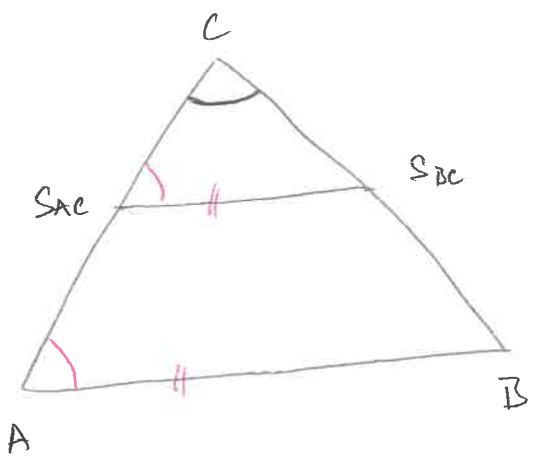
1)



Ozn. S_{AC}, S_{BC} středy po řadě AC, BC.

Pak $\triangle ABC \sim \triangle S_{AC}S_{BC}C$ (sus),

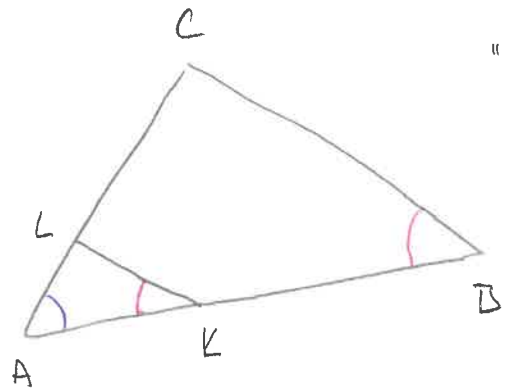
neboť $\dfrac{|AC|}{|S_{AC}C|} = \dfrac{|BC|}{|S_{BC}C|} = \dfrac{2}{1}$

$\Rightarrow \dfrac{|AB|}{|S_{AC}S_{BC}|} = \dfrac{2}{1}$, $|\sphericalangle CAB| = |\sphericalangle CS_{AC}S_{BC}|$

$\Rightarrow$ $AB \parallel S_{AC}S_{BC}$

↑ souhlasné úhly

2)



"$\Rightarrow$" $|AK|:|KB| = |AL|:|LC| \Longrightarrow |AK|:|AB| = |AL|:|AC|$

$\Rightarrow \triangle AKL \sim \triangle ABC$ (sus) $\Rightarrow$

$\Rightarrow |\sphericalangle AKL| = |\sphericalangle ABC| \Rightarrow KL \parallel BC$

↑ souhlasné $\sphericalangle$

"$\Leftarrow$" $KL \parallel BC \Rightarrow \triangle AKL \sim \triangle ABC$ (uu) $\Rightarrow |AK|:|AB| = |AL|:|AC|$

↑ rovnost souhl. úhlů: $|\sphericalangle AKL| = |\sphericalangle ABC|$

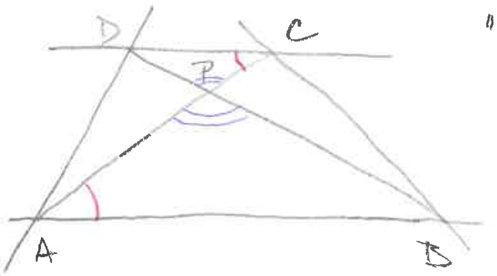
$\Downarrow$

$|AK|:|KB| = |AL|:|LC|$

$$\boxed{\frac{|AK|}{|AB|} = \frac{|AL|}{|AC|} \Leftrightarrow \frac{|AK|}{|AK|+|KB|} = \frac{|AL|}{|AL|+|LC|} \Leftrightarrow \ \cdots \ \frac{|AK|}{|KB|} = \frac{|AL|}{|LC|}}$$

$$\Leftrightarrow \frac{|AK|+|KB|}{|AK|} = \frac{|AL|+|LC|}{|AL|} \Leftrightarrow 1 + \frac{|KB|}{|AK|} = 1 + \frac{|LC|}{|AL|} \Leftrightarrow \frac{|KB|}{|AK|} = \frac{|LC|}{|AL|}$$

