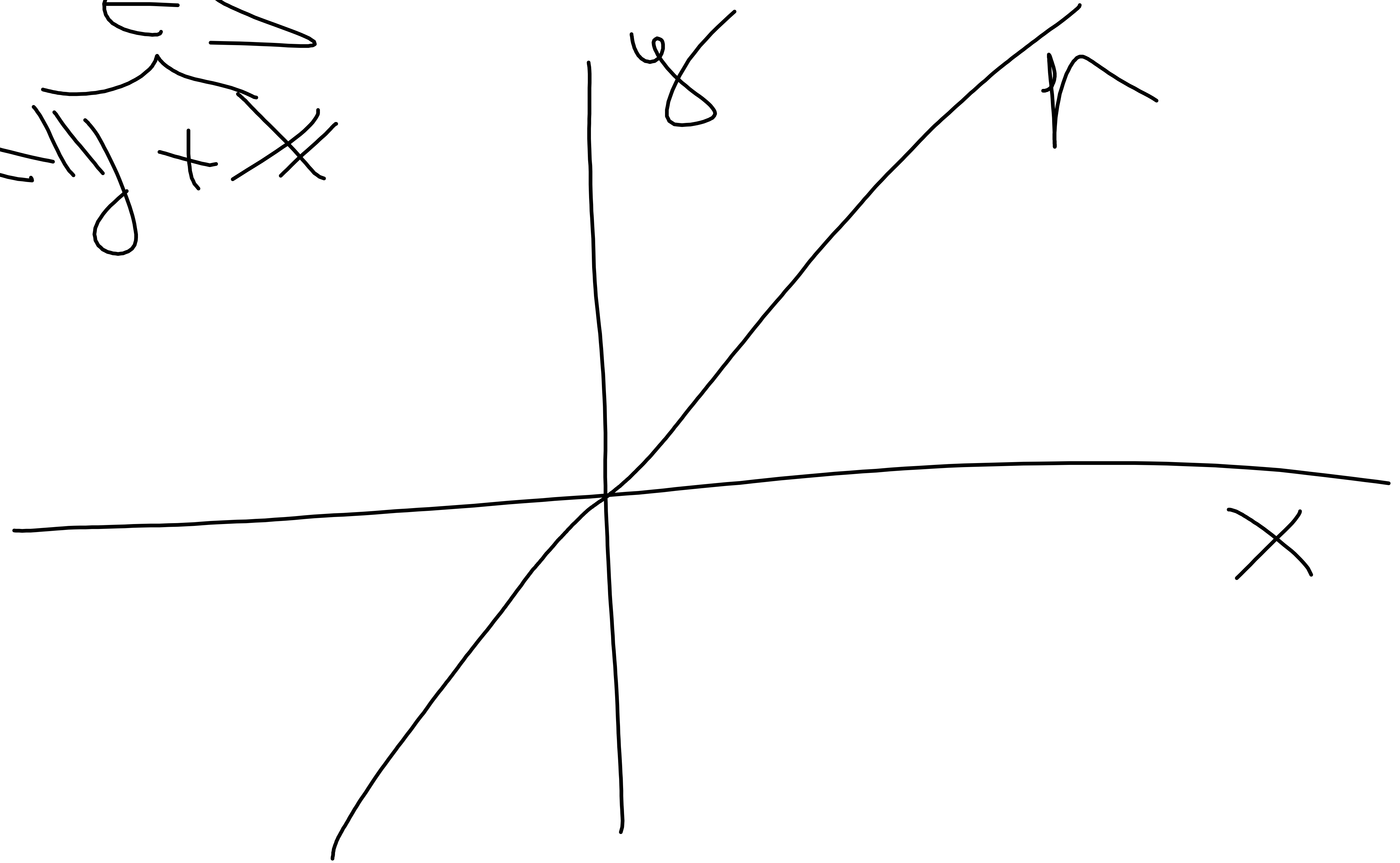
$$\overbrace{\underset{\in S}{\mathbb{X}}+\underset{\in S}{\mathbb{Y}}}^{\in S}=\overbrace{\mathbb{Y}+\mathbb{X}}^{\in S}$$



