- Multivariate functions = simple generalization of a functions with one real variable to more real variables
- Meaning of the words „more variables“ or „multivariate“ ?

Everything will be about the so-called mutlivariate functions

↓

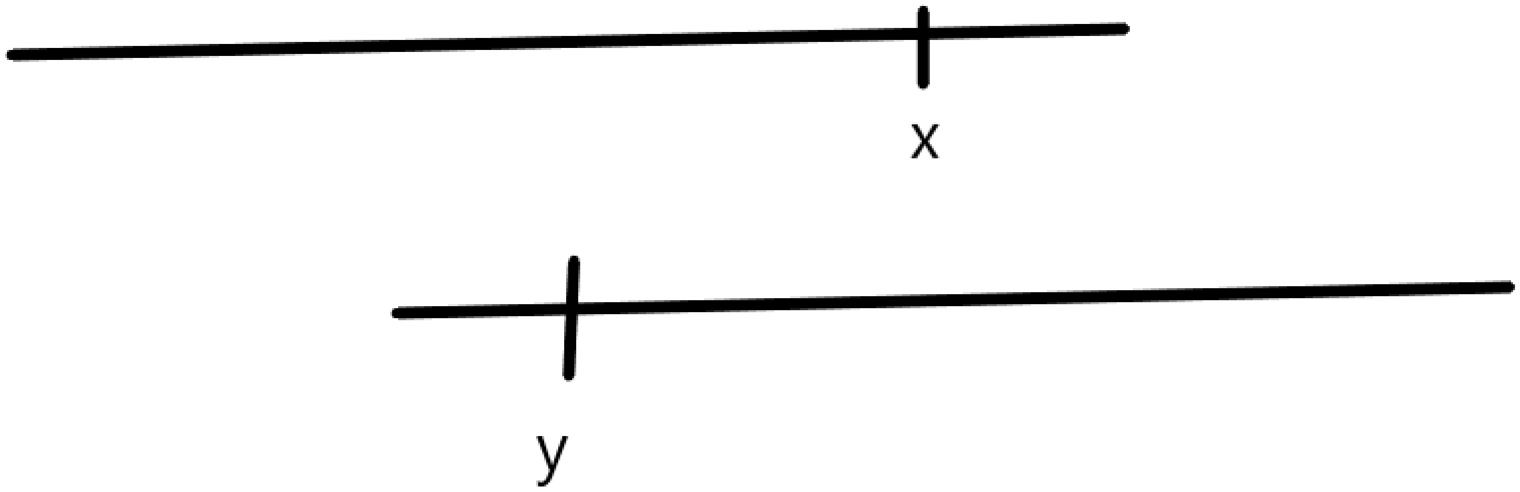
Does not answer what the word „multivariate“ actually means

- Example: take $f(x) = x^2$. Here we have 1 input: x and one output x^2
- Then a function of multiple variables have „more“ variables in its argument, as $f(x, y) = x^2 + y$. Here, the output corresponds to a number or it can also be vector

$$f(x, y) = \begin{pmatrix} 5x \\ 7y \end{pmatrix}$$

→ Interpretation or convention?

- Consider $f(x, y)$, where we can understand the inputs x and y as separate real numbers

