↓ Poslat odtud

7. $D_n = \begin{pmatrix} a+1 & a & & 0 \\ 1 & a+1 & a & \\ & 1 & \ddots & a \\ 0 & & 1 & a+1 \end{pmatrix} \sim (-1)^{n+n-1} \det \begin{pmatrix} a+1 & a & & \\ 1 & a+1 & a & \dots \\ & 1 & & \vdots \\ & 0 & & 0 \\ 0 & 0 & 0 \dots & a \end{pmatrix} + (a+1)(-1)^{2n} \cdot D_{n-1}$

→ není D_n

Laplace dle posledního ř.

dle 1. sčítku $= (a+1)(-1)^{1+1} D_{n-1} + a(-1)^{2+1} \det($

rozvoj dle posl. sloupce $a \cdot (-1)^{n-1+n-1} D_{n-2}$

$D_n = (a+1) D_{n-1} - a D_{n-2}$

$D_1 = a+1$

$D_2 = \det\begin{pmatrix} a+1 & a \\ 1 & a+1 \end{pmatrix} = (a+1)^2 - a = a^2+a+1$

$D_3 = (a+1)(a^2+a+1) - a(a+1) = a^3+a^2+a+a^2+a+1-a^2-a = a^3+a^2+a+1$

Hypotéza $D_n = a^n + a^{n-1} + \dots + a + 1$

Dokažme $D_n = (a+1)\sum_{i=0}^{n-1} a^i - a \cdot \sum_{i=0}^{n-2} a^i = (a^n + a^{n-1} + \dots + a) + (a^{n-1} + \dots + 1) - (a^{n-1} + \dots + a) = D_n$

Př 6.: $D(a_1, \dots, a_n) = \det \begin{pmatrix} a_1+x & x & x & & x \\ x & a_2+x & x & \dots & x \\ x & x & a_3+x & & \vdots \\ x & \vdots & & \ddots & x \\ x & x & \dots & & a_n+x \end{pmatrix}$

$D(a_1) = a_1 + x$

$D(a_1, a_n) = (a_1+x)(a_2+x) - x^2 = a_1a_2 + a_1x + a_2x$

$D(a_1,a_2,a_3) = \begin{vmatrix} a_1+x & x & x \\ x & a_2+x & x \\ x & x & a_3+x \end{vmatrix} = (a_1+x)(a_2+x)(a_3+x) + x^3 + x^3 - x^2(a_2+x) - x^2(a_1+x) - x^2(a_3+x)$

$= a_1a_2a_3 + a_1a_2x + a_1a_3x + a_2a_3x + a_1x^2 + a_2x^2 + a_3x^2 + x^3 + x^3 + x^3 - x^2a_2 - x^3 - x^2a_1 - x^3 - x^2a_3 - x^3 =$

$= a_1a_2a_3 + a_1a_2x + a_1a_3x + a_2a_3x$

odečtu poslední od všech:

$\det\begin{pmatrix} a_1 & 0 & 0 & & -a_n \\ 0 & a_2 & 0 & & -a_n \\ 0 & 0 & a_3 & & -a_n \\ & & & \ddots & -a_n \\ x & x & x & x \dots & a_n+x \end{pmatrix}$

$D(a_1, \dots, a_n) = \prod_{i=1}^{n} a^i + \sum_{i=1}^{n}$

rozvoj 1. sčítku $\rightarrow a_2 \cdot \dots \cdot a_{n-1}) \cdot (-1)^{n-1+1}$

Podle 1. sloupce $a_1 \cdot (-1)^{1+1} \det D(a_2, \dots, a_n) + x(-1)^{n+1} \det \begin{pmatrix} 0 & 0 & 0 & \dots & -a_n \\ a_2 & 0 & \dots & & -a_n \\ & a_3 & & & \\ & & & a_{n-1} & -a_n \\ 0 & 0 & & & -a_n \end{pmatrix}$

$D(a_1, \dots a_n) = a_1 D(a_2, \dots a_n) + x a_2 a_3 \cdots a_n = a_1 a_2 D(a_3, \dots, a_n) a_1 x a_3 a_4 \cdots a_n + x a_2 a_3 \cdots \dots \rightarrow$

$D(a_1, \dots, a_n) = D(a_{n+1}, \dots, a_n)$

$= a_1 \cdots a_k \, D(a_{k+1}, \dots, a_n) + x a_2 \cdots a_n + a_1 x a_3 \cdots a_n + \dots + a_1 \cdots a_{k-1} x a_{k+1} \cdots a_n$

