _____, the French, are renowned for the quality of their restaurants.
- 4) The company was declared bankrupt; _____, the CEO was imprisoned for fraud.
- 5) _____, on retirement, she began a second career as an author of detective novels.
- 6) Sales rapidly declined and, _____, Bailey Brothers were forced to close their factories.
- 7) Internet marketing can automatically measure its own success _____ websites provide convenient access data.
- 8) _____ understand the causes of the infection, a team of scientists began a series of laboratory tests.