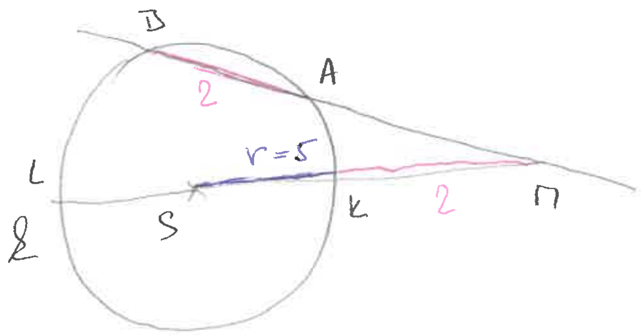
1)



$|MB| = |MA| + 2$

$m_M(k) = |MA| \cdot |MB| = |MK| \cdot |ML|$

$|MA| \cdot (|MA| + 2) = 2 \cdot 12$

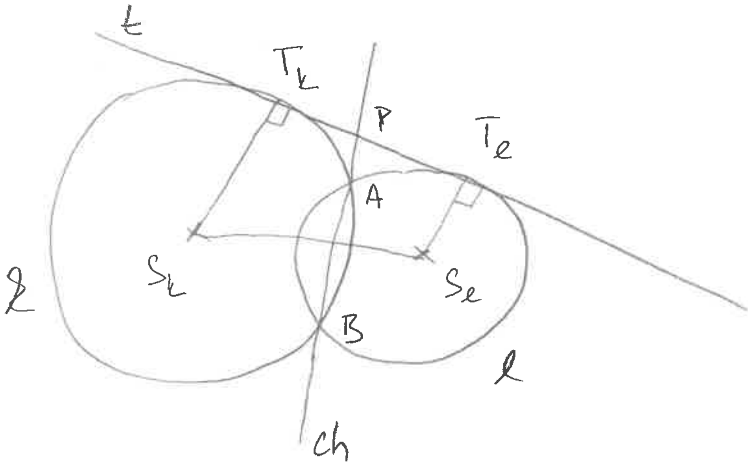
$|MA|^2 + 2|MA| - 24 = 0$

$(|MA| + 6)(|MA| - 4) = 0$

$|MA| = -6$ nelze

$\boxed{|MA| = 4 \text{ cm} \Rightarrow |MB| = 6 \text{ cm}}$

2)



Označme $P \in t \cap \overleftrightarrow{AB}$.

Chceme dokázat, že $|PT_k| = |PT_l|$

$m_P(k) = |PA| \cdot |PB| = |PT_k|^2$

$m_P(l) = |PA| \cdot |PB| = |PT_l|^2$

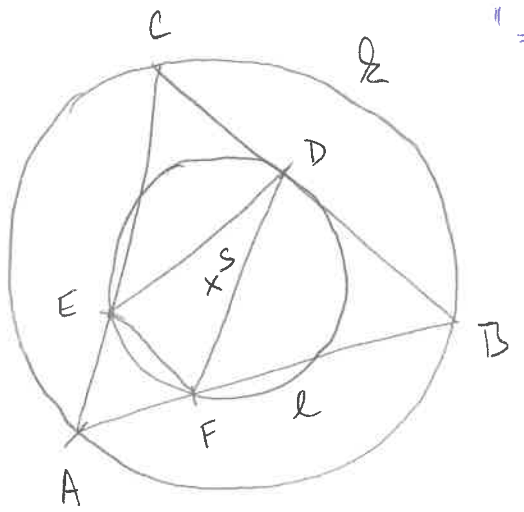
$\Rightarrow |PT_k|^2 = |PT_l|^2 \Leftrightarrow \boxed{|PT_k| = |PT_l|}$

3)

$k(S;r)$...

kruž. opsané

ΔABC



"$\Rightarrow$" Nechť $|DB| \cdot |DC| = |EC| \cdot |EA| = |FA| \cdot |FB| = m$

$m_D(k) = -|BD| \cdot |DC| = -(|DS| + r) \cdot (r - |DS|) \Rightarrow \boxed{|BD| \cdot |DC| = r^2 - |DS|^2}$ (1)

$\Rightarrow |DS| = \sqrt{r^2 - m}$

$m_E(k) = -|EC| \cdot |EA| = -(|ES| + r) \cdot (r - |ES|) \Rightarrow \boxed{|EC| \cdot |EA| = r^2 - |ES|^2}$ (2)

$\Rightarrow |ES| = \sqrt{r^2 - m}$

$m_F(k) = -|FA| \cdot |FB| = -(|FS| + r) \cdot (r - |FS|)$

$\Rightarrow \boxed{|FA| \cdot |FB| = r^2 - |FS|^2}$ (3) $\Rightarrow |FS| = \sqrt{r^2 - m}$

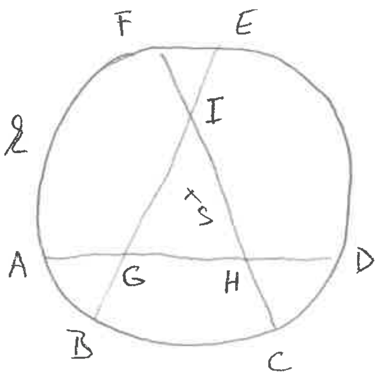
$\Rightarrow |DS| = |ES| = |FS| \Rightarrow$ S je střed kružnice opsané také ΔDEF.

