1 Př. Najděte kolineaci převádějící kružnici $(x^1)^2+(x^2)^2=1$ na parabolu $(y^1)^2+4y^2=0$

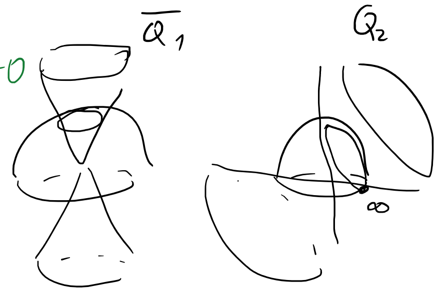
1) převést do homog. souř.: $(x^0 : x^1 : x^2) = \left(1 : \frac{x^1}{x^0} : \frac{x^2}{x^0}\right)$
a dosadíme do rovnice
a vynásobíme $(x^0)^2$

kružnice Q_1:

$$\left(\frac{x^1}{x^0}\right)^2 + \left(\frac{x^2}{x^0}\right)^2 = 1 \qquad / \cdot (x^0)^2$$

$$\overline{Q}_1 : (x^1)^2 + (x^2)^2 = (x^0)^2 \quad \leftarrow \quad \text{homogenizace}$$

lépe: $-(x^0)^2 + (x^1)^2 + (x^2)^2 = 0$



matice:

$$\begin{array}{c|ccc} & x^0 & x^1 & x^2 \\ x^0 & \multicolumn{3}{c}{\multirow{3}{*}{$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$}} \\ x^1 & & & \\ x^2 & & & \end{array} = A_1$$

parabola Q_2:

$$\left(\frac{y^1}{y^0}\right)^2 + 4\frac{y^2}{y^0} = 0 \qquad / \cdot (y^0)^2$$

$$\overline{Q}_2 : (y^1)^2 + 4y^0y^2 = 0 \quad \leftarrow \quad \text{trik: } (y^1)^2 + \underbrace{(y^0+y^2)^2 - (y^0-y^2)^2}_{4y^0y^2} = 0$$

$$\| $$

$$(x^1)^2 + (x^2)^2 - (x^0)^2 = 0$$

matice:

$$\begin{array}{c|ccc} & y^0 & y^1 & y^2 \\ y^0 & 0 & 0 & 2 \\ y^1 & 0 & 1 & 0 \\ y^2 & 2 & 0 & 0 \end{array} = A_2$$

$\rightarrow$ diagonalizace

Odpověď?
$x^0 = y^0 - y^2$
$x^1 = y^1$
$x^2 = y^0 + y^2$

$$(y^0 : y^1 : y^2) \in \overline{Q}_2 \implies (x^0 : x^1 : x^2) \in \overline{Q}_1$$

$$\| $$

$$(y^0 - y^2 : y^1 : y^0 + y^2)$$

Φ kolineace $(y^0 : y^1 : y^2) \longmapsto (y^0 - y^2 : y^1 : y^0 + y^2)$

$\| $

$[\varphi]$, $\varphi = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} y^0 \\ y^1 \\ y^2 \end{pmatrix}$ $\quad \mathcal{P}_2 = \overline{A}_2 \longrightarrow \overline{A}_2 = \mathcal{P}_2$

$$\Phi(\overline{Q}_2) = \overline{Q}_1 \qquad \text{parabola na kružnici}$$

Převeďme do afinních souřadnic

$$(1 : y^1 : y^2) \longmapsto (1 - y^2 : y^1 : 1 + y^2) \underset{\cdot \frac{1}{1-y^2}}{=} \left(1 : \frac{y^1}{1-y^2} : \frac{1+y^2}{1-y^2}\right)$$

$$[y^1, y^2] \longmapsto \left[\frac{y^1}{1-y^2}, \frac{1+y^2}{1-y^2}\right]$$

$$\left(\frac{y^1}{\ }\right)^2 + \left(\frac{1+y^2}{\ }\right)^2$$

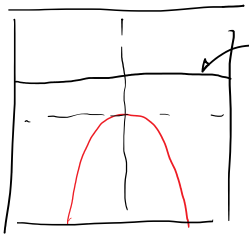
$$[y^1, y^2] \longmapsto \left[\frac{y^1}{1-y^2}, \frac{1+y^2}{1-y^2}\right]$$

$\Uparrow$ parabola

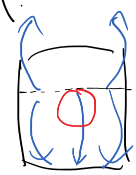
$\uparrow$ leží na kružnici ?