(I) $\Rightarrow$ (II) $S \neq \emptyset$ JASNÉ (II) $\Rightarrow$ (III)

$$\{\underbrace{\underbrace{a}_{\in K} \underbrace{x}_{\in S}}_{\in S} + b \underbrace{y}_{\in S}\} \in S \qquad \sum_{i=1}^{n=0} a_i x_i := 0 \in S$$

$$\exists u \in S \qquad \underbrace{1 \cdot u + (-1) u}_{\in S} = 0 \cdot u = 0 \in S$$

$n \implies n+1$:

$a_1, \ldots, a_{n+1} \in \mathbb{K}$, $x_1, \ldots, x_{n+1} \in S$

$$\sum_{i=1}^{n+1} a_i x_i = \underbrace{\left(\sum_{i=1}^{n} a_i x_i\right)}_{\in S} + \underbrace{a_{n+1} x_{n+1}}_{\in S} \in S$$

(iii) $\Rightarrow$ (i)

$n=0$ $\quad \sum_{i=1}^{0} a_i x_i = 0 \in S$

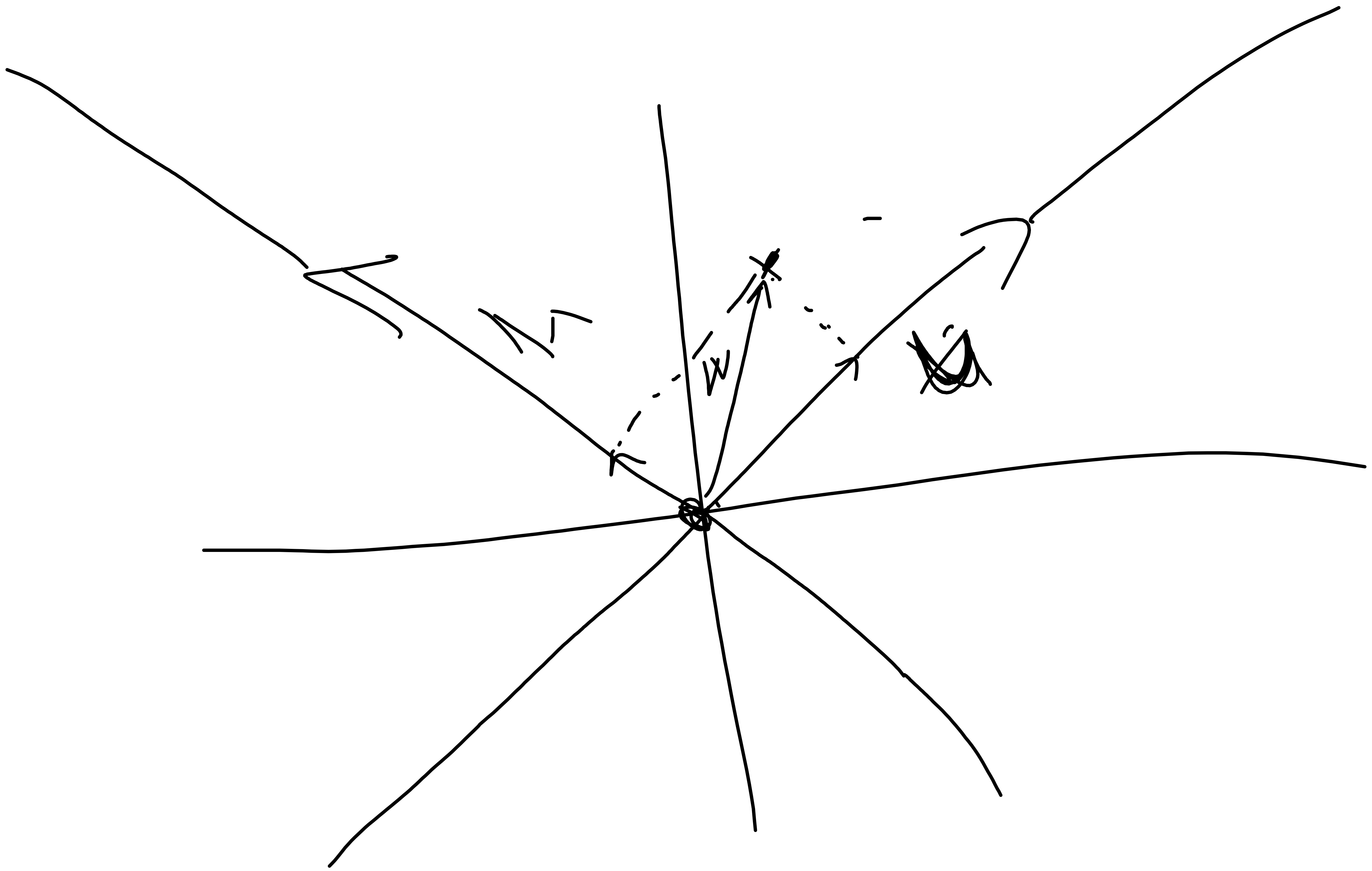
$n=1$ $\quad x \in S$, $a \in \mathbb{K}$ $\Rightarrow$ $ax \in S$

