$$\mathbb{y} = \sum_{i=1}^{m} c_i \mathbb{x}_i$$

$$\begin{pmatrix} \mathbb{x}_1 \,|\, \cdots \cdots \,|\, \mathbb{x}_m \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ \mathbb{y}_m \end{pmatrix}$$

$$\sum_{i=1}^{m} c_i x_i = 0$$

$X' = (X_{j_1}, \dots, X_{j_r})$ $\qquad X \sim Y$

$$X' \sim Y' = \left(\begin{array}{ccc|c} 1 & & 0 & \cancel{X} \\ & \ddots & & \\ 0 & & 1 & \end{array}\right)$$

&

$$X' \cdot \begin{pmatrix} c_1 \\ \vdots \\ c_r \end{pmatrix} = 0 \iff Y' \begin{pmatrix} c_1 \\ \vdots \\ c_r \end{pmatrix} = 0 \quad \boxed{\begin{array}{l} c_1 = 0 \\ c_2 = 0 \\ \vdots \\ c_r = 0 \end{array}}$$

$$(A \mid 0) \sim \left(\begin{array}{c} I_r \\ \hline 0 \end{array}\right)$$

$$(1, \ldots, 0)\begin{pmatrix} c_1 \\ \vdots \\ c_r \end{pmatrix} = 0$$

$$(0 \ldots 0)\begin{pmatrix} c_1 \\ \vdots \\ c_r \end{pmatrix}$$

$$X'' = (x_{j_1}, \dots, x_{j_l} \mid x_j) \qquad X \sim Y$$

$$X'' \sim Y'' = \left(\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & b_1 \\ 0 & 1 & & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & b_l \\ \hline & & 0 & & 0 \end{array}\right)$$

$$Y'' = (Y''' \mid b)$$

$$b_1 x_{j_1} + \dots + b_l x_{j_l} = x_j$$

$$[x_{j_1}, \ldots, x_{j_2}] = [x_1, \ldots, x_n]$$

$$\subseteq$$

$$\underbrace{u_0, u_1, \ldots, u_n}, \ldots$$

$(I) \iff (III)$

$$v = \sum_{i=1}^{m} c_i x_i \qquad x_i \in X$$

$X \cup \{v\}$ LZ $\iff$

$\{x_1, \dots, x_m, v\}$ LZ

$$\frac{W_1 \ldots W_m \qquad \mathrm{LN}\; u_1, \ldots, u_n}{[u_1 \ldots u_n, W_{n+1} \ldots W_m]} \qquad u_j \in [W_1, \ldots, W_m]$$

IB Steinitz. $\boxed{n = 0}$

$$[(v_1, \dots, v_m)] = [v_1, \dots, v_m]$$

$$n \Longrightarrow n+1 \qquad u_{n+1} \in$$

$$[(u_1, \dots, u_n, v_{n+1}, \dots, v_m)] = [v_1, \dots, v_m]$$

$$u_{n+1} \in [u_1, \dots, u_n, v'_{n+1}, \dots, v'_m]$$

$$u_{n+1} = \sum_{j=1}^{n} c_j u_j + \sum_{j=n+1}^{m} c_j v'_j$$

$$\exists\ n+1 \leq j\ m \quad c_j \neq 0$$

$$[u_1, \dots, u_n, u_{n+1}, v''_{n+2}, \dots, v''_m]$$

$(I) \Rightarrow (III)$ $\quad X = \{ x_1, \dots, x_m \}$

$\forall\ Y = (y_1, \dots\dots, y_m, \dots\dots)$
$\quad\ \ \|$

$[X] \ni y_1, \dots, y_{m+1}$ ; LN $\qquad$ SPOR

$$m+1 \leq m \Rightarrow 1 \leq 0$$

(II) ⟹ (I) SPOREM: Nezávislý zav. mn. g.