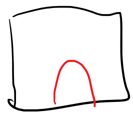
$$\left(\frac{y^1}{1-y^2}\right)^2 + \left(\frac{1+y^2}{1-y^2}\right)^2 = \frac{(y^1)^2 + 1 + 2y^2 + (y^2)^2}{1 - 2y^2 + (y^2)^2} = 1$$

leželo to na parabole $\longrightarrow$ $(y^1)^2 = -4y^2$

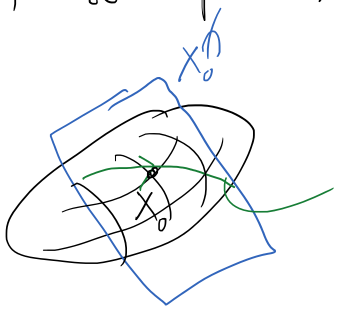


není def.





2 Př. Ukažte, že pro $X_0 \in Q$ je $X_0^{\pitchfork}$ tečnou nadrovinou Q v b. X_0.



$X: \mathbb{R} \longrightarrow A_3$, $X(0) = X_0$

t.ž. $X(t)$ splňuje rovnici

$X'(0)$ splňuje ??

$\hookrightarrow X'(0) \pitchfork X_0$

$X: \mathbb{R} \longrightarrow \mathbb{R}^{n+1}$

$X^0(t) = 1$, $X^1(t)$, …, $X^n(t)$ fce.

$$Q: \sum_{i,j=0}^{n} a_{ij} \cdot \underline{X^i(t) \cdot X^j(t)} = 0 \qquad \text{platí} \iff X(t) \in Q$$

$\longrightarrow$ derivací v $t = 0$

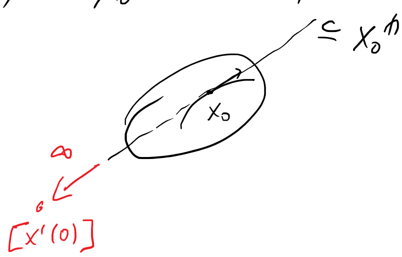
$$\sum a_{ij} \cdot (X^{i\prime}(0) \cdot X^j(0) + X^i(0) \cdot X^{j\prime}(0)) = 0$$

$$(X^{0\prime}(0) \; \dots \; X^{n\prime}(0)) \; A \cdot \begin{pmatrix} X^0(0) \\ \vdots \\ X^n(0) \end{pmatrix} + \underset{\text{ta samá hodnota}}{\text{sym}} = 0$$

$$\Rightarrow [X'(0)] \pitchfork [\underset{X_0}{X(0)}]$$

$\uparrow$ tečné vektory v X_0 jsou pol. sdr. s X_0

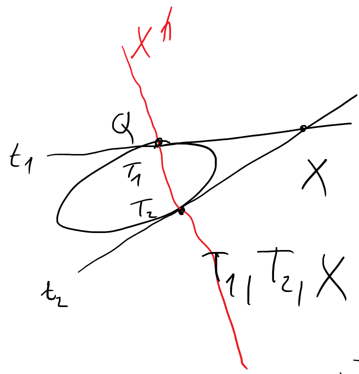
$$X_0 \pitchfork X_0 \quad \Rightarrow \quad X_0 + t \cdot X'(0) \pitchfork X_0$$



3 Př. Najděte tečny kuželosečky

$$Q: \; 2(x^1)^2 - 4x^1x^2 + (x^2)^2 \underbrace{-2x^1}_{-2x^0x^1} + 6x^2 \underbrace{-3}_{-3(x^0)^2} = 0$$