3)



Vrcholové úhly: $|\sphericalangle APB| = |\sphericalangle CPD|$

"$\Rightarrow$" $AB \parallel CD \Rightarrow$ střídavé úhly: $|\sphericalangle BAC| = |\sphericalangle DCA|$

$\quad\quad |\sphericalangle BAP| = |\sphericalangle DCP|$

$\Rightarrow \triangle ABP \sim \triangle CDP$ (uu) $\Rightarrow$

$\dfrac{|AB|}{|CD|} = \dfrac{|BP|}{|DP|} = \dfrac{|AP|}{|CP|} \Longrightarrow |BP|\cdot|CP| = |AP|\cdot|DP|$

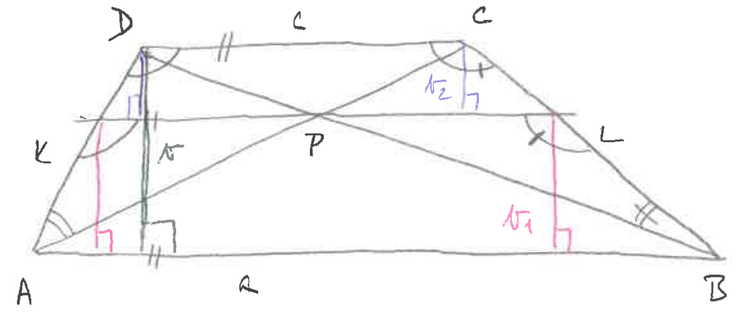
"$\Leftarrow$" $|AP|\cdot|DP| = |BP|\cdot|CP| \Rightarrow \dfrac{|AP|}{|CP|} = \dfrac{|BP|}{|DP|} \Rightarrow$

$\Rightarrow \triangle ABP \sim \triangle CDP$ (sus) $\Rightarrow$

$\Rightarrow |\sphericalangle BAP| = |\sphericalangle DCP|$, tj. $|\sphericalangle BAC| = |\sphericalangle DCA| \Rightarrow AB \parallel CD$

↑ rovnost střídavých úhlů

4)



Nechť $K \in AD$, $L \in BC$ jsou takové body, že $P \in KL$ a $KL \parallel AB$.

- Dk, že $|KP| = |PL|$
- Vyjádřete $|KL|$ pomocí $a = |AB|$, $c = |CD|$

$$KL \parallel AB \Rightarrow |K\overleftrightarrow{AB}| = |L\overleftrightarrow{AB}| \overset{\text{ozn.}}{=} \underline{v_1}$$

$$KL \parallel DC \Rightarrow |D\overleftrightarrow{KL}| = |C\overleftrightarrow{KL}| \overset{\text{ozn.}}{=} \underline{v_2}$$

$$AB \parallel CD \Rightarrow |D\overleftrightarrow{AB}| = |C\overleftrightarrow{AB}| \overset{\text{ozn.}}{=} \underline{v}$$

$$\Rightarrow v = v_1 + v_2$$

$\triangle AKP \sim \triangle ADC$ (uu)

$$\frac{|KP|}{|DC|} = \frac{v_1}{v}$$

$\triangle BLP \sim \triangle BCD$ (uu)

$$\frac{|LP|}{|CD|} = \frac{v_1}{v}$$

$$\Rightarrow \underline{\underline{|KP| = |LP|}} \;//$$

ozn. $x = |KL| \Rightarrow |KP| = |LP| = \frac{x}{2}$;

$$\frac{\frac{x}{2}}{c} = \frac{v_1}{v} = \frac{x}{2c} \quad (1)$$

$\triangle DKP \sim \triangle DAB$ (uu)

$$\frac{|KP|}{|AB|} = \frac{v_2}{v} = \frac{\frac{x}{2}}{a} = \frac{x}{2a} \quad (2)$$