$\underline{n=2}$ $\quad x, y \in S$ $\quad x+y = 1 \cdot x + 1 \cdot y \in S$

$\mathbb{K}^X \quad \underline{\underline{VP}}$

$\mathbb{K}^{(X)} = \left\{ f \in \mathbb{K}^X : \underset{\text{je konečná}}{\{x \in X : f(x) \neq 0\}} \right\}$

$2^n \quad 3^n$



$$X \subseteq W \qquad \Big(\sum^{m} a_i \underbrace{x_i}_{\in X}\Big) + \Big(\sum^{m} b_j \underbrace{y_j}_{\in X}\Big) \in [X]$$

$$[X] = \Big\{ \sum_{i=1}^{n} a_i x_i : n \in \mathbb{N},\ a_1, \dots, a_n \in K,\ x_1, \dots, x_n \in X \Big\}$$

$$n = 0 \qquad \sum_{i=1}^{n} a_i x_i = 0 \in [X]$$

$$c \cdot \Big(\sum a_i x_i\Big) = \sum (c \cdot a_i)\, x_i \in [X],$$

$$x \in X$$

$$x = 1 \cdot x \in [X]$$

$$u \in [X], \quad u = \sum_{i=1}^{m} \overset{\in Y}{a_i} \underset{\in X}{x_i} \in [Y]$$

(d) X VR $\Longrightarrow X = [X]$

$$\subseteq$$

$u \in [X]$

$$u = \sum_{i=1}^{n} a_i x_i \in [X]$$

$\in X \qquad \in X$

2.1 $X \subseteq S$, S i VP $\overset{?}{\Rightarrow}$ $[X] \subseteq S$

$u \in [X]$, $u = \sum_{i=1}^{n} a_i \underbrace{\overset{\in X}{x_i}}_{\in S} \in \underline{\underline{S}}$

2.3. (e) $[X] \subseteq [[X]]$

Zbytek pomocí (d).

$$\Rightarrow: w \in [X] \overset{?}{\Rightarrow} [X \cup \{w\}] = [X]$$

$$\supset$$

$$u \in [X \cup \{w\}]$$

$$u = \sum_{i=1}^{n} a_i \underbrace{x_i}_{\in X} + b\, w = \underbrace{\sum_{i=1}^{m} a_i x_i + b \sum_{j=1}^{m} c_j \underbrace{y_j}_{\in X}}_{\in [X]}$$

$S_\alpha,\ \alpha \in \Lambda$ VP $\Rightarrow \underset{\alpha\in\Lambda}{\overset{0 \in}{\bigcap}} S_\alpha$ ist VP

$\overset{\in}{0}$

$x, y \in \bigcap_{\alpha\in\Lambda} S_\alpha \in S_\beta\ \forall \beta \in \Lambda$

$a, b \in \mathbb{K}$ $\overset{?}{\Longrightarrow} a \cdot x + b y \in \bigcap S_\alpha$