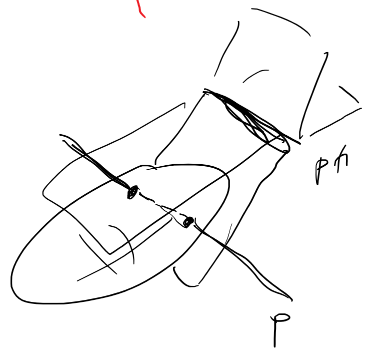
procházející bodem $[3,4]$.



$T_1 \pitchfork T_1$ ... protože $T_1 \in Q$

$T_2 \pitchfork T_2$

$$X \in T_1^\pitchfork \iff \begin{matrix} X \pitchfork T_1 \\ X \pitchfork T_2 \end{matrix} \implies X^\pitchfork = p$$



Postup: spočítáme $\underline{X^\pitchfork}$ a jeho průsečíky s Q, tj. $T_1, T_2 \rightsquigarrow t_i = \overleftrightarrow{T_iX}$

$$X \pitchfork \underset{(1,\,x^1,\,x^2)}{T}^{\in X^\pitchfork} \iff (1,3,4)\cdot A \cdot \begin{pmatrix} 1 \\ x^1 \\ x^2 \end{pmatrix} = 0 \leftarrow \text{rovnice } X^\pitchfork$$

$$A = \begin{pmatrix} -3 & -1 & 3 \\ -1 & 2 & -2 \\ 3 & -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 6 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x^1 \\ x^2 \end{pmatrix} = 0$$

$$6 - 3x^1 + x^2 = 0 \qquad (1)$$

$\to$ průsečíky s Q: $\quad 2(x^1)^2 - 4x^1x^2 + (x^2)^2 - 2x^1 + 6x^2 - 3 = 0 \qquad (2)$

$\overset{(1)}{\implies} \quad x^2 = 3x^1 - 6 \quad$ dosadíme do (2)

$$2(x^1)^2 - 4x^1(3x^1-6) + (3x^1-6)^2 - 2x^1 + 6(3x^1-6) - 3 = 0$$

$$-(x^1)^2 + 4\cdot x^1 - 3 = 0$$

$$(x^1)^2 - 4x^1 + 3 = 0$$

$$(x^1-1)(x^1-3) = 0$$

$T_1 = (1,1,-3) \implies \overleftrightarrow{XT_1} = t_1$

$T_2 = (1,3,3) \implies \overleftrightarrow{XT_2} = t_2$

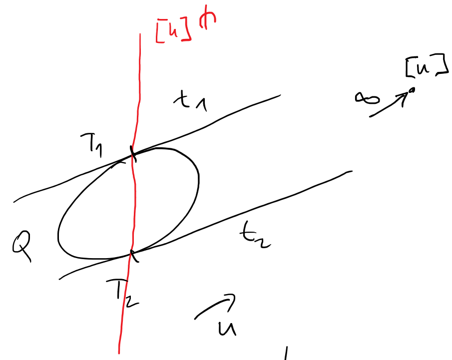
4

Př. Určete tečny kuželosečky

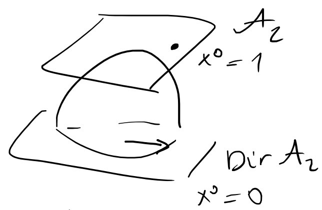
$$\overset{4x^0x^1}{4x^1} + 2x^2 - 4x^1x^2 - 4 = 0$$

rovnoběžné se směrem vektoru

$u = (1,2)$.