$\exists u_1 \neq 0 \in V$

$[u_1] \neq V$ $\exists u_2 \in V$

$u_2 \notin [u_1]$

$[u_1, u_2] \neq V$ $\exists u_3 \notin [u_1, u_2]$

LN

(a) $[\mathcal{T}_1, \ldots, \mathcal{T}_m]$

$\psi$

$\mathcal{U}_1, \ldots, \mathcal{U}_k$ LN

$[\mathcal{U}_1, \ldots, \mathcal{U}_k, \ldots, \mathcal{U}_m]$ LN

$\in \{\mathcal{T}_1, \ldots, \mathcal{T}_m\}$

(b) $$[\mathbb{V}_1, \dots, \mathbb{V}_n] = [\mathbb{V}_{j_1}, \dots, \mathbb{V}_{j_\ell}]$$

LN

$[W_1, \ldots, W_m] = V$

$dk\,(h)$ $\quad //$

$[W_{j_1}, \ldots, W_{j_l}]$

LN

$u_1, \ldots, u_m$

báze $V$

LN

Steinitz

$v_1, \ldots, v_n$

báze $V$,

generující

$$n \leq m$$

$$m \leq n \implies n = m$$

$\mathbb{C}$ nad $\mathbb{R}$ $\quad 1, i \quad \dim_{\mathbb{R}} \mathbb{C} = 2$

$\mathbb{C}$ nad $\mathbb{C}$ $\quad 1 \quad \underline{\dim_{\mathbb{C}} \mathbb{C} = 1}$

---

(i) + (ii) : JSEM BÁZE $\Longrightarrow$ $\underline{n = m}$.

$\Downarrow$

(iii)

$(i) + (iii) \Longrightarrow (ii)$

JSEM BÁZE

$(ii) + (iii) \Longrightarrow (i)$ ZŘEJMÉ (ZMENGENÍ!

1.6.

$$\Rightarrow x \in [u_1, \dots, u_n] = V$$

$$x = \sum_{j=1}^{n} c_j u_j = \sum_{j=1}^{n} d_j u_j$$

$$\sum_{j=1}^{n} (c_j - d_j) u_j = 0 \Rightarrow c_j - d_j = 0 \ \forall j$$

$$c_j = d_j$$

$\Leftarrow \quad x \in [u_1, \ldots, u_n]$

?LN

$$\sum_{j=1}^{m} c_j \cdot u_j = 0 = \sum_{j=1}^{m} 0 \cdot u_j$$

$$\Rightarrow c_j = 0$$

$$(-)_\alpha : \mathbb{V} \to \mathbb{K}^n$$

$$\mathbb{x} = \sum_{j=1}^{n} c_j \mathbb{u}_j \qquad (\mathbb{x})_\alpha = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$$

$$(\mathbb{x})_\alpha = (\mathbb{y})_\alpha \implies \mathbb{x} = \mathbb{y}$$

$$\mathbb{x} = \alpha \cdot (\mathbb{x})_\alpha = \alpha \cdot (\mathbb{y})_\alpha = \mathbb{y} \checkmark$$

$$\forall \mathbf{v} \in \mathbb{K}^n \; \exists \mathbf{x} \quad (\mathbf{x})_\alpha = \mathbf{v} \; ?$$

$$\boxed{\mathbf{x} = \alpha \cdot \mathbf{v} = \alpha \cdot (\mathbf{x})_\alpha}$$

$$\alpha \cdot (a\mathbf{x} + b\mathbf{y})_\alpha = a\mathbf{x} + b\mathbf{y}$$

$$\alpha \cdot \left( a(\mathbf{x})_\alpha + b(\mathbf{y})_\alpha \right) = a \cdot \alpha(\mathbf{x})_\alpha + b\,\alpha(\mathbf{y})_\alpha$$