(1) + (2) :

$$\frac{v_1}{v} + \frac{v_2}{v} = \frac{x}{2c} + \frac{x}{2a}$$

$$\frac{v_1 + v_2}{v} = x\left(\frac{1}{2c} + \frac{1}{2a}\right)$$

$$\frac{v}{v} = x \cdot \frac{a+c}{2ac}$$

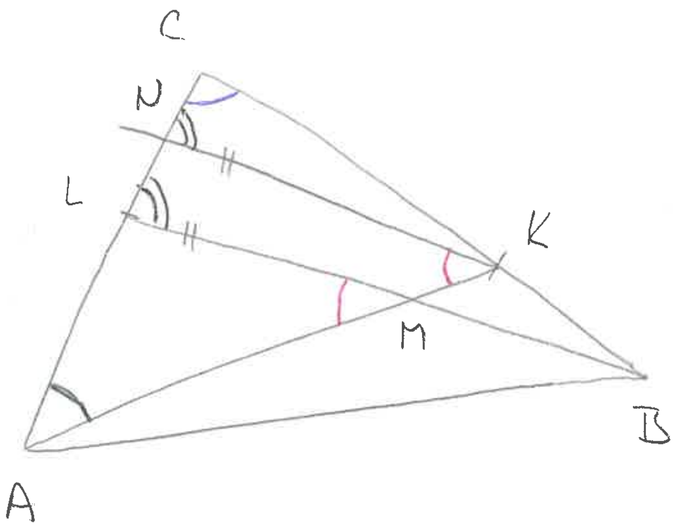
$$1 = x \cdot \frac{a+c}{2ac}$$

$$\boxed{x = \frac{2ac}{a+c}}$$

… jde tedy o harmonický průměr délek a, c

5)

$p, q \in \mathbb{R}^+$



$|BK| : |KC| = 1 : p$ (1)

$|AL| : |LC| = 1 : q$ (2)

Ozn. $N \in CL$ tak, aby $KN \parallel BL$

$|AM| : |MK| = \ldots$ určit

$\Delta AML \sim \Delta AKN$ (uu)

$$\frac{|AM|}{|AK|} = \frac{|AL|}{|AN|} \Leftrightarrow \frac{|AM|+|MK|}{|AM|} = \frac{|AL|+|LN|}{|AL|} \Leftrightarrow 1+\frac{|MK|}{|AM|} = 1+\frac{|LN|}{|AL|} \Leftrightarrow$$

$$\Leftrightarrow \frac{|AM|}{|MK|} = \frac{|AL|}{|LN|} \quad (*)$$

(přímo lze z $ML \parallel KN$ tvrdit dle výsledku př 2)

$\Delta CNK \sim \Delta CLB$ (uu)

$$\frac{|CN|}{|CL|} = \frac{|CK|}{|CB|} \Leftrightarrow \frac{|CN|+|NL|}{|CN|} = \frac{|CK|+|KB|}{|CK|} \Leftrightarrow 1+\frac{|NL|}{|CN|} = 1+\frac{|KB|}{|CK|} \Leftrightarrow$$

$$\Leftrightarrow \frac{|CN|}{|NL|} = \frac{|CK|}{|KB|} \overset{(1)}{=} p \Rightarrow |CN| = p\,|NL| \quad (3)$$

přímo z $NK \parallel LB$ dle př. 2

$$\frac{|LC|}{|AL|} = \frac{|LN|+|NC|}{|AL|} \overset{(3)}{=} \frac{|LN|+p|NL|}{|AL|} = \frac{|NL|(1+p)}{|AL|} \Rightarrow |LC| = |NL|(1+p) \quad (4)$$

(2): $\frac{|LC|}{|AL|} = q \Rightarrow |LC| = |AL|\cdot q$ (5)