$= a_1 a_2 \cdots a_{n-1}(a_n + x) + x a_2 \cdots a_n + a_1 \cdots a_{n-2} x a_n$ ✓

3. sloupec: $x \quad \dots \quad x \quad \dots \quad x(-1)^{n+3}$
$$\left|\begin{array}{cc|ccc} a_1 & & & & \\ & a_2 & & & \\ \hline & & 0 & & -a_n \\ & & a_4 & & \\ & & & \ddots & \\ & & & & a_{n-1} \end{array}\right.$$

$\to a_1 a_2 \times a_3 a_4 \dots$

$x(-1)^{n+3} a_1 a_2 \dots (-a_n)(-1)^{n-3+1} a_4 \dots$

$A = (a_{ij}), \ \det A \neq 0$

$A^{-1} = (b_{ij}), \ b_{ij} = \dfrac{\tilde{a}_{ji}}{\det A}$

$A^{-1} = \dfrac{1}{\det A} (\tilde{a}_{ij})^T$

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \det A = 4 - 6 = -2$

$\tilde{a}_{11} = (-1)^{1+1} \cdot 4 = 4 \quad \tilde{a}_{12} = -3 \quad \tilde{a}_{21} = -2 \quad \tilde{a}_{22} = 1$

$\tilde{A} = \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} \quad A^{-1} = \frac{1}{-2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \quad A^{-1} \cdot A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix} \quad \det A = 6 + 6 + 6 - 27 - 8 - 1 = 18 - 27 - 8 - 1 = -18$

$\tilde{a}_{11} = 1 \cdot 5 \quad \tilde{a}_{12} = -1 \cdot 1 = -1 \quad \tilde{a}_{13} = 1 \cdot (-7) = -7$

$\tilde{a}_{21} = -1 \cdot 1 \quad \tilde{a}_{22} = 1 \cdot (-7) = -7 \quad \tilde{a}_{23} = -1 \cdot (-5) = 5$

$\tilde{a}_{31} = 1 \cdot (-7) \quad \tilde{a}_{32} = -1 \cdot (-5) = 5 \quad \tilde{a}_{33} = 1 \cdot (-1)$

$\tilde{A} = \begin{pmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & 1 \end{pmatrix} = \tilde{A}^T$

$A^{-1} = \tilde{A}^T \cdot \frac{1}{\det A} = \tilde{A} \cdot \frac{1}{-18}$

$A \cdot A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Cv. 13 DÚ: Pomocí algebr. doplňků i Gauss. inverzní mat.

2\. $n \times n$ $\begin{pmatrix} 1 & x & \cdots & \\ 0 & 1 & x & \vdots \\ \cdots & & & x \\ \cdots & & & 1 \end{pmatrix}$ $\to \tilde{a}_{21}, \tilde{a}_{12}$

Teorie ze zkoušky: Napište def. determinantu...

$U = \{A \in Mat_{2\times2}(\mathbb{R}),$ řádky A LZ$\}$

Je U VPP? Bod za zdův.

$$\underset{\in U}{\begin{pmatrix} a_1 & a_2 \\ ka_1 & ka_2 \end{pmatrix}} + \underset{\in U}{\begin{pmatrix} b_1 & b_2 \\ nb_1 & nb_2 \end{pmatrix}} = \underset{\notin U}{\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ ka_1+nb_1 & ka_2+nb_2 \end{pmatrix}} \qquad \begin{matrix} a_1+b_1 \\ k(a_1+b_1)+(n-k)(b_1) \end{matrix}$$

$$m \cdot \begin{pmatrix} a_1 & a_2 \\ ka_1 & ka_2 \end{pmatrix} = \begin{pmatrix} ma_1 & ma_2 \\ mka_1 & mka_2 \end{pmatrix} \leftarrow \text{stále LZ}$$

Mat 3×3 s nulovým det.

4\. $\varphi: \mathbb{R}^{50} \to Mat_{7\times7}(\mathbb{R})$

$\psi\ Mat_{7\times7} \to \mathbb{R}^{60}$

$\Big\}\ \mathbb{R}^{50} \to \mathbb{R}^{60}$ Může být složené zobr. $\psi \circ \varphi$ prosté?