"$\Leftarrow$" Nechť $|DS| = |ES| = |FS| = s \Rightarrow$

$$\left.\begin{aligned} (1):&\ |BD|\cdot|DC| = r^2 - s^2 \\ (2):&\ |EC|\cdot|EA| = r^2 - s^2 \\ (3):&\ |FA|\cdot|FB| = r^2 - s^2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow |BD|\cdot|DC| = |EC|\cdot|EA| = |FA|\cdot|FB|$$

4)



$m_G(k) = -|AG|\cdot|DG| = -|BG|\cdot|GE| =$

$\Rightarrow |AG|\cdot 2\cdot|AG| = |BG|\cdot 2|BG| \Rightarrow |AG| = |BG|$

$\Downarrow$

$m_H(k) = -|CH|\cdot|HF| = -|DH|\cdot|AH| \Rightarrow$ $|AD| = |BE|$

$\Rightarrow |CH|\cdot 2|CH| = |DH|\cdot 2|DH| \Rightarrow |CH| = |DH|$

$\Rightarrow |AD| = |CF|$

Nechť $|AG| = |GH| = |HD| \Rightarrow |DG| = 2|AG|$, $\Rightarrow |AD| = 3|AG|$, $|AH| = 2|DH|$, $|AD| = 3|DH|$

$|BG| = |GI| = |IE| \Rightarrow |EG| = 2|BG|$, $|BE| = 3|BG|$

$|CH| = |HI| = |IF| \Rightarrow |FH| = 2|CH|$, $\cancel{|CH|}$ $|CF| = 3|CH|$

$$\Rightarrow |AD| = |BE| = |CF| \;/\!/$$

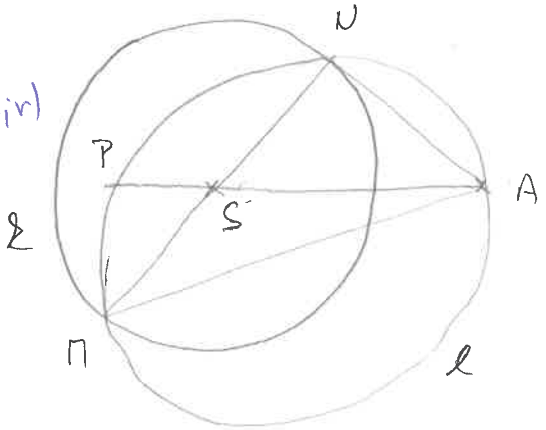
5)

$k: k(S; r)$

A

$|AS| = d$

$d > r$



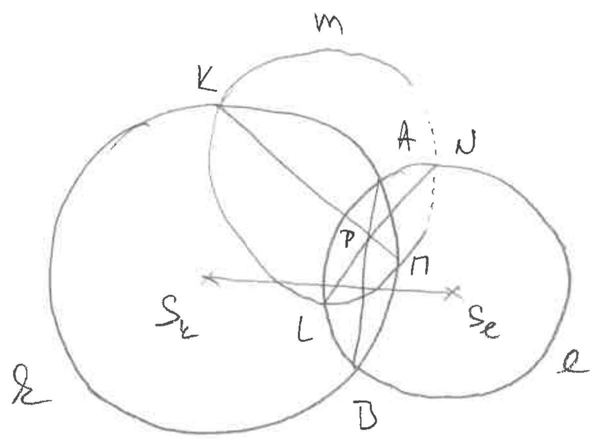
ozn. $P \in l \cap \overleftrightarrow{AS}$, $P \neq A$

$m_S(l) = -\underbrace{|MS|}_{= r}\cdot\underbrace{|NS|}_{= r} = -\underbrace{|AS|}_{= d}\cdot|PS|$

$\Rightarrow \boxed{|PS| = \frac{r^2}{d} = \text{konst.}} \Rightarrow$

poloha P se nezmění při změně průměru MN

6)



Ozn. m kružnici ops. $\triangle KLM$

$$\left.\begin{aligned} m_P(k) &= -|AP|\cdot|BP| = -|KP|\cdot|PM| \\ m_P(\ell) &= -|AP|\cdot|BP| = -|LP|\cdot|PN| \end{aligned}\right\} \Rightarrow |KP|\cdot|PM| = |LP|\cdot|PN|$$

$\Rightarrow N \in m$

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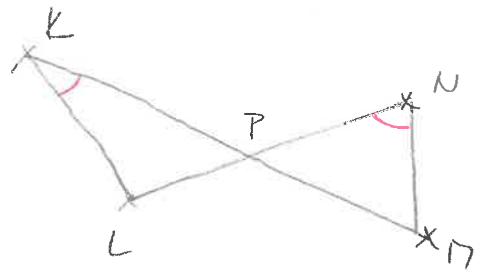
dk. $\Rightarrow \dfrac{|KP|}{|PN|} = \dfrac{|LP|}{|PM|}$



+ vrcholové úhly:
$|\sphericalangle KPL| = |\sphericalangle NPM|$

$\triangle KPL \sim \triangle NPM$ (sus)

$\Rightarrow |\sphericalangle LKP| = |\sphericalangle MNP|$
$|\sphericalangle LKM| \qquad |\sphericalangle MNL|$

$\Rightarrow$ z K i N (ležících v téže polorovině s hran. přímkou $\overleftrightarrow{LM}$ vidíme LM pod stejnými (obvodovými) úhly $\Rightarrow$

body K, L, M, N leží na téže kružnici m