$f = g^{-1}$ $\qquad$ $f \circ g = id$

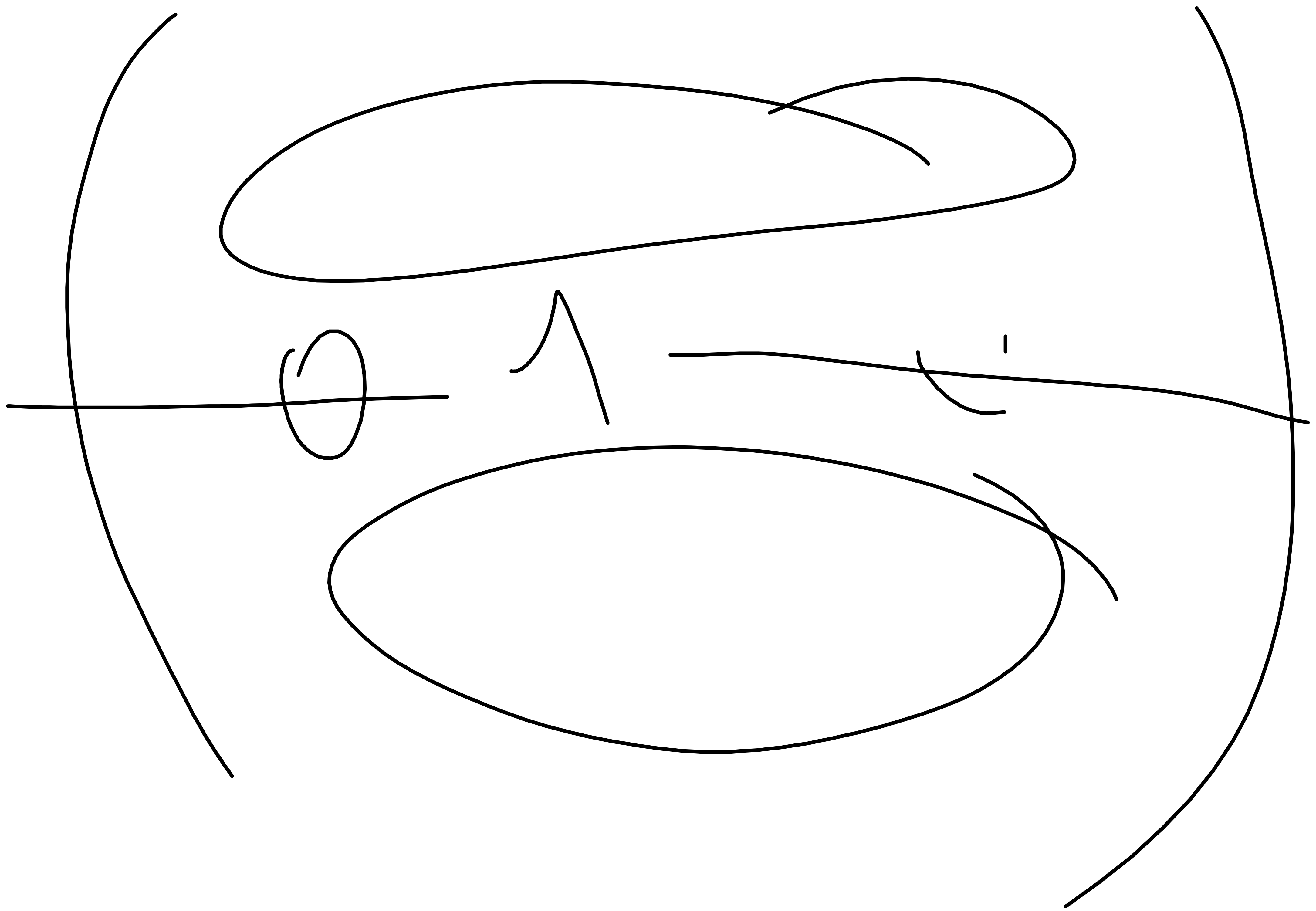
$g \circ f = id$

$\alpha(\mathbb{X})_\alpha = \mathbb{X}$ $\qquad$ $(-)_\alpha^{-1} \circ (-)_\alpha = id_{\mathbb{X}}$

$(\underbrace{\alpha \cdot \mathbb{C}}_{\mathbb{X}})_\alpha = \mathbb{C}$ $\qquad$ $(-)_\alpha \circ (-)_\alpha^{-1} = id_{\mathbb{K}^n}$

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \dots \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \dots + x_n \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$




$M, N \subseteq \Omega$

$|M \cup N| =$

$= |M| + |N| - |M \cap N|$

báze $S \cap T$ $\quad u_1, \dots, u_k, v_{k+1}, \dots, v_m, u_{k+1}, \dots, u_n$

$u_1 \dots u_k$

LN

báze $S$ $\quad$ báze $T$

$u_1 \dots, u_k, \dots, u_n \qquad u_1, \dots, u_k, v_{k+1}, v_m$

$$\sum_{j=1}^{k} c_j u_j + \sum_{j=k+1}^{m} c_j u_j + \sum_{l=k+1}^{m} d_j v_j = 0$$

$$\Leftrightarrow \quad u - \sum d_j v_j \in T$$

$$c_1 = c_2 = c_k = 0$$