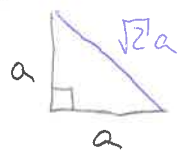
(4)+(5) $\Rightarrow$ $|NL|(1+p) = |AL|\cdot q$

$$\boxed{\frac{1+p}{q} = \frac{|AL|}{|NL|} \overset{(*)}{=} \frac{|AM|}{|MK|}}$$

$\Rightarrow$

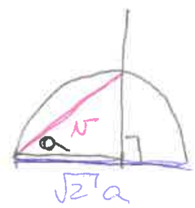
$\boxed{7e}$

$v = a\sqrt[4]{2} = \sqrt{(a\cdot\sqrt{2})\cdot a}$

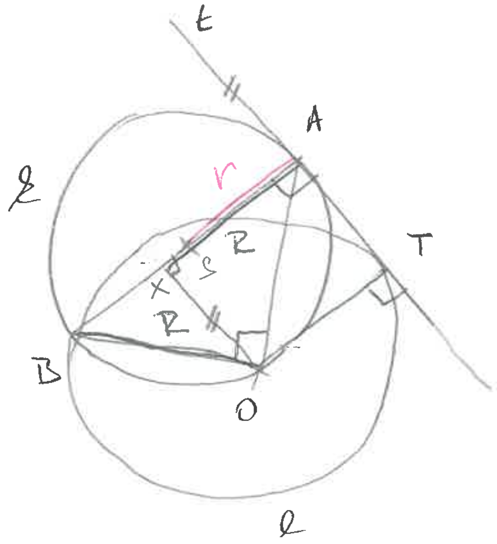


P.U.

E.V.O.



6)



- $OTAX$ je obdélník o stranách $|OT| = |AX| = R$, $|XO| = |AT|$
- $\Delta\, BOA$ je pravoúhlý (z O vidíme AB pod pravým úhlem – Thaletova věta)

X značí kolmý průmět O na AB

E.U.O.: $R^2 = 2r \cdot (2r - R)$

$$R^2 + 2rR - 4r^2 = 0$$

$$(R+r)^2 = 5r^2$$

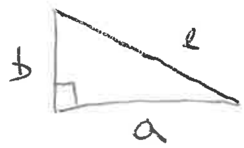
$$R = \sqrt{5}\,r - r$$

$$R = (\sqrt{5} - 1)\,r$$

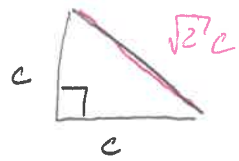
7) Dány délky a, b, c, d

a) sestr. $x = \sqrt{a^2 + b^2 + 2c^2}$

ozn. $e^2 = a^2 + b^2$

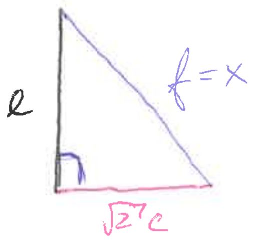


Pyth. věta (konstrukce přepony)

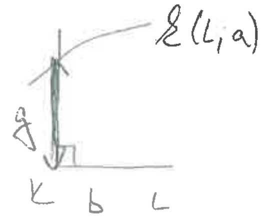


ozn. $f^2 = e^2 + (\sqrt{2}\,c)^2 \Rightarrow f = \sqrt{e^2 + 2c^2} = \sqrt{a^2 + b^2 + 2c^2}$