$\in S_\beta\ \forall \beta$

$$S+T = \{ x+y : x \in S,\ y \in T \}$$

$$0 \in S \quad 0 \in T$$

$$0+0 = 0 \in S+T$$

$$a\cdot(x+y) = ax + ay \in S+T$$

$$\underset{\in S}{ax} \quad \underset{\in T}{ay}$$

$$(\overset{\in S}{x_1} + \overset{\in T}{y_1}) + (\overset{\in S}{x_2} + \overset{\in T}{y_2}) = (\overset{\in S}{x_1 + x_2}) + (\overset{\in T}{y_1 + y_2}) \in S+T$$

$S \subseteq S+T$, $T \subseteq S+T$

s

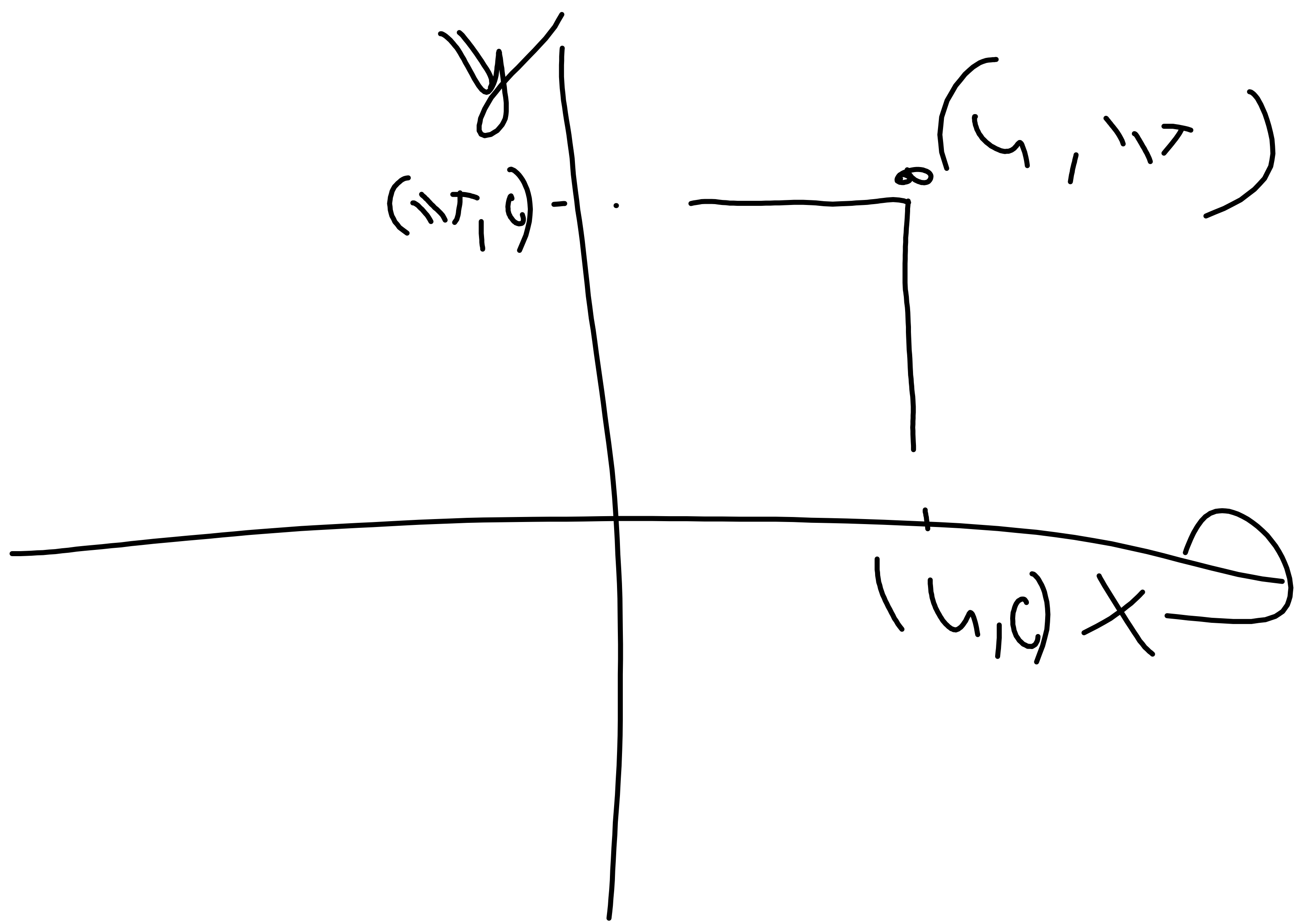
$s = s + 0 \in S+T$

$S \cup T \subseteq S+T$ $\quad S+T \subseteq [S \cup T]$

Różne

$\parallel$

$[S \cup T] \subseteq S+T$, $\quad 1 \cdot s + 1 \cdot t$



y
(4,0)
x

$\Rightarrow$: $r \in S+T$

$$r = \underset{\in S}{x} + \underset{\in T}{y} = \underset{\in S}{x'} + \underset{\in T}{y'}$$

$$0 = \underset{\substack{\cap \\ S}}{x - x'} = \underbrace{y' - y}_{\in T} = 0$$

$\Leftarrow$: $\mathbb{R} \in S \cap T \subseteq S, T$

$$\mathbb{R} = \underset{\in S}{\mathbb{R}} + \underset{\in T}{0} = \underset{\in S}{0} + \underset{\in T}{\mathbb{R}}$$

$\Rightarrow \mathbb{R} = 0$

(a) $u \neq 0$

$c \cdot u = 0 \Rightarrow \boxed{c = 0}$

$u = 2v$

$1 \cdot u + (-2) v = 0$

(b) u, v L Z $\Rightarrow (c, d) \neq (0, 0)$

$c u + d v = 0 \quad | \frac{1}{c} \quad c \neq 0$

$u + \frac{d}{c} v = 0 \qquad u = \left(-\frac{d}{c}\right) v$

(c)