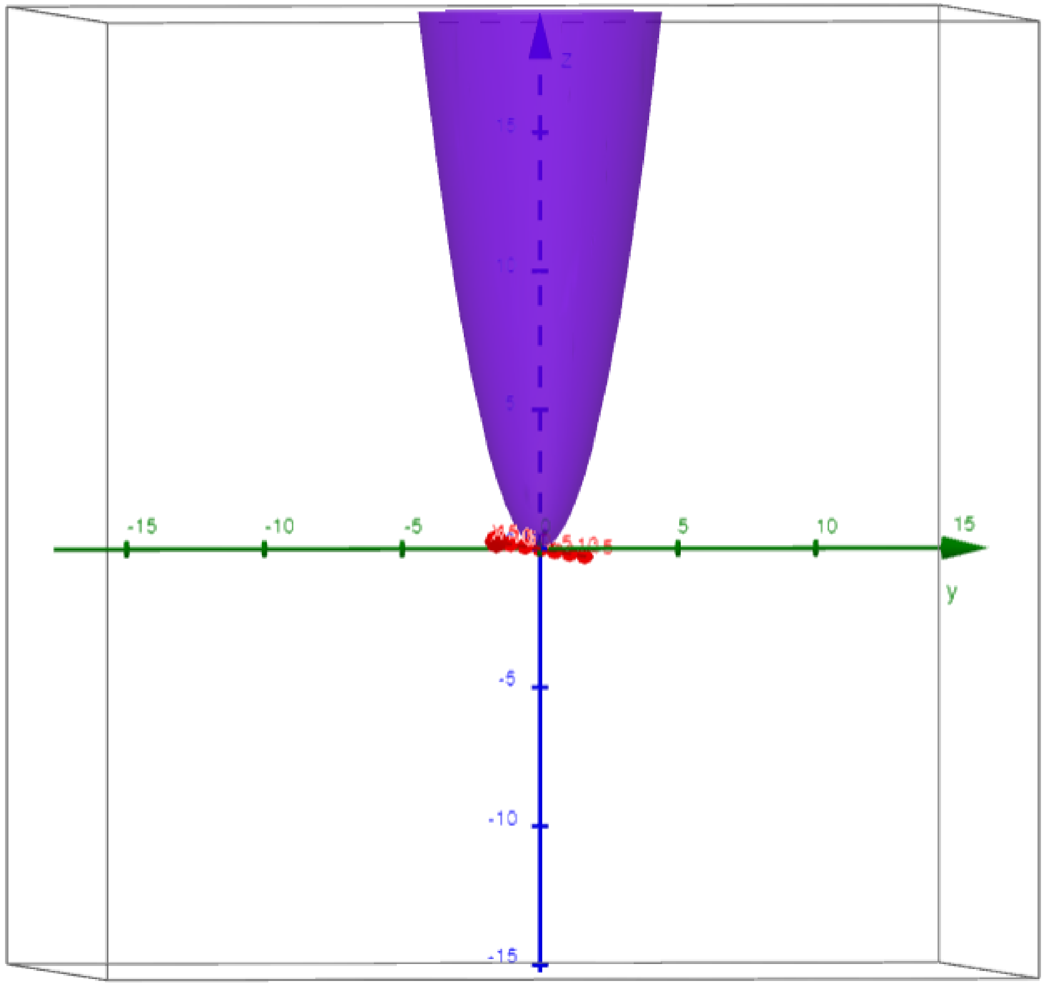


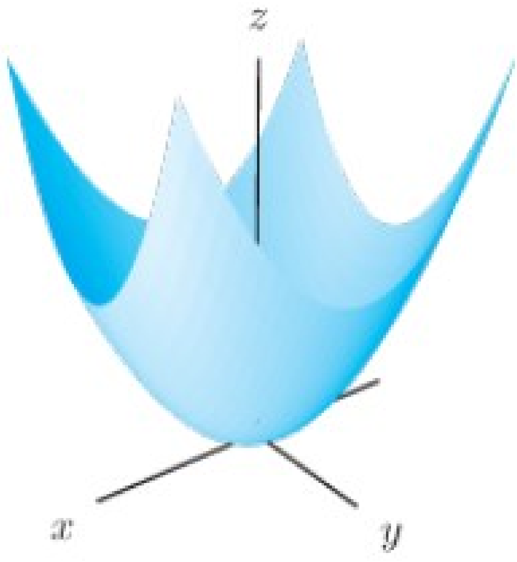
- Really not the case – consider the pair (x, y) as a point in space

Reformulate the name as the multi – dimensional calculus

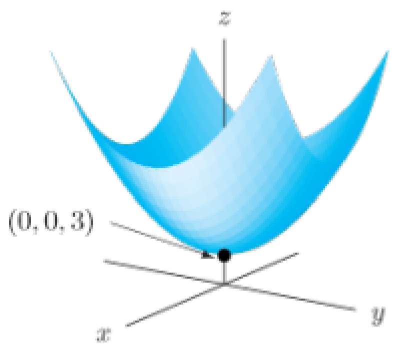
- Point → Number
- Point → Vector

How to visualize them?

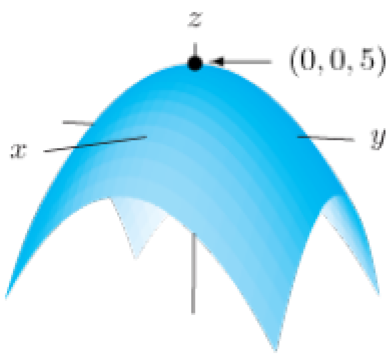




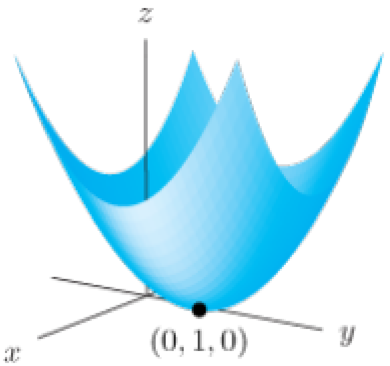
- Transformations: reflection, translation, rotation, composition…