$$A = \left(\begin{array}{c|cc} -4 & 2 & 1 \\ \hline 2 & 0 & -2 \\ 1 & -2 & 0 \end{array}\right)$$



opet. $[u] \in T_i^{\perp}$

$[u] \cap T_i$

$$[u]^{\perp}: \quad \underbrace{\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -4 & 2 & 1 \\ 2 & 0 & -2 \\ 1 & -2 & 0 \end{pmatrix}} \begin{pmatrix} 1 \\ x^1 \\ x^2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 4 & -4 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ x^1 \\ x^2 \end{pmatrix} = 0$$

$$4 - 4x^1 - 2x^2 = 0$$

$$2 - 2x^1 - \underline{\underline{x^2}} = 0 \Rightarrow x^2 = 2 - 2x^1$$

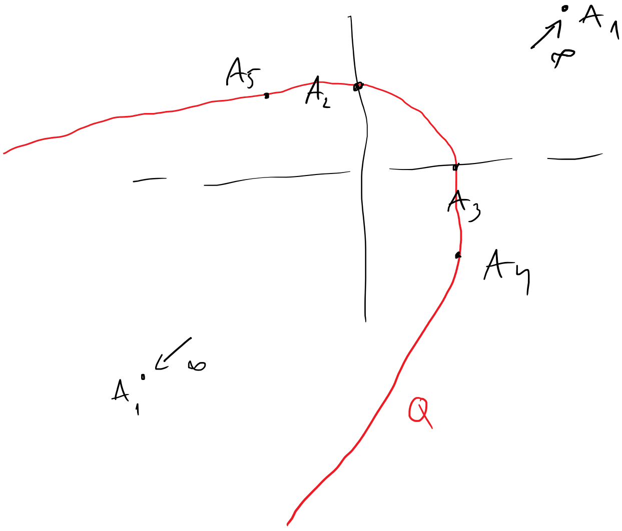
$$4x^1 + 2(2 - 2x^1) - 4x^1(2 - 2x^1) - 4 = 0$$

$$8(x^1)^2 - 8x^1 = 0$$

$$x^1(x^1 - 1) = 0 \quad < \quad \begin{array}{l} (1,0,2) = T_1 \to t_1 = T_1 + [u] \\ (1,1,0) = T_2 \to t_2 = T_2 + [u] \end{array}$$

5 **Př.** Určete kuželosečku procházející body

$A_1 = (0:1:1)$, $A_2 = [0,1]$, $A_3 = [1,0]$, $A_4 = [1,-1]$, $A_5 = (1:-1:1) = [-1,1]$



$\underline{\underline{Q}}$: $a\cdot(x^1)^2 + b\cdot(x^2)^2 + c\cdot x^1\cdot x^2 + d x^1 \underset{x^0}{\wedge} + e x^2 \underset{x^0}{\wedge} + f \underset{(x^0)^2}{\wedge} = 0$

hledáme a,b,c,d,e,f tak, aby

$A_1, \dots, A_5 \in Q$

$A_2 \in Q$: po dosazení $x^1 = 0$, $x^2 = 1$ musí platit

$b + e + f = 0$

$A_3 \in Q$: $a + d + f = 0$

$A_4 \in Q$: $a + b - c + d - e + f = 0$

$A_5 \in Q$: $a + b - c - d + e + f = 0$

$A_1 \in Q$: $a + b + c = 0$

$$\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

L body jsou v tzv. "obecné poloze" $\Rightarrow$ hodnost 5

$\Rightarrow$ řešení ex. jediné až na násobek

$\Rightarrow$ Q je jediná

Pozn.

– pro kubické křivky

6 Př. Určete kanonickou rovnici kuželosečky

$$3(x^1)^2 + 10\, x^1 x^2 + 3(x^2)^2 - 2x^1 - 14x^2 - 13 = 0$$

7 Př. Určete kanonickou rovnici kuželosečky

$$7(x^1)^2 + 6x^1x^2 - (x^2)^2 + 28x^1 + 12x^2 + 28 = 0$$

8 Př. Určete kanonickou rovnici kvadriky

$$(x^1)^2 + (x^2)^2 + 5(x^3)^2 - 6x^1x^2 - 2x^1x^3 + 2x^2x^3 - 6x^1 + 6x^2 - 6x^3 + 9 = 0$$