$$\begin{array}{cccc} & w_1 & \ldots & , w_n \\ & \downarrow & \| & \downarrow \\ & & 0 & \\ & & \downarrow & \\ x: & 0 & 1 & 0 \end{array}$$

(d)

$$\begin{array}{cccc} u_1, \dots, & u_k, \dots & u_l, \dots & u_n \\ \downarrow & & & \downarrow \\ 0 & 1 & (-1) & 0 \end{array}$$

$(I) \Rightarrow (II)$ $\quad u_1, \dots, u_n$ LZ

$\exists (c_1, \dots, c_n) \neq 0$

$$\sum_{i=1}^{n} c_i u_i = 0$$

$l = \max\{i : c_i \neq 0\}$

$$\sum_{i=1}^{l} c_i u_i = 0$$

$$u_l = -\frac{1}{c_l} \sum_{i=1}^{l-1} c_i u_i$$

$j > l$

$\Rightarrow c_j = 0$

$(I) \Rightarrow (II)'$: ZREJME, $(II) \Rightarrow (III)$, $(II)' \Rightarrow (III)$

(iii) $\Rightarrow$ (i) $\quad u_\ell = \sum_{\substack{i=1 \\ i \neq \ell}}^{m} c_i u_i$

$$\sum_{i=1}^{m} c_i u_i = 0, \quad c_\ell = -1$$

$\Rightarrow$: $\underline{u_1 \ldots u_n \text{ LN}}$

$x \in [u_1, \ldots, u_n]$

$x = \sum_{i=1}^{m} c_i u_i = \sum_{i=1}^{m} d_i u_i$

$\Rightarrow \sum_{i=1}^{m} (c_i - d_i) u_i = 0 \Rightarrow c_i - d_i = 0$

$c_i = d_i$

$\Uparrow$

$\forall i$

$$\Leftarrow \quad \sum_{i=1}^{m} c_i \cdot u_i = 0 = \sum_{i=1}^{r} 0 \cdot u_i \in [u_1, .. u_r]$$

$$c_i = 0 \quad \forall i$$

$u_1, \dots, u_n, v_1, \dots, v_m$

1. $v_1 \overset{?}{\in} [u_1, \dots, u_n]$

ANO

$[u_1, \dots, u_n] = [u_1, \dots, u_n, v_1]$

NE : $\Rightarrow$ $u_1, \dots, u_n, v_1$ LN