tedy $f = x$



b) sestr. $y = \dfrac{a\sqrt{a^2 - b^2}}{c}$ $(a > b)$

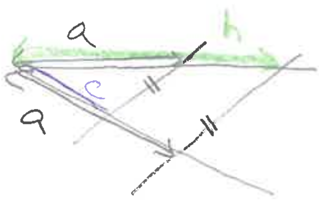


ozn $g^2 = a^2 - b^2 \Leftrightarrow g = \sqrt{a^2 - b^2}$

$$\frac{y}{g} = \frac{a}{c}$$

c) $z = \dfrac{a^2 b}{cd}$

ozn. $h = \dfrac{a^2}{c} \Leftrightarrow \dfrac{h}{a} = \dfrac{a}{c}$



$z = \dfrac{hb}{d} \Leftrightarrow \dfrac{z}{h} = \dfrac{b}{d}$

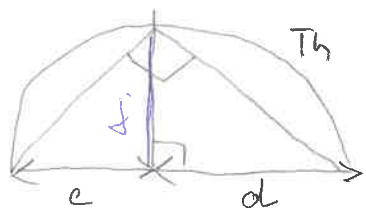
d) $w = \sqrt{ab + cd} = \sqrt{i^2 + j^2}$ (P.V.)

ozn. $i^2 = ab$

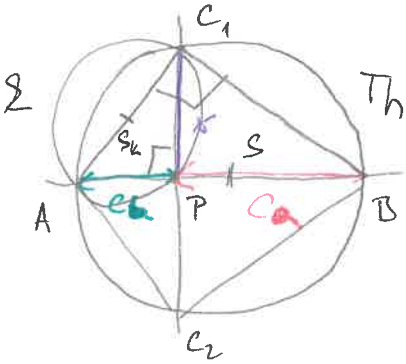
(E.V.) $j^2 = cd$

o odvěsně

o výšce



8)



$\triangle ABC_1 \cong \triangle ABC_2$ (sss) (jsou osově souměrné podle $\overleftrightarrow{AB}$)

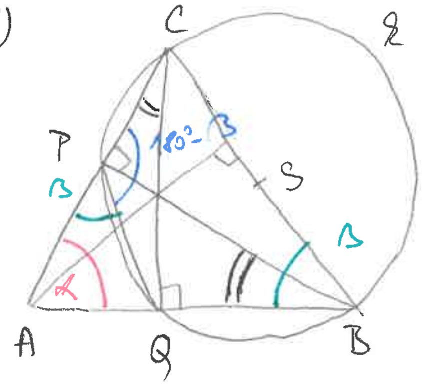
$\Rightarrow$ P je střed C_1C_2

$m_P(Th) = -|C_1P|\cdot|C_2P| = -|AP|\cdot|BP|$

$\boxed{v^2 = c_b \cdot c_a}$ $\boxed{a^2 = c\cdot c_a}$

ℓ... kružnice s průměrem AC_1 – prochází bodem P, přímka BC_1 je její tečnou $\Rightarrow$ $m_B(\ell) = |BC_1|^2 = |BA|\cdot|BP|$

9)



($90° - \alpha = |\sphericalangle ABP| = |\sphericalangle ACQ|$ → nevyužito)

$\boxed{\ell\text{... Thaletova kružnice nad } BC \text{ – prochází body } P, Q}$

$\Rightarrow$ QBCP je tětivový čtyřúhelník $\Rightarrow$

$180° = \underbrace{|\sphericalangle QBC|}_{|\sphericalangle ABC| = \beta} + |\sphericalangle QPC| \Rightarrow |\sphericalangle QPC| = 180° - \beta$

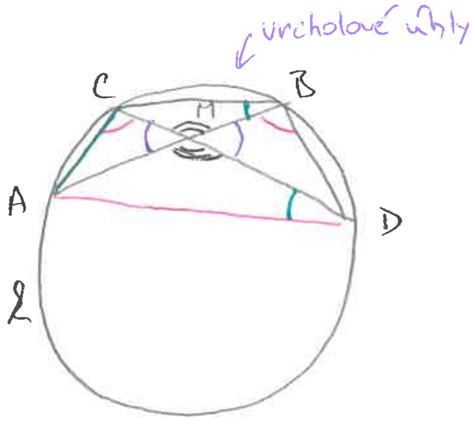
$\Downarrow$ vedlejší $\sphericalangle$

$|\sphericalangle APQ| = \beta$

$\Rightarrow \triangle ABC \sim \triangle APQ$ (uu)