Graph of $g(x, y) = x^2 + y^2 + 3$

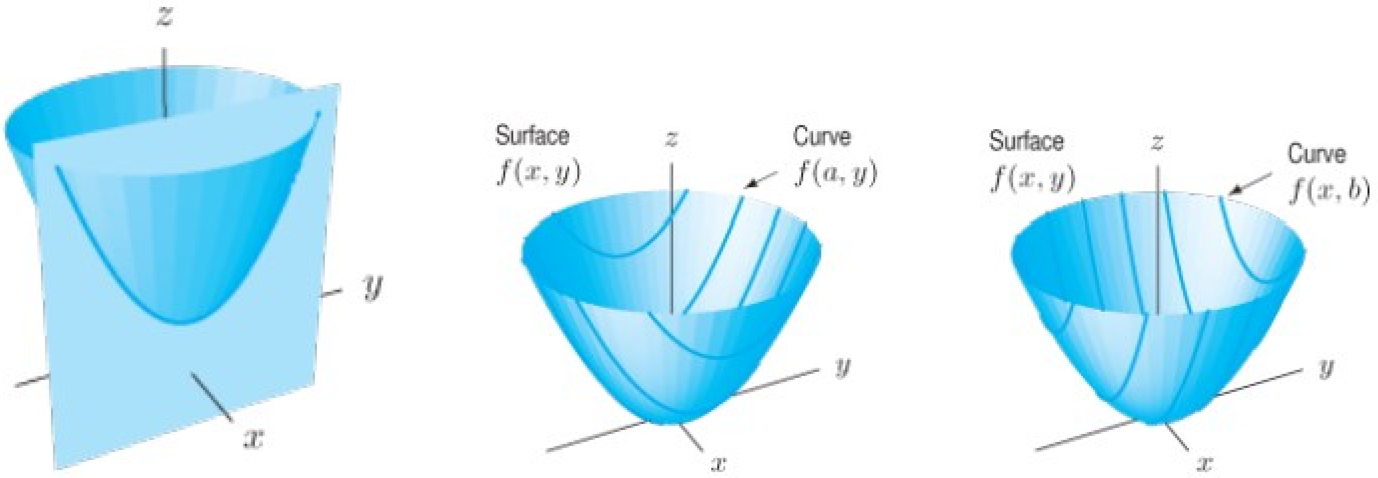


Graph of $h(x, y) = 5 - x^2 - y^2$

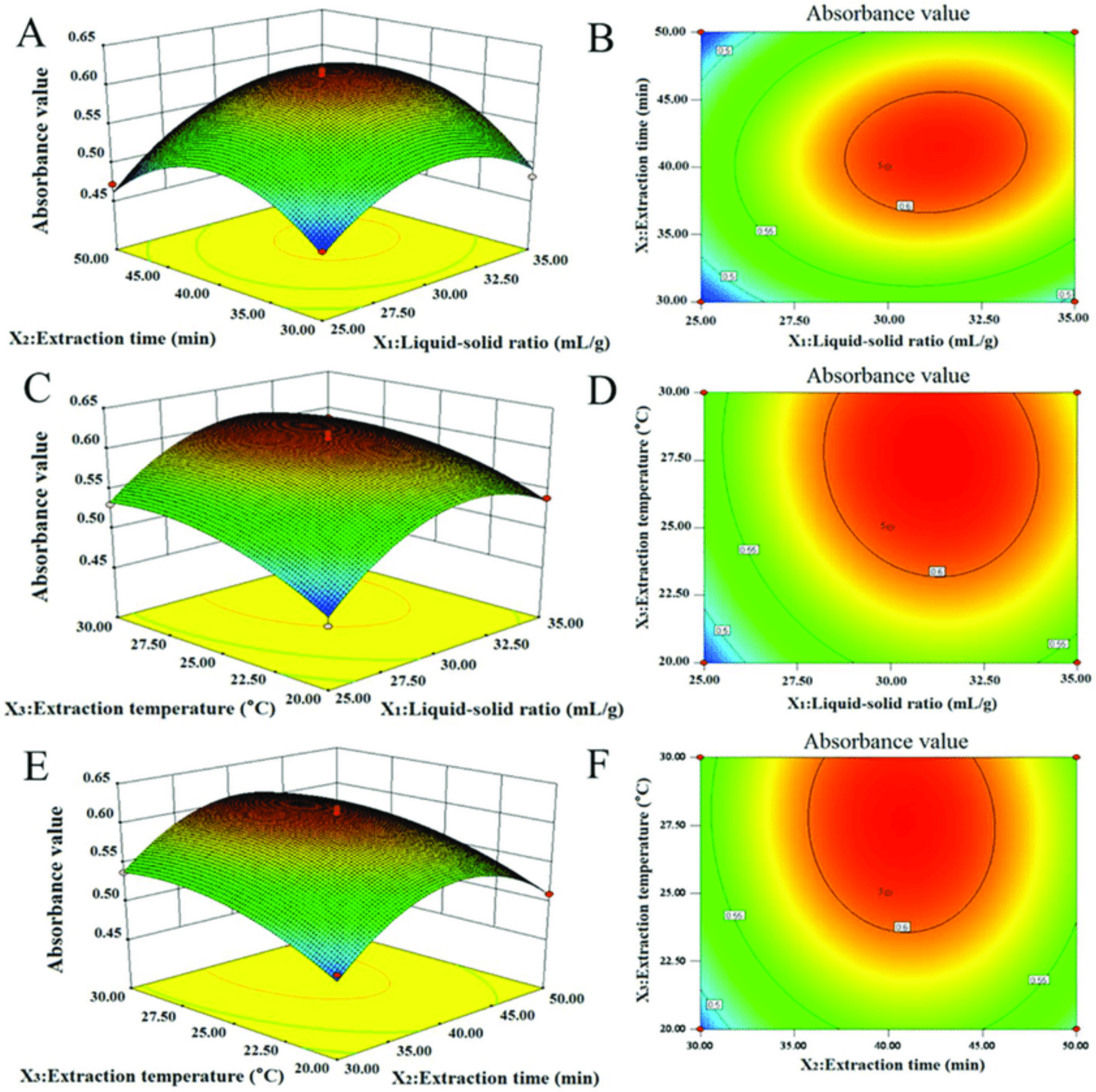


Graph of $k(x, y) = x^2 + (y - 1)^2$

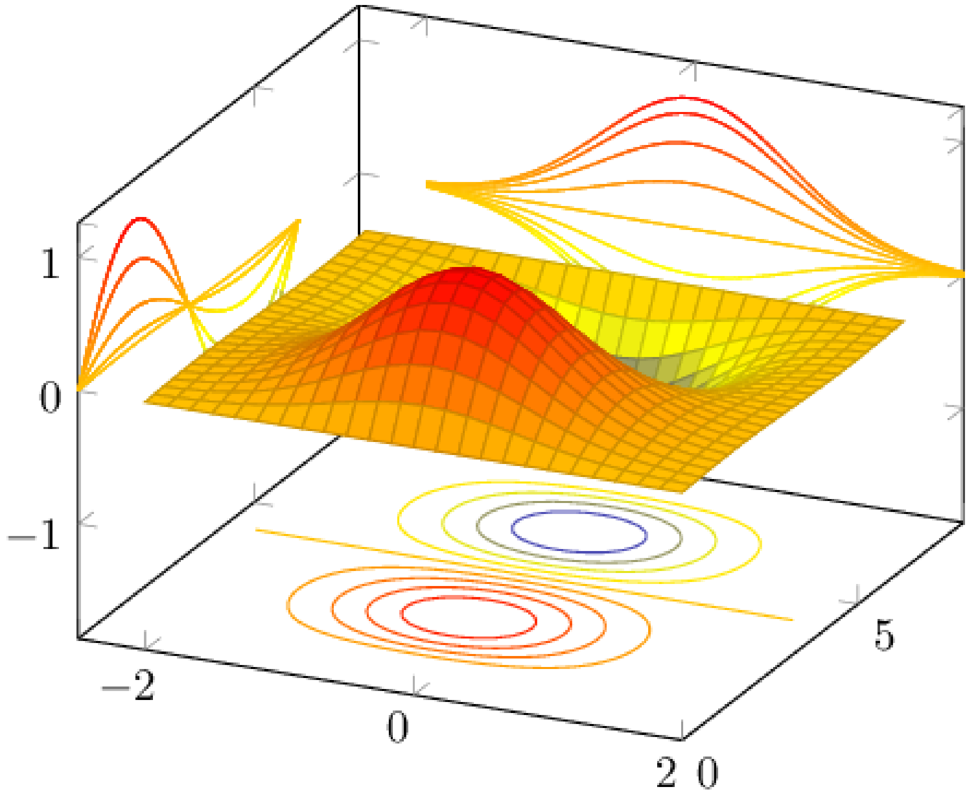
- Possible to visualize $3D$ graphs with their cross-sections in $2D$
- Space of all inputs → color for each point



- Contour lines, where the size of the output is proportional to the given color



A
Absorbance value
X₂:Extraction time (min)
X₁:Liquid-solid ratio (mL/g)
B
Absorbance value
X₂:Extraction time (min)
X₁:Liquid-solid ratio (mL/g)
C
Absorbance value
X₃:Extraction temperature (°C)
X₁:Liquid-solid ratio (mL/g)
D
Absorbance value
X₃:Extraction temperature (°C)
X₁:Liquid-solid ratio (mL/g)
E
Absorbance value
X₃:Extraction temperature (°C)
X₂:Extraction time (min)
F
Absorbance value
X₃:Extraction temperature (°C)
X₂:Extraction time (min)



1
0
−1
−2
0
2
0
5

- Plots in $3D$ can be also visualized as a mapping of some other $2D$ objects (surfaces)

- Mapping of $2D$ onto $3D$ output

Parametric surfaces