Ne, protože dim $Mat_{7\times7} = 49$

Prostota souvisí s jádrem

Prosté $\Rightarrow$ (nutná podmínka) dim ker $\varphi = 0 \Rightarrow$ dim im $\varphi = 50 \Rightarrow$ Nelze, protože

dim im $\varphi \leq$ dim $Mat_{7\times7}$

$\to$ dim ker $\varphi \neq 0 \Rightarrow$ Není prosté!!

dim ker φ = dim $\mathbb{R}^{50}$ − dim im φ

ker $\psi \circ \varphi \neq \{0\} \Rightarrow$ nemůže být prosté

Vzorec pro dim. jádra a obrazu

5. Předpis lin zob

$\mathbb{R}^{20} \to R_9[x] \quad \varphi(a_1 a_2, \dots, a_{20}) = \dots$ tak, že

$\dim \operatorname{im} \varphi = 5, \quad \varphi(1,0,1,0,\dots,1,0) = x^5$

Napište bázi obrazu

$\cancel{(a_1 - a_3 + a_5 - a_7 + a_9 - a_{11} + a_{13} - a_{15} + a_{17} - a_{19})} \, x^5$

$a_1 x^5 + a_2 x^4 + a_4 x^3 + a_6 x^2 + a_7 x$

báze $[x^5, x^4, x^3, x^2, x]$

6. $R_1[x] \quad \alpha = (p_1, p_2)$
$\beta = (q_1, q_2)$

$p_1(x) = 1 - 2x \qquad (id)_{\beta,\alpha} = \begin{pmatrix} 2 & -7 \\ 3 & 4 \end{pmatrix}$
$q_2(x) = 3 - 2x$

Najděte $p_2(x)$ a $q_1(x)$

$p_1 = 2q_1 + 3q_2$

$1 - 2x = 2q_1 + 3(3 - 2x) \Rightarrow q_1(x) = 2x - 4$

$p_2(x) = -7q_1 + 4q_2 = -7(2x-4) + 4(3-2x) = -22x + 40$

Platí pro každé 2 čtvercové matice A, B

$(A+B)^2 = A^2 + 2AB + B^2$

$AB + BA$

$A^2 = \begin{pmatrix} a_1^2 + a_2 a_3 & \\ & \end{pmatrix} \quad 2AB = 2 \cdot \begin{pmatrix} a_1 b_1 + a_2 b_3 & \\ & \end{pmatrix} \quad B^2 = \begin{pmatrix} b_1^2 + b_2 b_3 & \\ & \end{pmatrix}$

$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$

$\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix} \circ \begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix} = \begin{pmatrix} (a_1+b_1)^2 + (a_2+b_2)(a_3+b_3) & \\ & \end{pmatrix}$

$A=\begin{pmatrix}1&1\\0&0\end{pmatrix}$ $B=\begin{pmatrix}3&2\\1&0\end{pmatrix}$ $(A+B)^2=\begin{pmatrix}4&3\\1&0\end{pmatrix}^2=\begin{pmatrix}19&12\\4&3\end{pmatrix}$

$A^2=\begin{pmatrix}1&1\\0&0\end{pmatrix}$

$\neq$

$2AB=\begin{pmatrix}4&2\\0&0\end{pmatrix}\cdot 2$ $A^2+2AB+B^2$

$B^2=\begin{pmatrix}11&6\\3&2\end{pmatrix}$

Proč to neplatí?

$$(A+B)\cdot(A+B)=AA+BA+AB+BB$$ a obecně se $AB\neq BA$

$\alpha=(u_1,u_2,u_3)$

$v=\begin{pmatrix}1\\0\\0\end{pmatrix}$ $(v)_\alpha=\begin{pmatrix}1\\1\\1\end{pmatrix}\Rightarrow v=u_1+u_2+u_3$

$z=\begin{pmatrix}1\\1\\1\end{pmatrix}$ $(z)_\alpha=\begin{pmatrix}1\\0\\0\end{pmatrix}\Rightarrow z=u_1$

$u_1=\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}+1\underbrace{\begin{pmatrix}?\\ \cdot\\ \cdot\end{pmatrix}}_{u_2}+1\underbrace{\begin{pmatrix}?\\ \cdot\\ \cdot\end{pmatrix}}_{u_3}$$

např $\begin{pmatrix}0\\-1\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\-1\end{pmatrix}$