$m_A(\ell) = |AQ|\cdot|AB| = |AP|\cdot|AC| \Rightarrow \frac{|AB|}{|AP|} = \frac{|AC|}{|AQ|} \Rightarrow \triangle ABC \sim \triangle APQ$ (sus)

10)



$m_M(\ell) = -|AM|\cdot|BM| = -|CM|\cdot|DM|$

$\frac{|AM|}{|DM|} = \frac{|CM|}{|BM|} \Rightarrow \triangle AMC \sim \triangle DMB$ (sus)

(uu)

nebo: vrcholové, obvodové úhly

$|\sphericalangle ACM| = |\sphericalangle ACD| = |\sphericalangle ABD| = |\sphericalangle DBM|$

$\Rightarrow \boxed{\frac{|AC|}{|DB|} = \frac{|AM|}{|DM|} \quad (1)}$

podobně: $|\sphericalangle AMD| = |\sphericalangle BMC|$ vrcholové

$|\sphericalangle ADM| = |\sphericalangle ADC| = |\sphericalangle ABC| = |\sphericalangle CBM|$ $\Rightarrow$

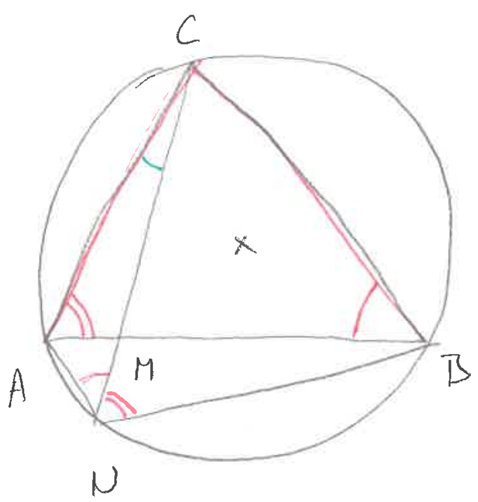
↑ obvodové k $\overset{\frown}{AC}$

$\Rightarrow \triangle ADM \sim \triangle CBM$ (uu) $\Rightarrow \boxed{\frac{|AD|}{|CB|} = \frac{|DM|}{|BM|} \quad (2)}$

$(1)\cdot(2)$: $\frac{|AC|}{|DB|}\cdot\frac{|AD|}{|CB|} = \frac{|AM|}{\cancel{|DM|}}\cdot\frac{\cancel{|DM|}}{|BM|} = \frac{|AM|}{|BM|}$

$\Leftrightarrow \boxed{\frac{|AC|\cdot|AD|}{|AM|} = \frac{|BC|\cdot|BD|}{|BM|}}$

11)



$|\sphericalangle ACM| = |\sphericalangle NCA|$

Dk. $|CM| \cdot |CN| = |AC|^2 \Leftrightarrow |AC| = |BC|$

$$\frac{|CM|}{|CA|} = \frac{|CA|}{|CN|}$$

"$\Rightarrow$"

$\triangle CMA \sim \triangle CAN$ (sus)

$\Rightarrow |\sphericalangle CAM| = |\sphericalangle CNA| = |\sphericalangle CBA|$

$\quad\; \| $

$|\sphericalangle CAB|$

obvodové k $\widehat{AC}$

$\Rightarrow \alpha = \beta \Rightarrow |AC| = |BC|$

"$\Leftarrow$" $|AC| = |BC| \Rightarrow |\sphericalangle CAB| = |\sphericalangle CBA| = |\sphericalangle CNA|$

$\quad\; \|$

$|\sphericalangle CAM|$

obvodové k $\widehat{AC}$

$\Rightarrow \triangle CMA \sim \triangle CAN$ (uu)

$$\Rightarrow \frac{|CM|}{|CA|} = \frac{|CA|}{|CN|} \Rightarrow |CM| \cdot |CN| = |AC|^2$$